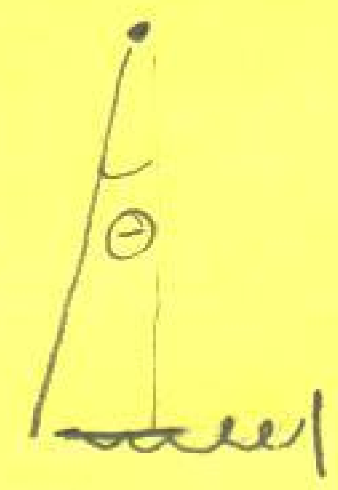
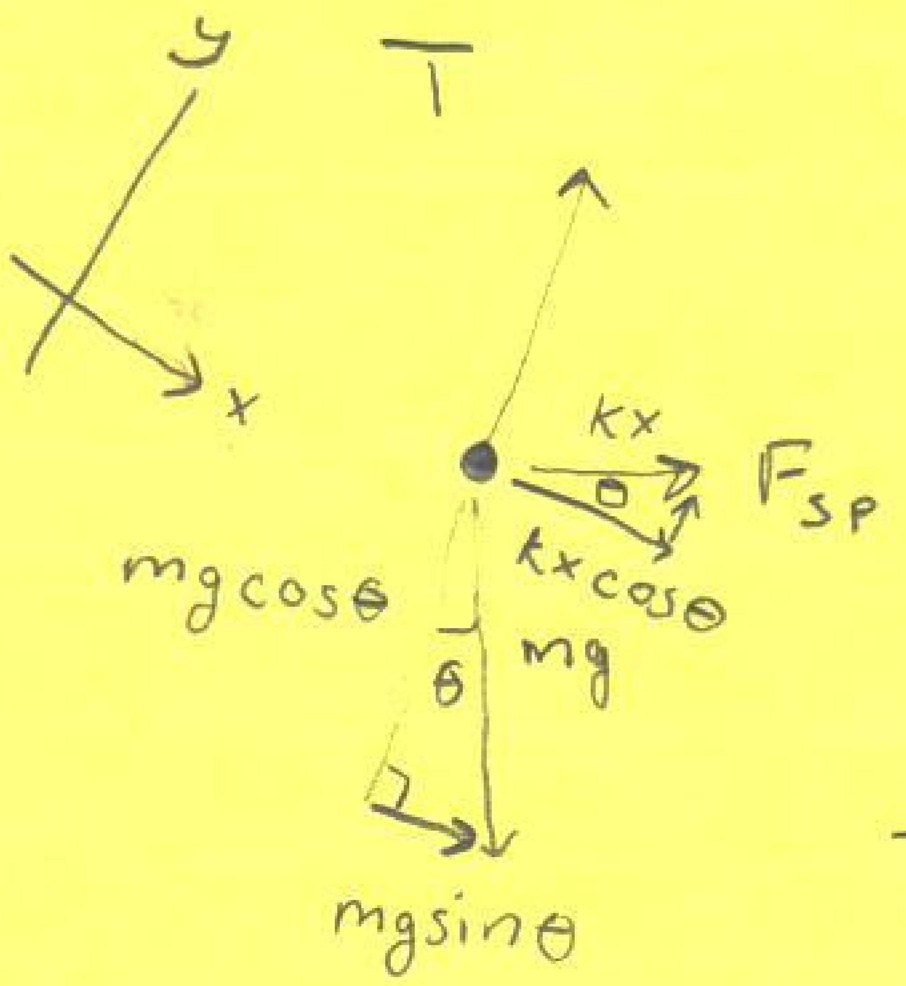
Additional Problems

P.59



draw a free body diagram



$$\sum F_y = 0$$

$$\sum F_x = m a_x$$

$$-mg\sin\theta - kx\cos\theta = m a_x$$

$$\sin\theta \approx \frac{x}{L} \approx \tan\theta \approx \theta$$

$$\cos\theta \approx 1$$

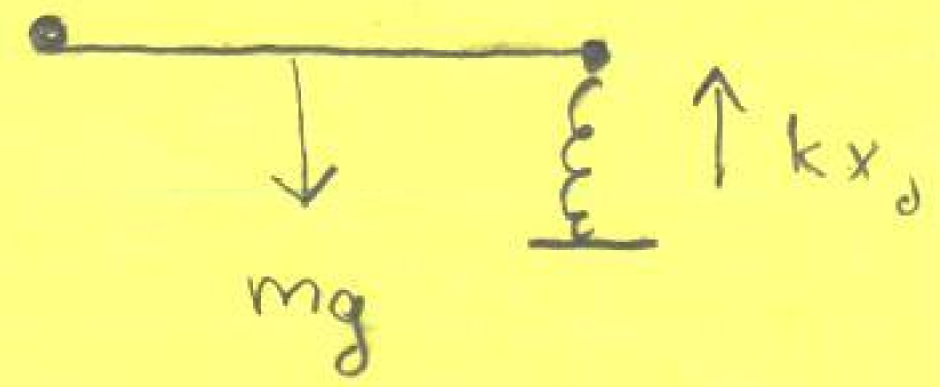
$$-mg\frac{x}{L} - kx = m\frac{d^2x}{dt^2}$$

$$-\left(\frac{g}{L} + \frac{k}{m}\right)x = \frac{d^2x}{dt^2} \qquad \text{compare } -\omega_0^2 x = \frac{d^2x}{dt^2}$$

$$\text{so } \omega_0^2 = \frac{g}{L} + \frac{k}{m} \Rightarrow \omega_0 = \sqrt{\frac{g}{L} + \frac{k}{m}}$$

Problem 61

Equilibrium

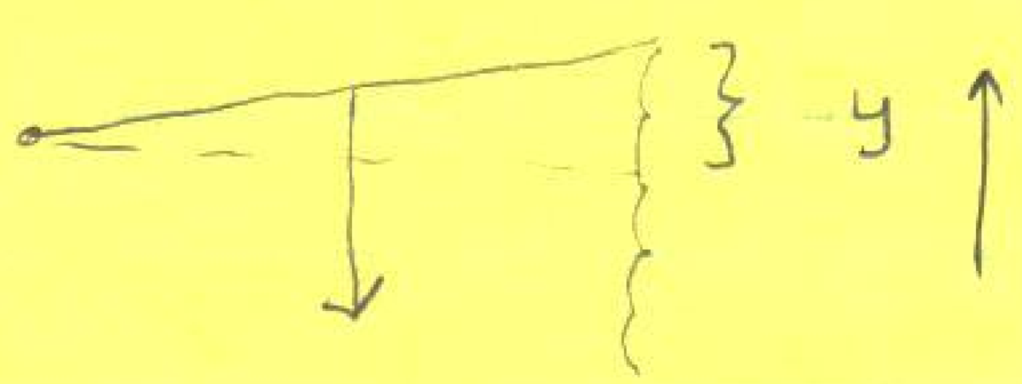


$$\sum \tau = I\alpha = 0$$

$$-mg\frac{L}{2} + kx_0 L = 0$$

$$+\frac{mg}{2} = x_0$$

Now displace from Equilibrium



$$F_{sp} = k(x_0 - y)$$

$$\sum \tau = I\alpha$$

$$-mg\cos\theta\,\frac{L}{2} + k(x_0 - y)L\cos\theta = I\frac{d^2\theta}{dt^2}$$

$$\cos\theta \approx 1$$

$$\underbrace{\cancel{-mg\frac{L}{2} + kx_0}}_{\text{see above}} - kyL = I\frac{d^2\theta}{dt^2}$$

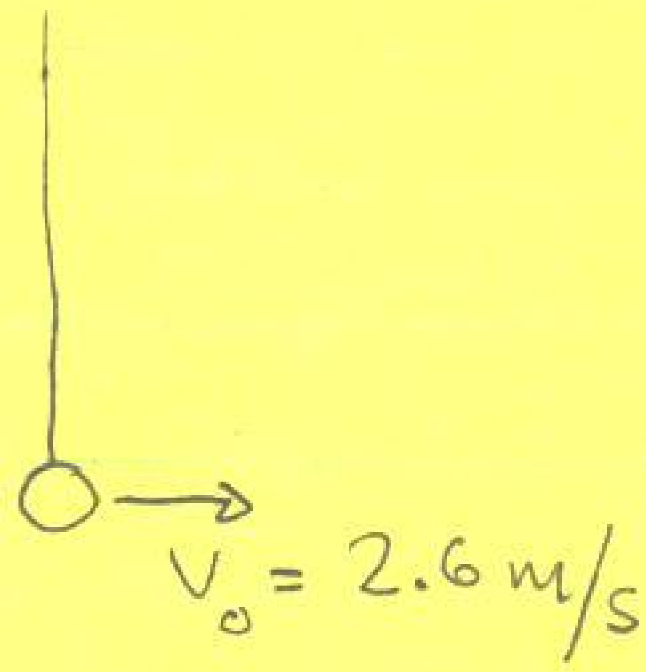
$\theta \approx \frac{y}{L}$, so

$$-k\theta L^2 = I\frac{d^2\theta}{dt^2}$$

$$-\frac{kL^2}{I}\theta = \frac{d^2\theta}{dt^2} \longrightarrow \text{compare } -\omega_0^2 x = \frac{d^2x}{dt^2}$$

$$\omega_0^2 = +\frac{kL^2}{I} \qquad \omega_0 = \sqrt{\frac{kL^2}{\frac{1}{3}ML^2}} = \sqrt{\frac{3k}{m}}$$

P63



$m = 6.74\,kg$

a) $\theta(t) = A\cos(\omega_0 t + \phi)$

$\omega_0 = \sqrt{\frac{g}{l}} \qquad T = \frac{2\pi}{\omega_0} = 2.997 \approx 3.0\,s$

b) $KE_i + \cancel{PE_i}^{0} = E_{TOT}$

$\frac{1}{2} m v_0^2 = E_{TOT} = 14.3\,J$

c) $\theta(t) = A\cos(\omega_0 t + \phi)$

$\dot{\theta}(t) = -A\omega_0 \sin(\omega_0 t + \phi)$

Initial Conditions

① $\theta(0) = 0 = A\cos\phi$

② $\dot{\theta}(0) = \frac{V}{L} = -A\omega_0 \sin\phi$

So From (1) $\phi = -\pi/2$, i.e.

$$\Theta(t) = A \sin(\omega_0 t) \qquad \dot{\Theta}(t) = A\omega_0 \cos(\omega_0 t$$

Then (2) becomes

$$\frac{V}{L} = -A\omega_0 \sin\left(-\frac{\pi}{2}\right) = +A\omega_0$$

$$A = \frac{V}{L\omega_0} = \frac{V}{L} \cdot \frac{1}{\sqrt{\frac{g}{L}}} = \frac{V}{\sqrt{gL}}$$

$$A = 0.44 \text{ radians} \approx 25.2^\circ$$

Remark, could have taken $\phi = +\frac{\pi}{2}$

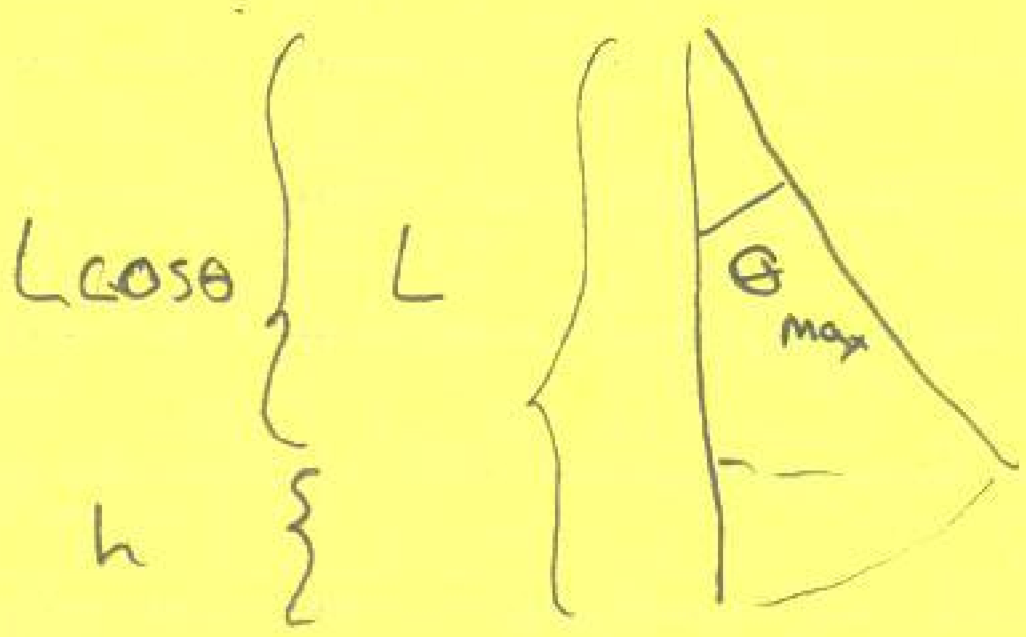
Then (2) becomes

$$\frac{V}{L} = -A\omega_0 \qquad A = -\frac{V}{\sqrt{gL}}$$

$$\Theta_{max} = \frac{V_0}{\sqrt{gL}} = 0.44 \text{ radian} \approx 25.2^\circ$$

$$\Theta(t) = -\frac{V_0}{\sqrt{gL}} \cos(\omega_0 t + \pi/2)$$

Alternate Solution



$$KE_i + PE_i = \cancelto{0}{KE_f} + PE_f$$

$$\frac{1}{2} m v_0^2 = mgh$$

$$\frac{1}{2} m v_0^2 = mg(L - L\cos\theta)$$

$$\frac{v_0^2}{2gL} = 1 - \cos\theta$$

$$\cos\theta = 0.9029$$

$$\theta = 0.44 \text{ radians}$$

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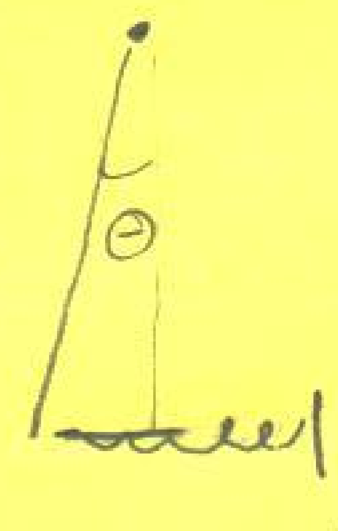
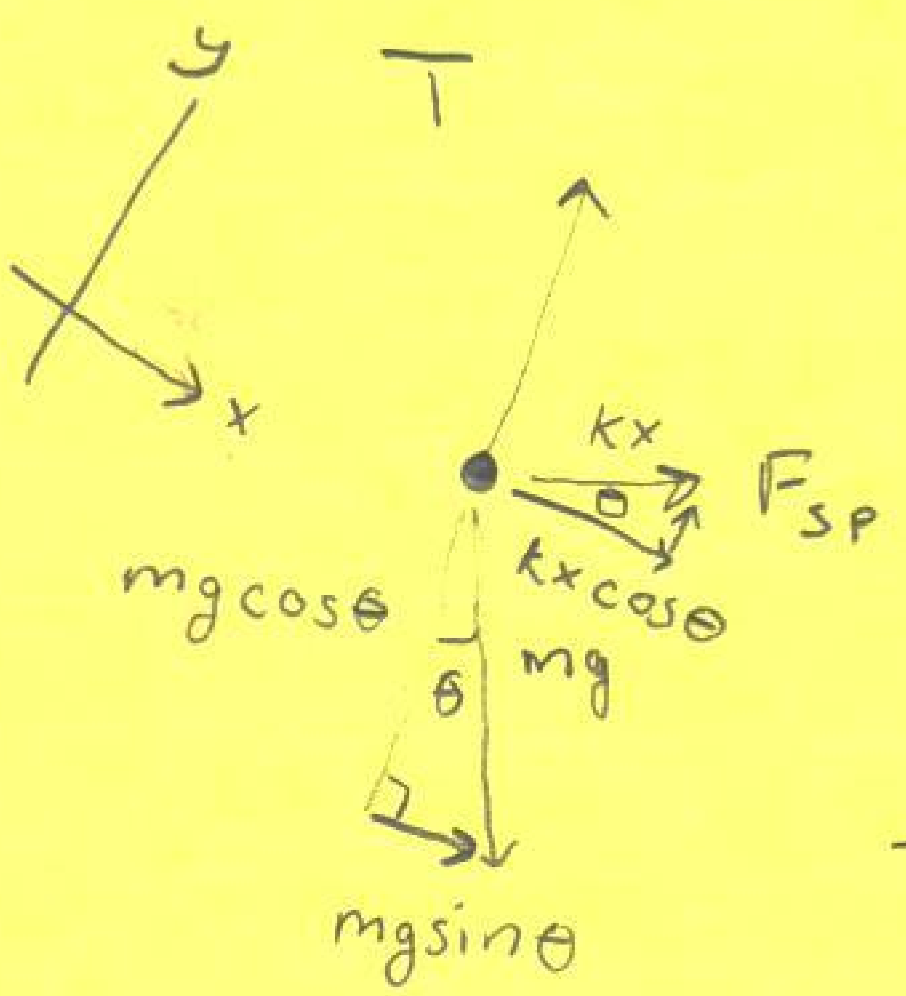


draw a free body diagram



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$$-mg\frac{x}{L} - kx = m\frac{d^2x}{dt^2}$$

$$-\left(\frac{g}{L} + \frac{k}{m}\right)x = \frac{d^2x}{dt^2}$$ compare $-\omega_0^2 x = \frac{d^2}{dt}$

so $\omega_0^2 = \frac{g}{L} + \frac{k}{m} \Rightarrow \omega_0 = \sqrt{\frac{g}{L} + \frac{k}{m}}$