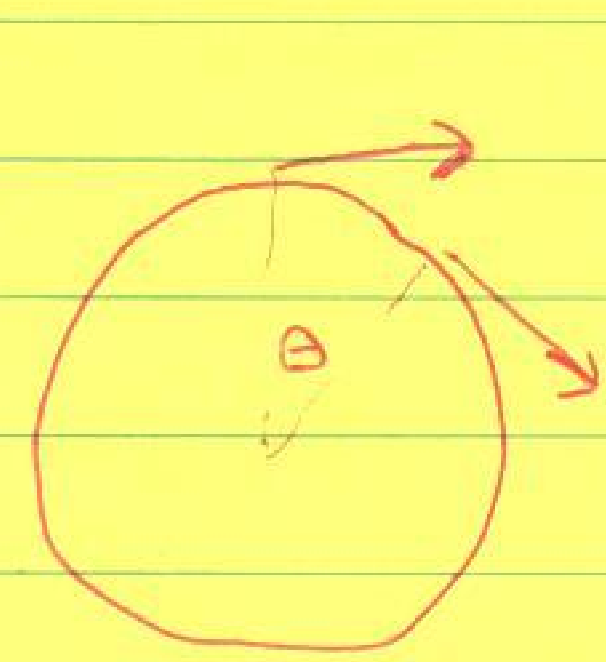
Centripetal Acceleration

Circular Motion:



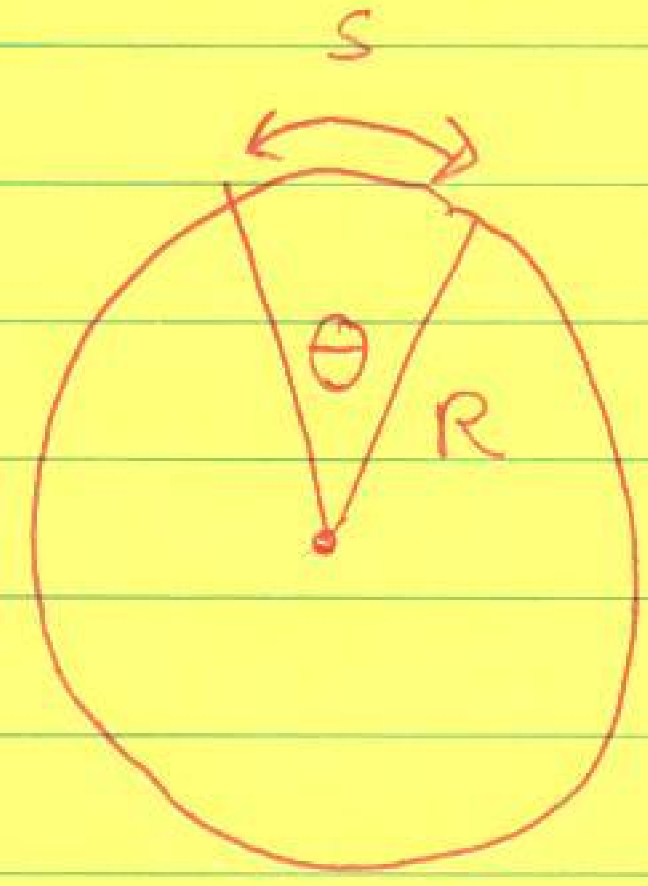
① The direction is changing

② Velocity has magnitude and direction

So

$$a = \frac{\Delta \vec{V}}{\Delta t} \quad \text{is non-zero}$$

Facts about Radian measure:



$$\theta = \frac{S}{R}$$

length

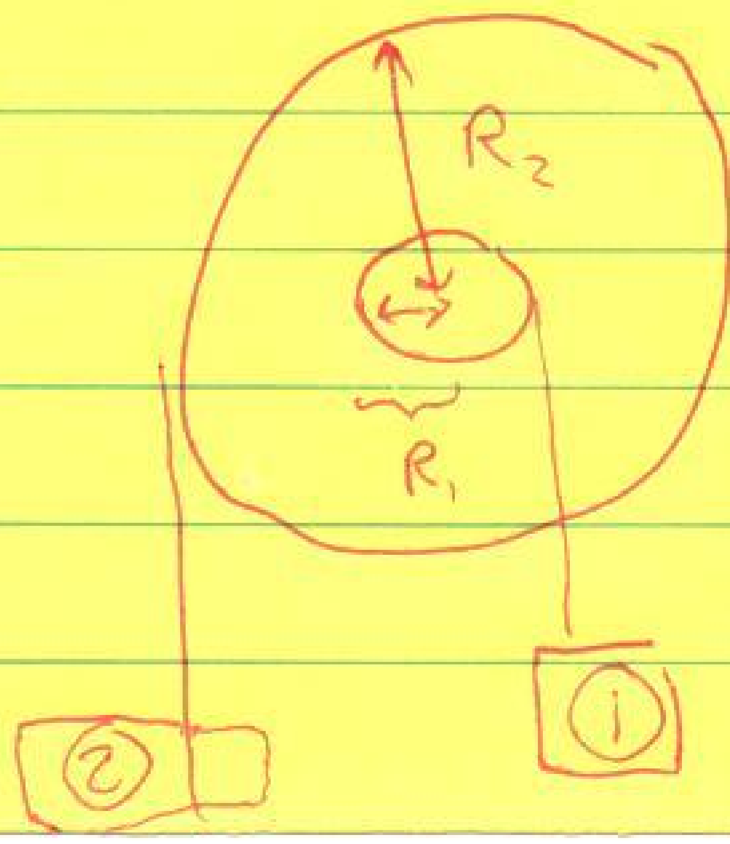
angle in radians

Radius

Problem:

$R_1 = 2cm$

$R_2 = 3cm$



If block ① descends 15 cm how much does block ② go up

$$\frac{s_1}{R_1} = \theta = \frac{s_2}{R_2}$$

amount block ① goes ~~up~~ down

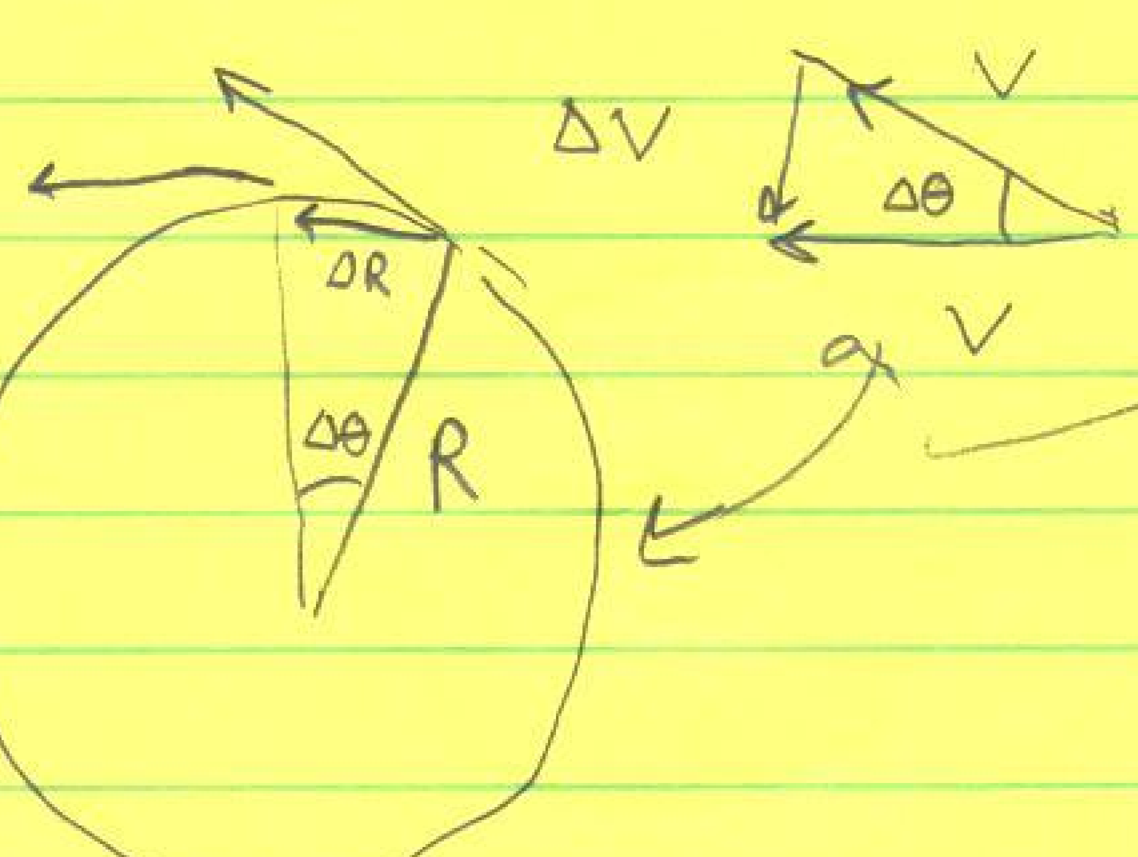
amount block ② goes up

$$s_2 = \frac{R_2}{R_1}\,\vartheta_1 = \frac{3\,cm}{2\,cm}\cdot 15\,cm = 22.5\,cm$$

Small Angles:

$$\Delta\vec{r} \simeq \Delta\theta\, R$$

Derivation:

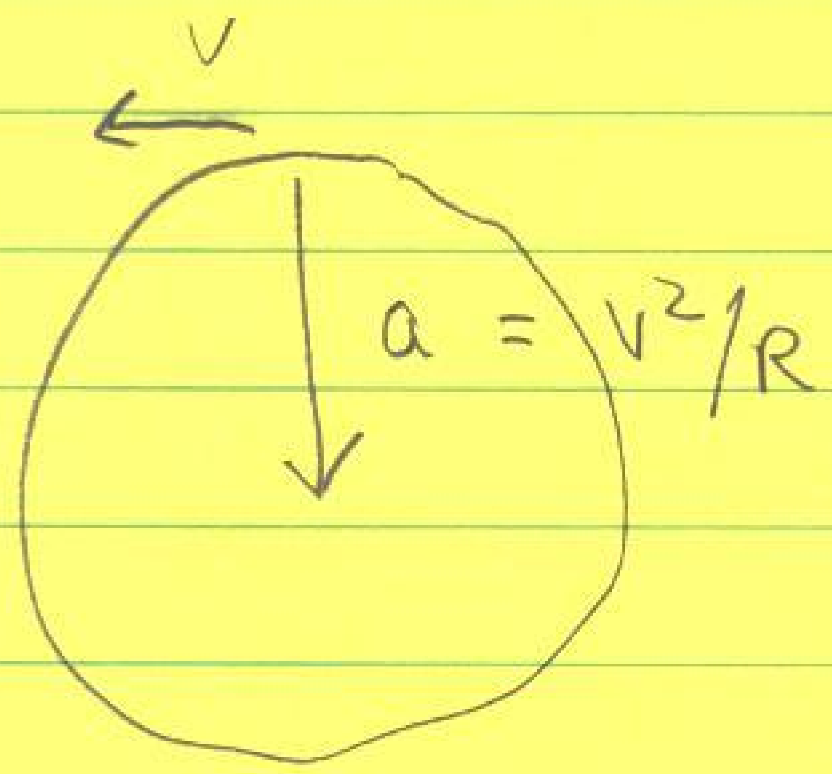


These are similar Tri

$$\frac{\Delta V}{V} = \frac{\Delta r}{r}$$

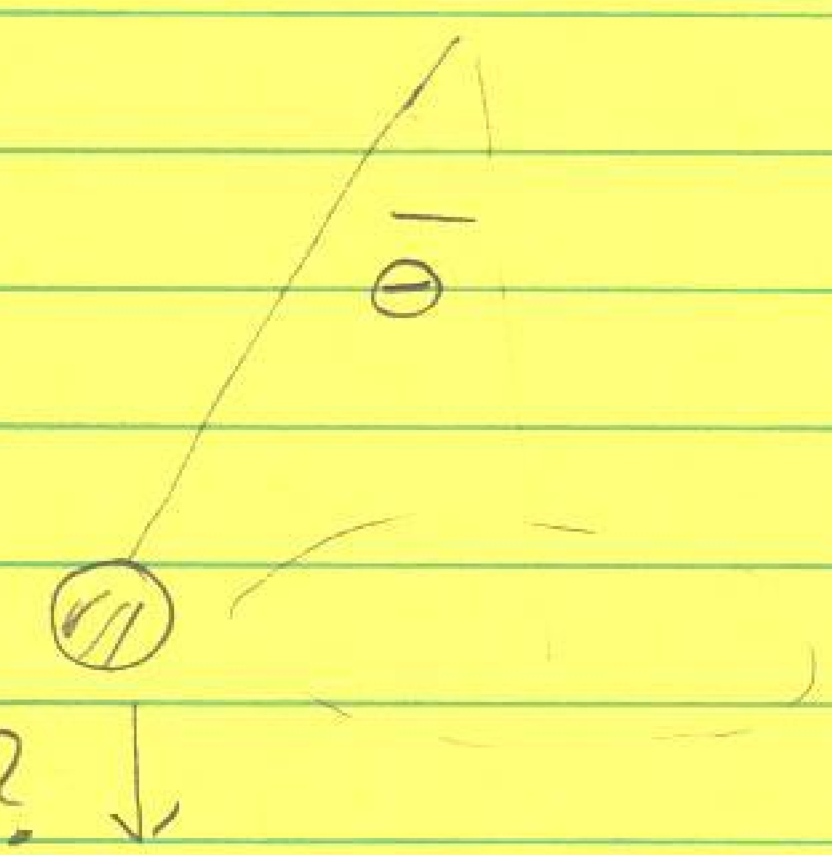
$$a = \frac{\Delta V}{\Delta t} = \frac{\Delta r}{\Delta t}\,\frac{V}{R} = \frac{V^2}{R}$$

Summary

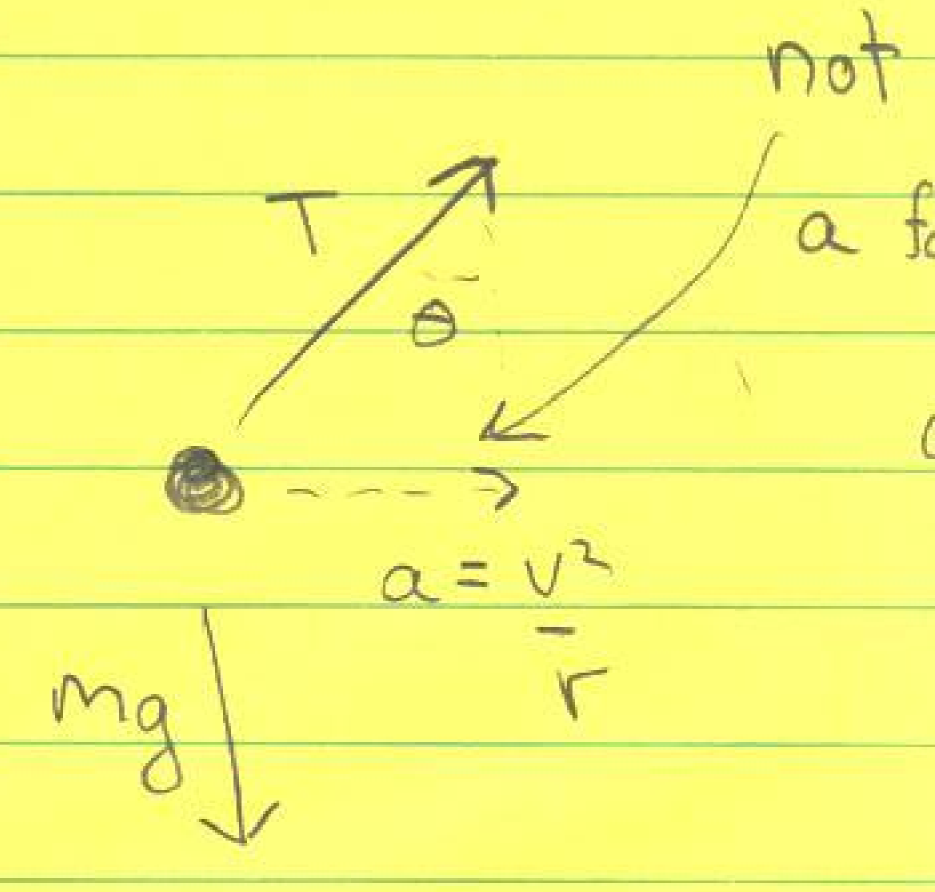


For a particle going around in a circle with speed v there is an acceleration pointing inward toward the center

Example ①



Solution: Draw all forces:



$$\sum F^x = ma^x$$

$$T\sin\theta = m\frac{v^2}{r}$$

Then:

$$\sum F^y = m\cancel{a^y}^{\,0}$$

$$T\cos\theta - mg = 0$$

$$T = \frac{mg}{\cos\theta}$$

$$\frac{\cancel{m} g \sin\theta}{\cos\theta} = \cancel{m}\frac{V^2}{R}$$

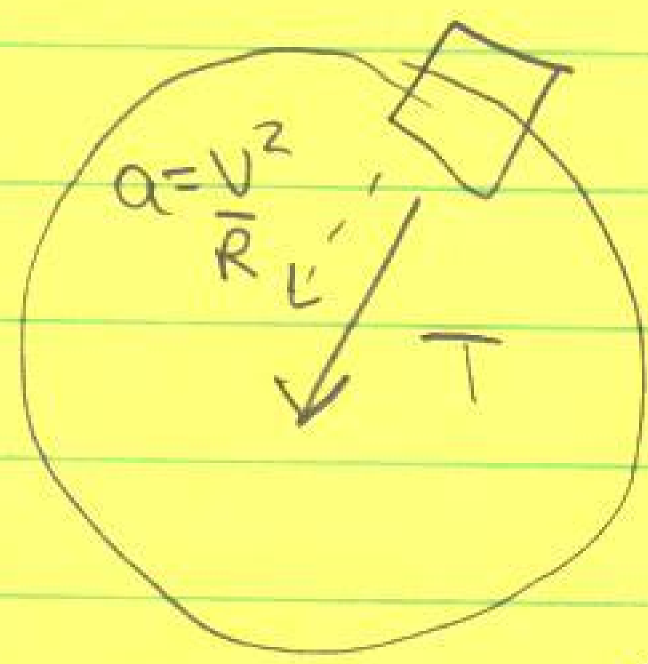
$$\sqrt{gR\tan\theta} = V$$

Example 2

The maximum tension a rope can withstand is 50 N

How fast can you spin a paint can which weighs m=0.5kg

Solution:



$$\sum F^{\perp} = ma^{\perp}$$

Newtons Law perpendicular to velocity

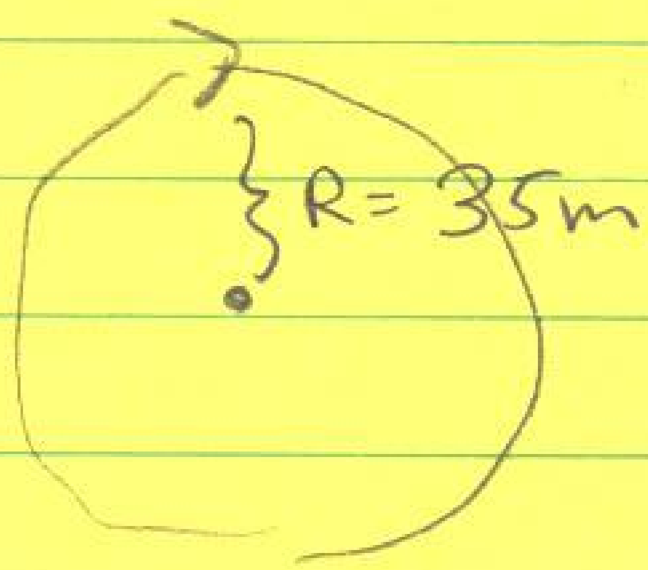
$$T_{max} = m\frac{V_{max}^2}{R}$$

$$v_{max} = \sqrt{\frac{R\,T_{max}}{m}} = 12.2\,m/s$$

Example ③

Car going around a curve, his coefficient of static friction $\mu_s = 0.5$

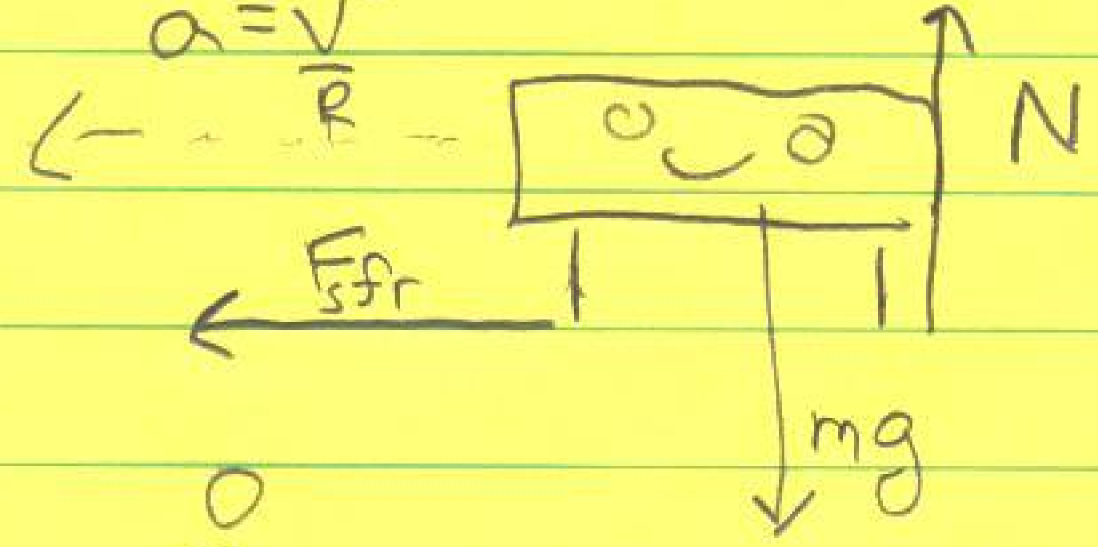
Top view:



Side View

What is the maximum speed he can go without slipping?

Solution - Free body diagram



$$\sum F^y = m\overset{0}{\cancel{a^y}} \qquad\qquad \sum F^x = ma^x$$

$$N - mg = 0 \qquad\qquad \underbrace{\mu_s N}_{\text{stat fric max}} = m\frac{v_{max}^2}{R}$$

So

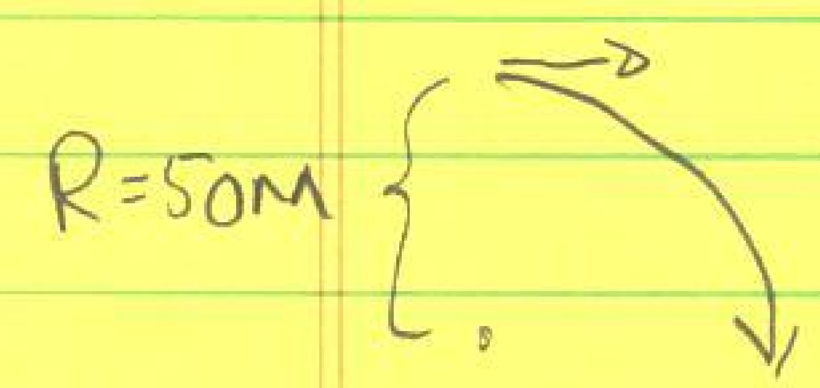
$$\mu_s \cancel{m} g = \cancel{m}\frac{v^2}{R}$$

$$\sqrt{\mu_s g R} = v_{max} = 13.1 \, m/s$$

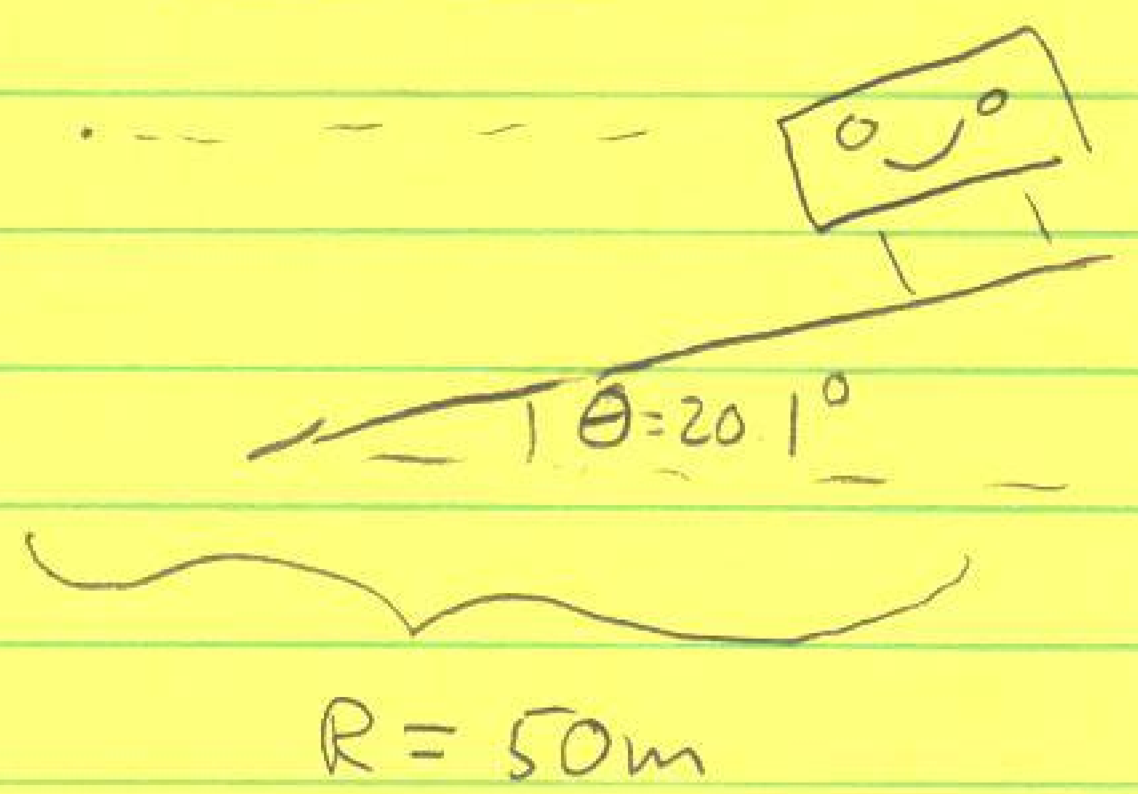
<u>Banked Road</u>

Car going round curve, which is banked with angle $\theta = 20.1^\circ$ (Like an exit ramp)

<u>Top view:</u>

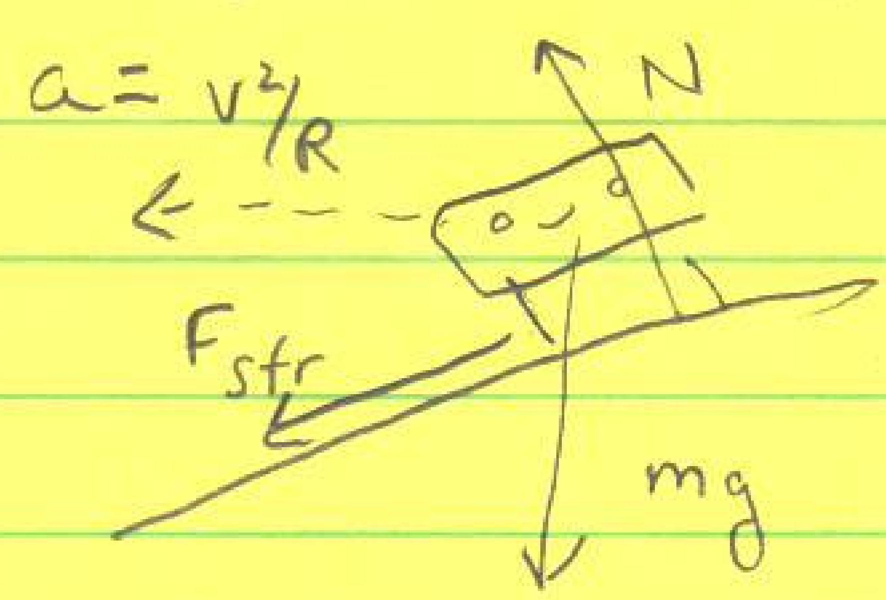


<u>Side View:</u>

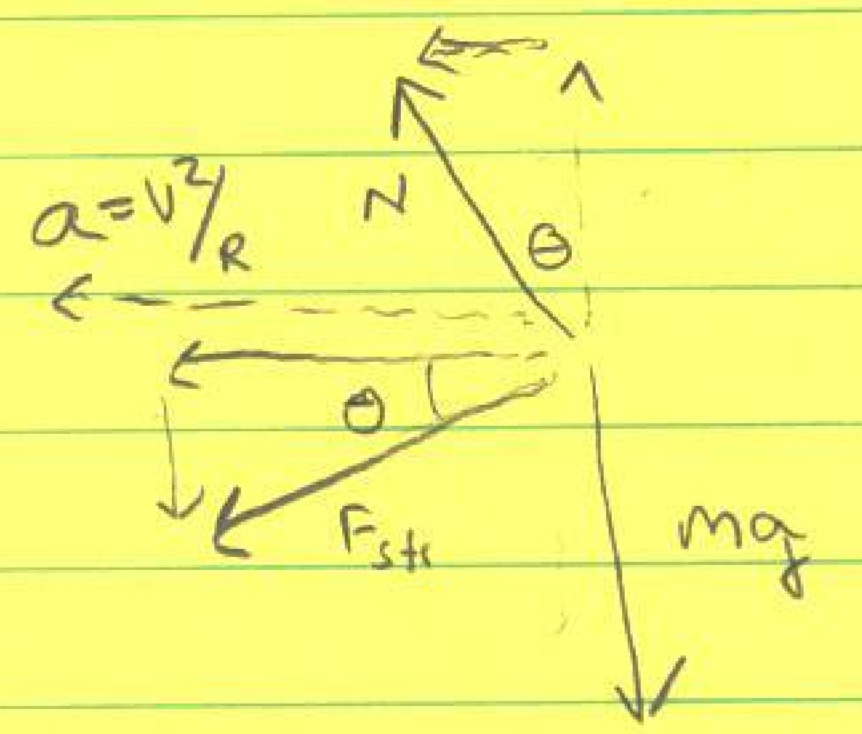


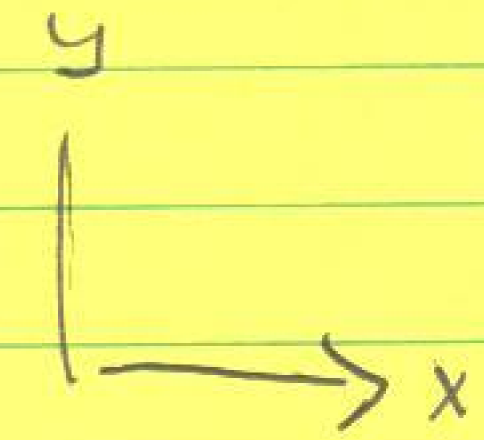
How fast can he go w/out slipping?

<u>Solution:</u> Draw a free body diagram:



Break up forces into x & y and write newtons law.





$\sum F^y = ma^y$ $\qquad\qquad$ $\sum F^x = ma^x$

$N\cos\theta - mg - F_{sfr}\sin\theta = \cancel{ma^y}$ $\qquad$ $-F_{sfr}\cos\theta - N\sin\theta = -m\frac{v^2}{R}$

Now $F_{sfr} = \mu_s N$ so

acceleration points in ne x directio

(1) $N\cos\theta - mg - \mu_s N\sin\theta = 0$ and

(2) $-\mu_s N\cos\theta - N\sin\theta = -\frac{mv^2}{R}$

Two Equations two unknowns —

$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}$ (from (1))

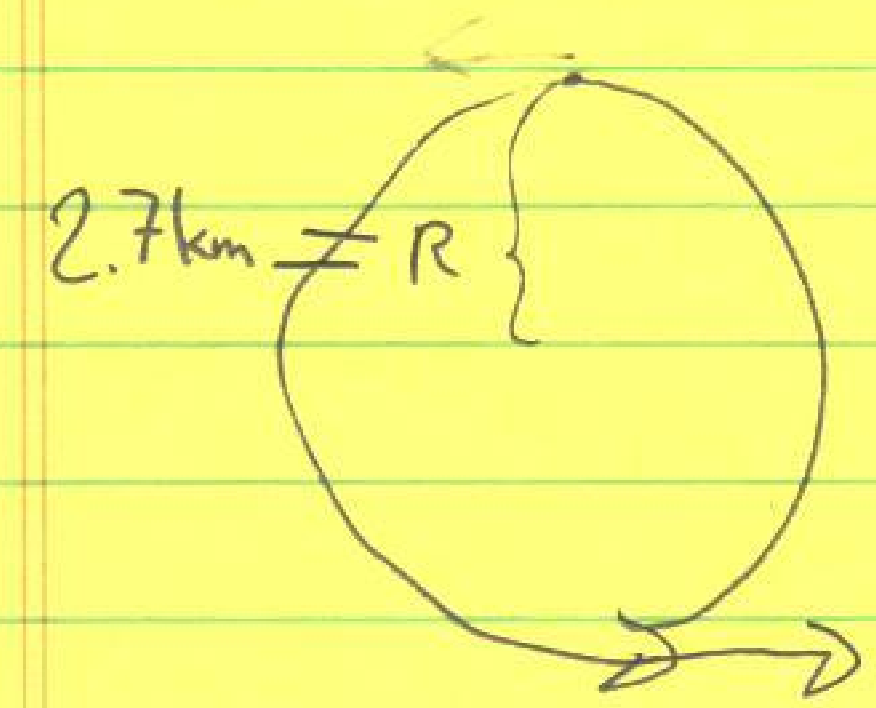
$(\mu_s\cos\theta + \sin\theta)N = \frac{mv^2}{R}$ (from (2))

So we have

$$\frac{\cancel{m} v_{max}^2}{R} = (\mu_s \cos\theta + \sin\theta) \frac{\cancel{m} g}{\cos\theta - \mu_s \sin\theta}$$

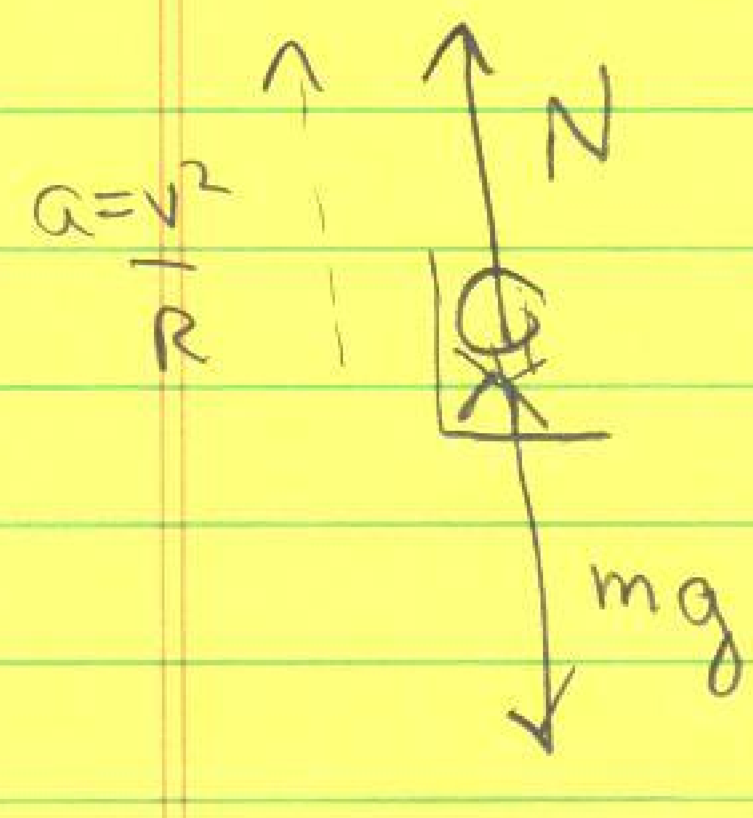
$$v_{max} = \sqrt{gR} \left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta} \right)^{1/2}$$

An airplane goes around in a loop:



Find the ratio of his apparent weight at the bottom to his weight at rest.

$v_0 = 225$ m/s



$$\Sigma F^y = m a^y$$

$$N - mg = m\frac{v^2}{R}$$

$$N = \frac{mv^2}{R} + mg$$

his apparent weight

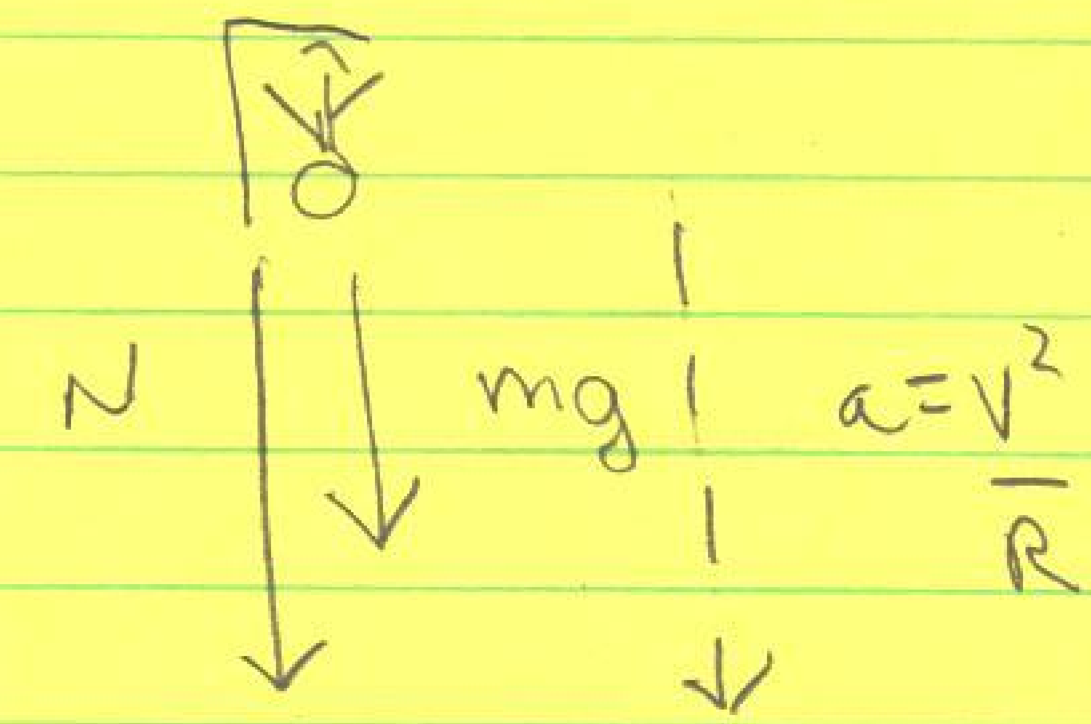
$$\frac{N}{mg} = \frac{\text{his apparent weight}}{\text{his weight at rest}} = \frac{\cancel{mg}}{\cancel{mg}}\left(1 + \frac{v^2}{rg}\right)$$

Then

units!

$$\frac{N}{mg} = 1 + \frac{(225\,m/s)^2}{(2.7\,km)(9.8\,m/s^2)} = 2.91$$

Find the ratio between his apparent weight and his weight at rest at the top



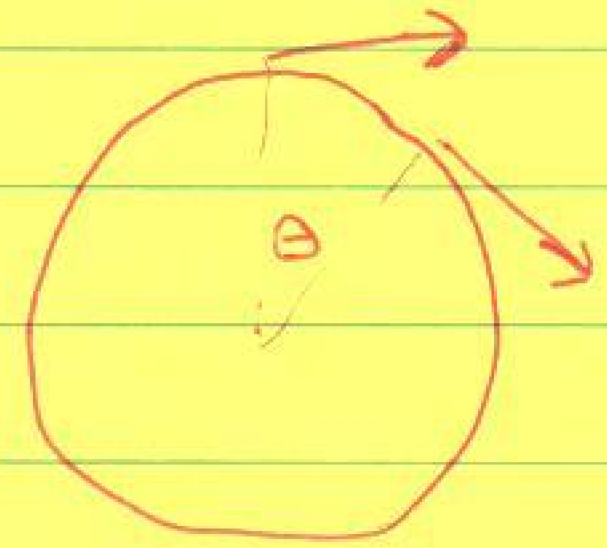
$$\sum F^y = ma^y$$

$$-N - mg = -m\frac{v^2}{R} \Rightarrow N = -mg + m\frac{v^2}{R}$$

$$\frac{N}{mg} = 0.91$$

Centripetal Acceleration

Circular motion:



① The direction is danging

② Velocity has magnitude and direction

So

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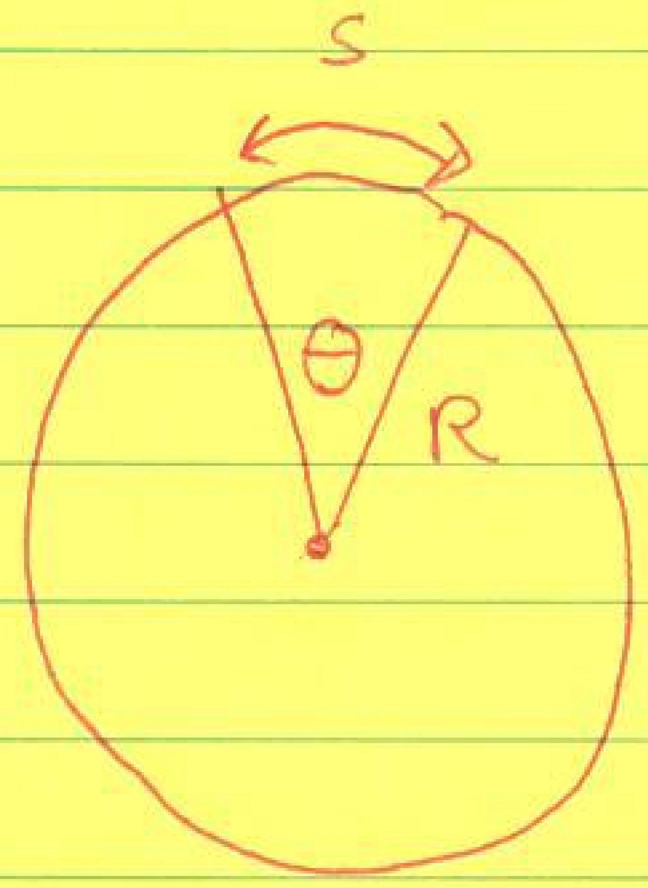
Facts about Radian measure:



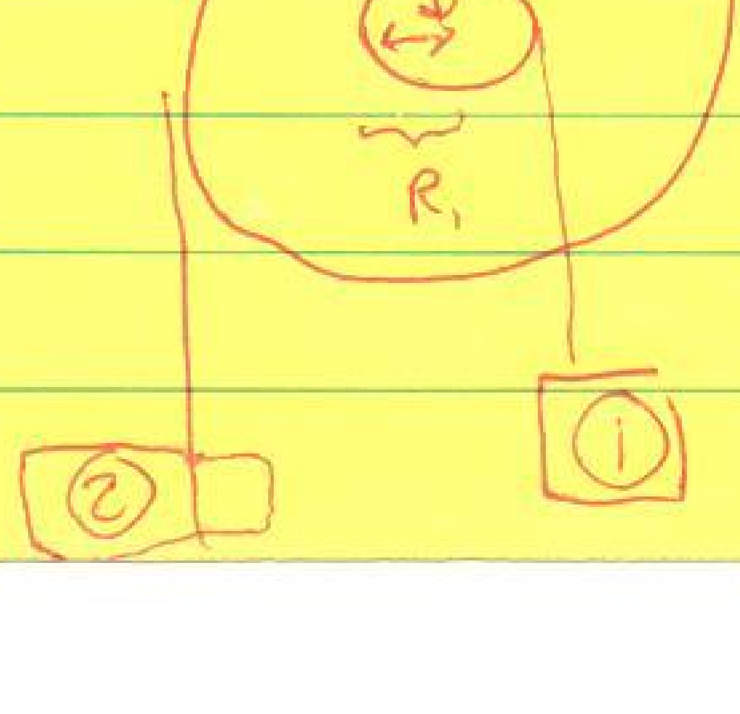
$$\theta = \frac{S}{R}$$

length

angle in radians

Radius

Problem:



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$R_2 = 3cm$