Last Time:

(1) Rotational KE – Energy Stored in rotation

$$KE = \frac{1}{2} I \omega_r^2 \qquad PE_g = Mg\, y_{cm}$$

moment of Inertia

how fast you spin

Like all mass is at the cm

$$= \text{"}\frac{1}{2} M v^2\text{"}$$

(2) Calculate Moments of Inertia

$$I = \sum m_i r_{\perp,i}^2 = M \langle r_\perp^2 \rangle$$

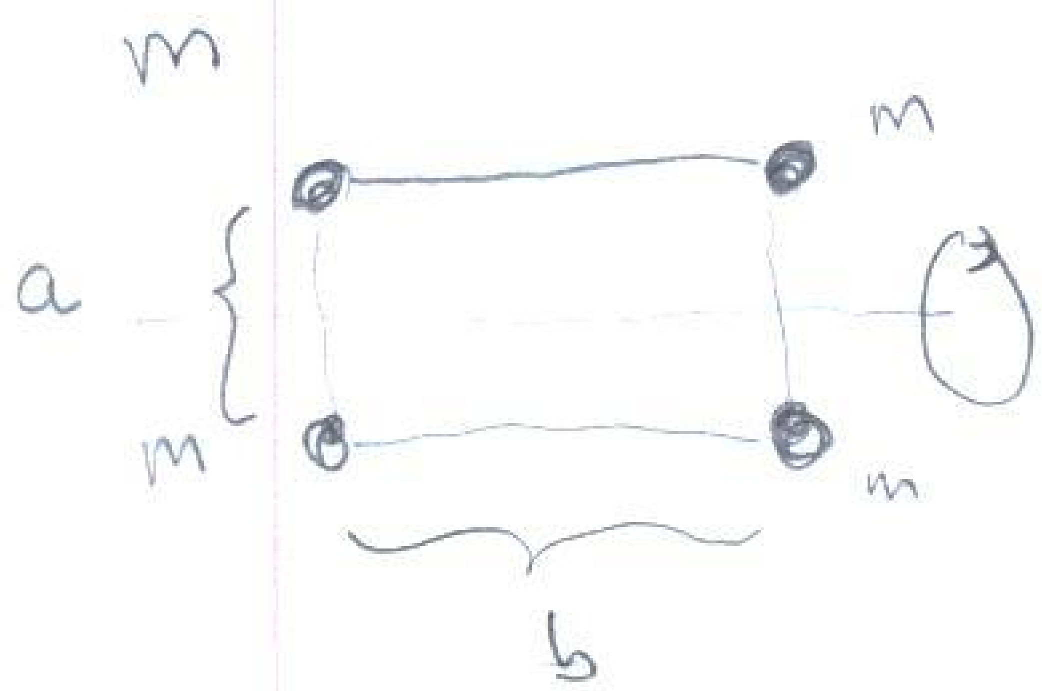
(3) What makes things rotate

$$\sum \tau = I \alpha$$

$$\tau = R F_\perp = R F \sin\theta$$

$$\text{"} \; F = M a$$

Example: Computation of Moment of Inertia



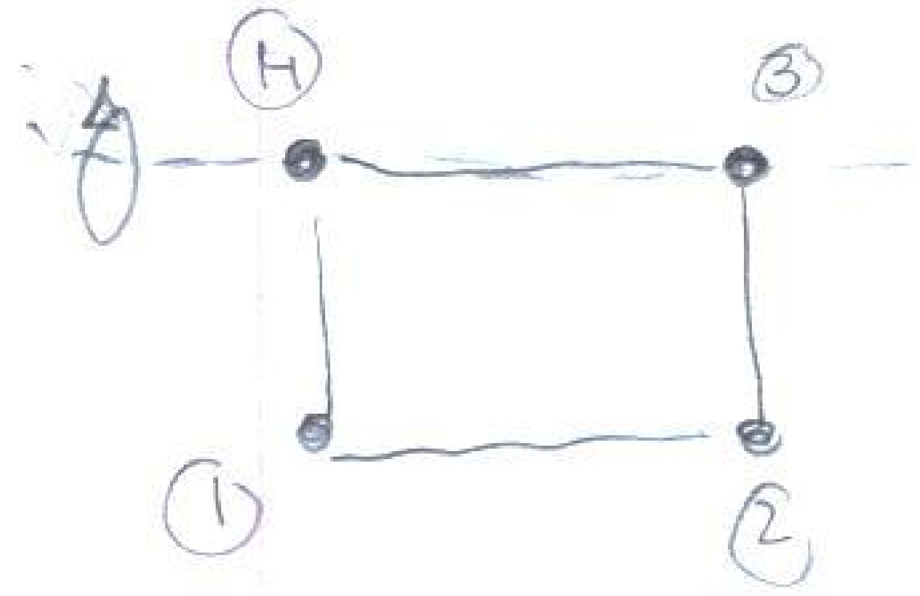
Compute the moment of Inertia:

So:

$$\bar{I}_{cm} = \sum m_i r_{\perp i}^2$$

$$\bar{I}_{cm} = m\left(\frac{a}{2}\right)^2 + m\left(\frac{a}{2}\right)^2 + m\left(\frac{a}{2}\right)^2 + m\left(\frac{a}{2}\right)$$

$$\bar{I}_{cm} = 4m\left(\frac{a}{2}\right)^2 = ma^2$$



$$I_A = \sum m_i r_{i\perp}^2$$

$$= \underset{1}{ma^2} + \underset{2}{ma^2} + \underset{3}{0} + \underset{4}{0}$$

$$= 2ma^2$$

Then we see tha

$$I_A > I_{cm}$$

The parrallel axis theorem!

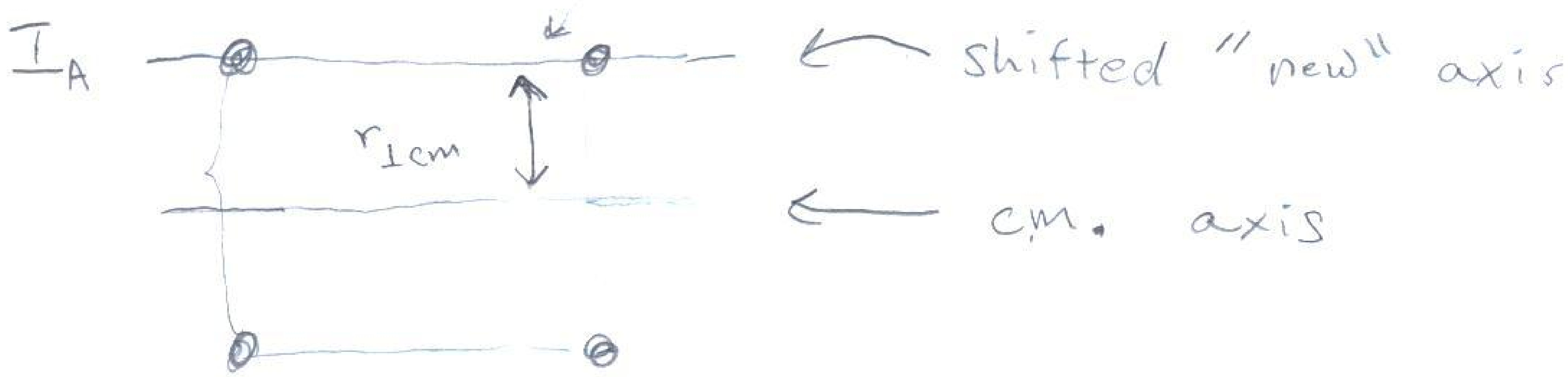
Total mas

$$I_A = I_{cm} + M r^2_{\perp, cm}$$

moment of inertia around "new" parallel axis

moment I_{cm} around c.m.

$\perp$ distance from cm to the other axis



$$I_A = I_{cm} + M r^2_{\perp cm}$$

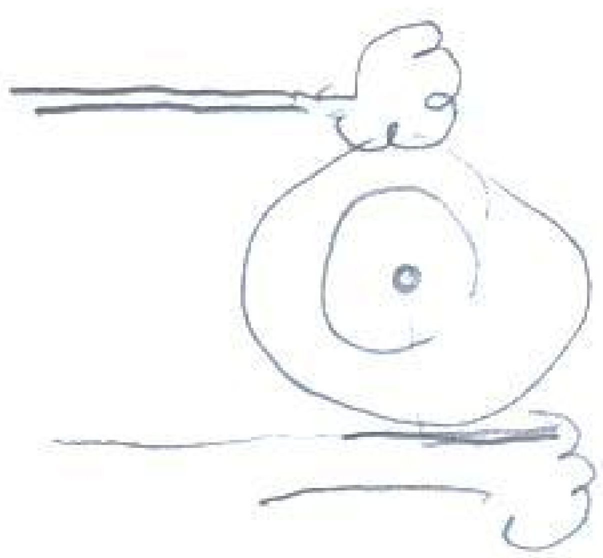
$$I_A = ma^2 + 4m\left(\frac{a}{2}\right)^2 = 2ma^2 \checkmark$$

agrees with direct co

Simple Example Involving Torque and Friction

$W_{ext} = \Delta KE + \Delta PE$

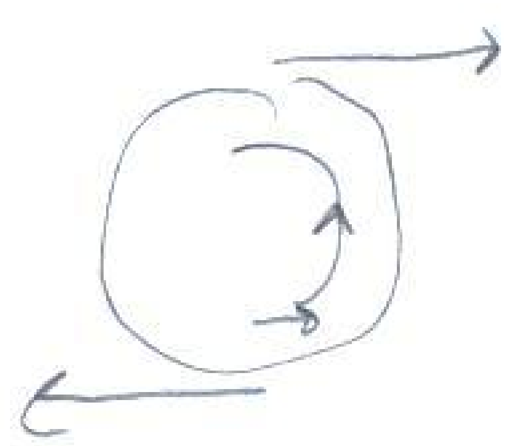
A horizontal wheel is spinning at a constant rate



mass $m = 2kg$ $R = 0.2m$
$\omega = 2 rev/s$

To slow it down a person presses and applies a friction force of 10N / han
How far does the wheel turn, before coming to rest?

(1) Draw a Free body diagram



$W_{hand} = \vec{F} \cdot \vec{d}$

Force + displa
opposit

$W_{hand} = F R \Delta\theta_r \times (-)$

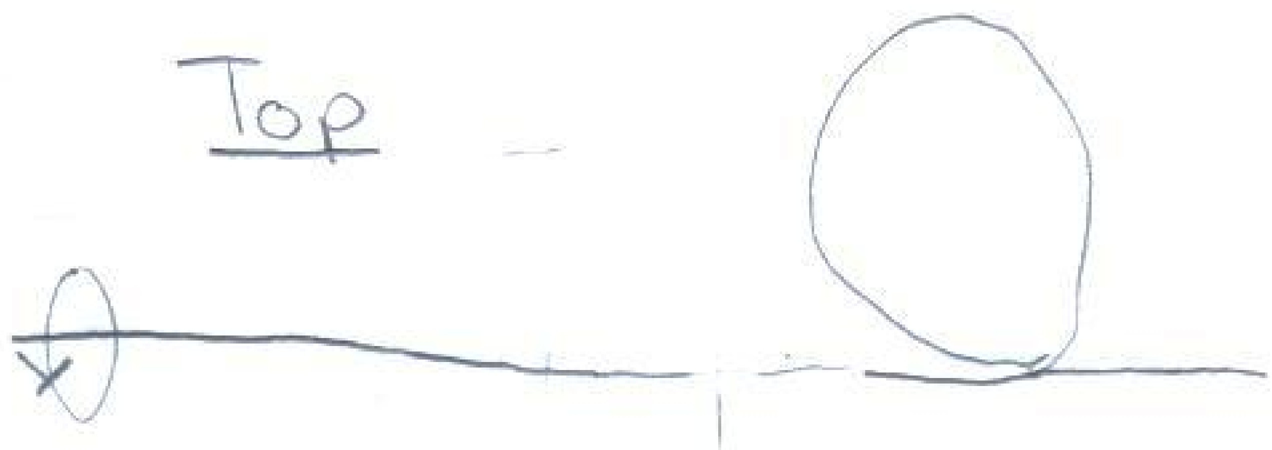
$W_{hand} = \tau \cdot \Delta\theta_r \, (-)$

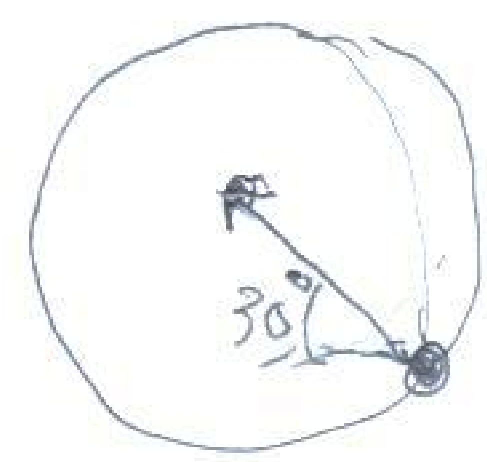
minus because f

$W = \tau \, \Delta\theta \times \underbrace{\text{sign}}$

+ if Torque and $\Delta\theta$ in same rotational
− if Torque and $\Delta\theta$ in — rotational

One More Example :





mass M, diameter D

<u>Problem</u>

A disc is attatched to a rod which can spin

(1) Find the initial angular acceleration ← Involves accelerat or time
use forces & Torq

(2) Find the velocity at the bottom ; ← Does not indue time or
- use E-consv.

Lets do (2) First!

→ Pretend we know I
- we will use the parallel axis theorem to find it shortly

two hands

$$W_{ext} = \Delta KE + \cancel{\Delta PE}^{\,0}$$

$$2W_{hand} = \cancel{KE_f}^{\,0} - KE_i$$

$$+2\tau\,\Delta\theta = +\frac{1}{2} I \omega_{ri}^2$$

$$\omega_r \text{ rad} = \omega$$

$$\Delta\theta_r = \frac{1}{2 \cdot 2\tau} I \omega_{ri}^2$$

$$\omega_r \cancel{\text{rad}} = 2 \times 2\pi r$$

$$\Delta\theta_r = \frac{1}{4FR} \cdot \frac{1}{2} M R^2 \omega_{ri}^2$$

$$\omega_r = \frac{4\pi}{s}$$

$$\Delta\theta_r = \frac{1}{4}\,\frac{1}{10N} \cdot \frac{1}{0.2m} \cdot \frac{1}{2}(2kg)(0.2m)^2\left(\frac{4\pi}{s}\right.$$

$$\Delta\theta_r = \left(0.79 \right) kg\,\frac{m^2}{s^2}\,\frac{1}{\underbrace{kg\,\frac{m}{s^2}}_{1N} \cdot m}$$

$$\Delta\theta_r = 0.79$$

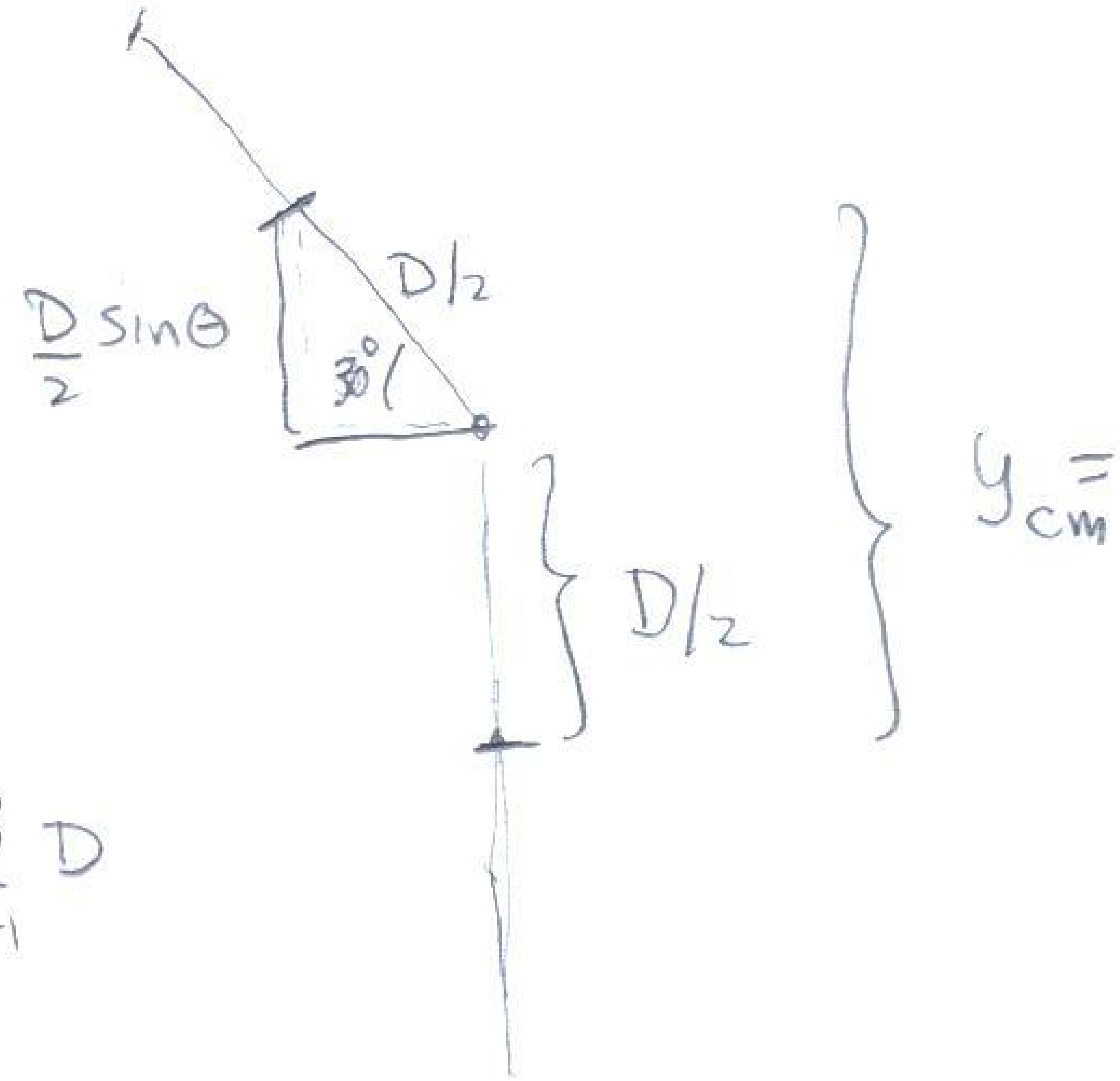
$$\Delta\theta = \Delta\theta_r \text{ rad} = 0.79 \text{ rad}$$

$W_{ext} = \Delta KE + \Delta PE$ (Pretend we know I)

$$0 = KE_f - \cancel{KE_i}^{\,0} + \cancel{PE_f}^{\,0} - PE_i$$

$$PE_i = KE_f$$

$$Mg\, y_{cm} = \frac{1}{2} I_A\, \omega_f^2$$



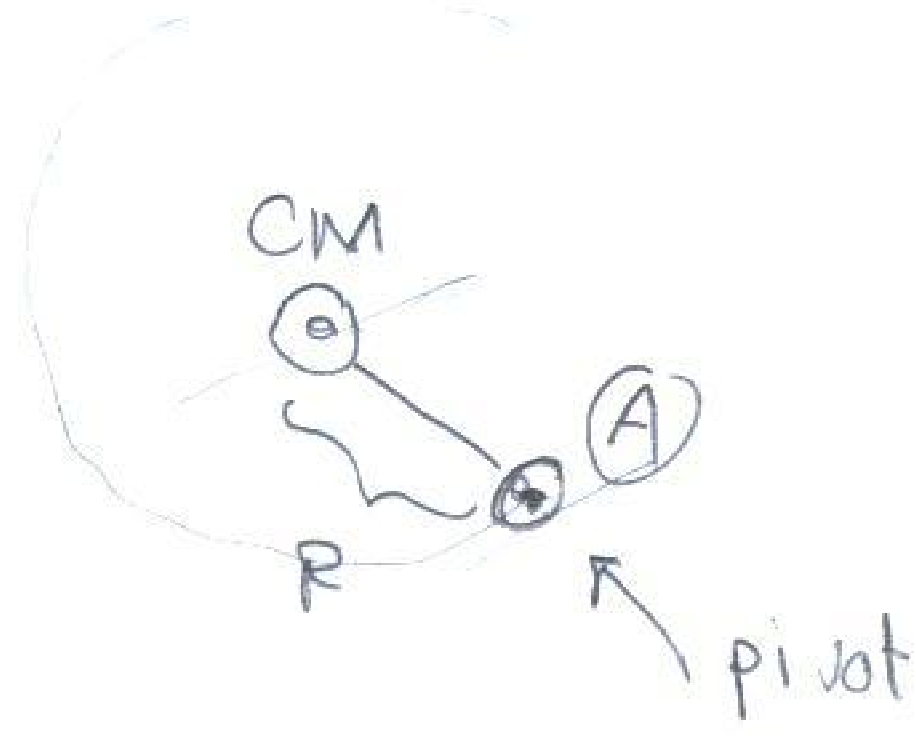
So $y_{cm} = \frac{D}{2} + \frac{D}{2}\sin 30^\circ$

$$y_{cm} = \frac{D}{2} + \frac{D}{2}\cdot\frac{1}{2} = \frac{3}{4}D$$

$$Mg\frac{3}{4}D = \frac{1}{2}(I)\,\omega_f^2$$

$$\sqrt{\frac{Mg\frac{3}{2}D}{I}} = \omega_f$$

Now we have to find I



$$I_{A} = I_{cm} + M r_{cm\perp}^2$$

$$= \frac{1}{2}MR^2 + MR^2$$

$$I_{A} = \frac{3}{2}MR^2$$

So we have

$$I_{A} = \frac{3}{2} M \left(\frac{D}{2}\right)^2 = \frac{3}{8} M D^2$$

Then

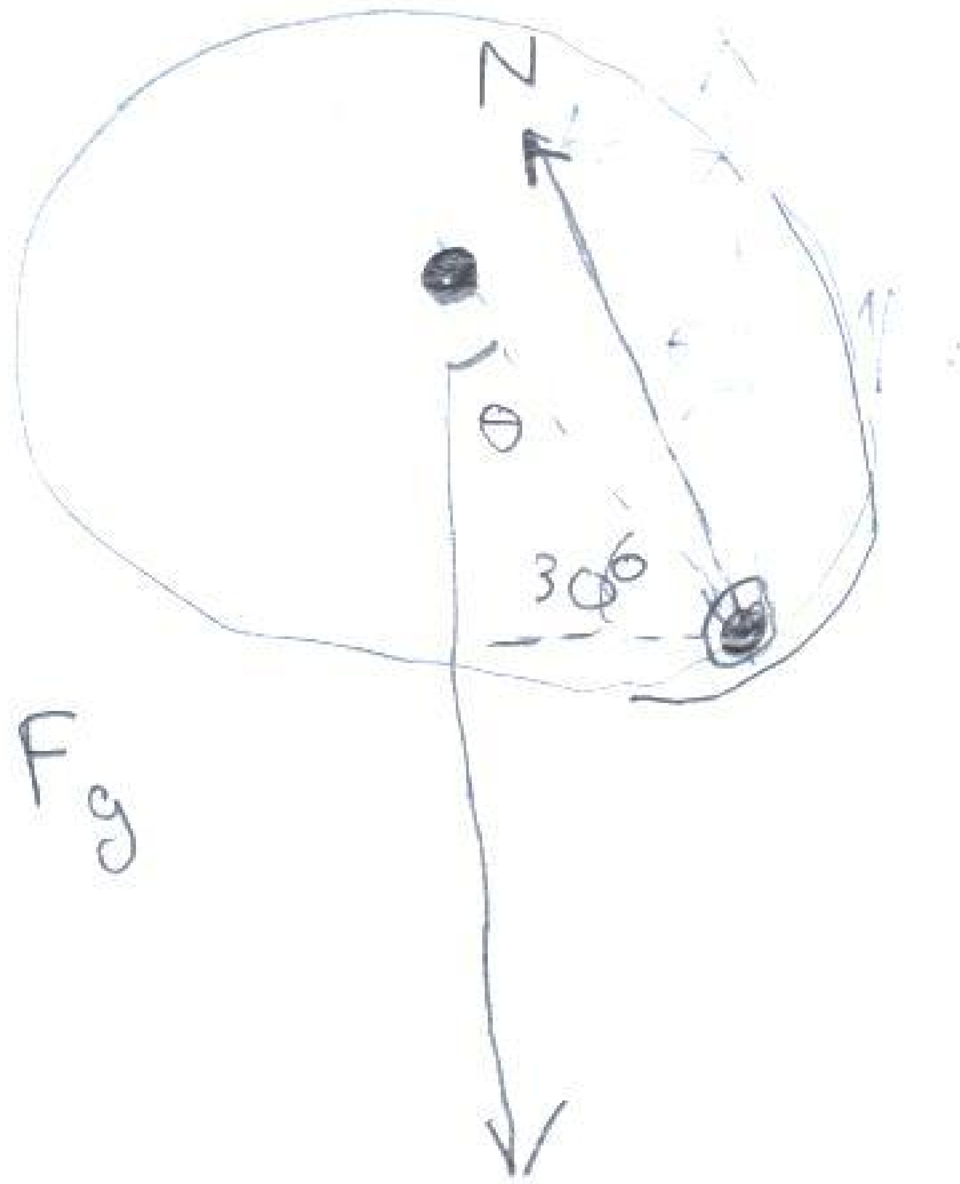
$$\omega_r = \left(\frac{Mg}{I} \frac{3}{2} D\right)^{1/2} = \left(\frac{Mg}{\frac{3}{8} M D^2} \cdot \frac{3}{2} D\right)^{1/2}$$

$$\omega_r = \sqrt{\frac{4g}{D}}$$

If we are interested in the v_{el} of cm we have

$$V_{cm} = \omega_r R = \sqrt{\frac{4g}{D}} \cdot \frac{D}{2} = \sqrt{gD}$$

Next Compute The initial acceleration



$Mg = F_g$

$$\sum \vec{F} = M\vec{a}_{cm}$$

Inolves Normal

$$\sum \tau = I\alpha_r$$

$$\tau_g + \cancel{\tau_N}^{0} = I\alpha_r$$

$$\tau_N = N\,\cancel{R}^{0}\sin\theta$$

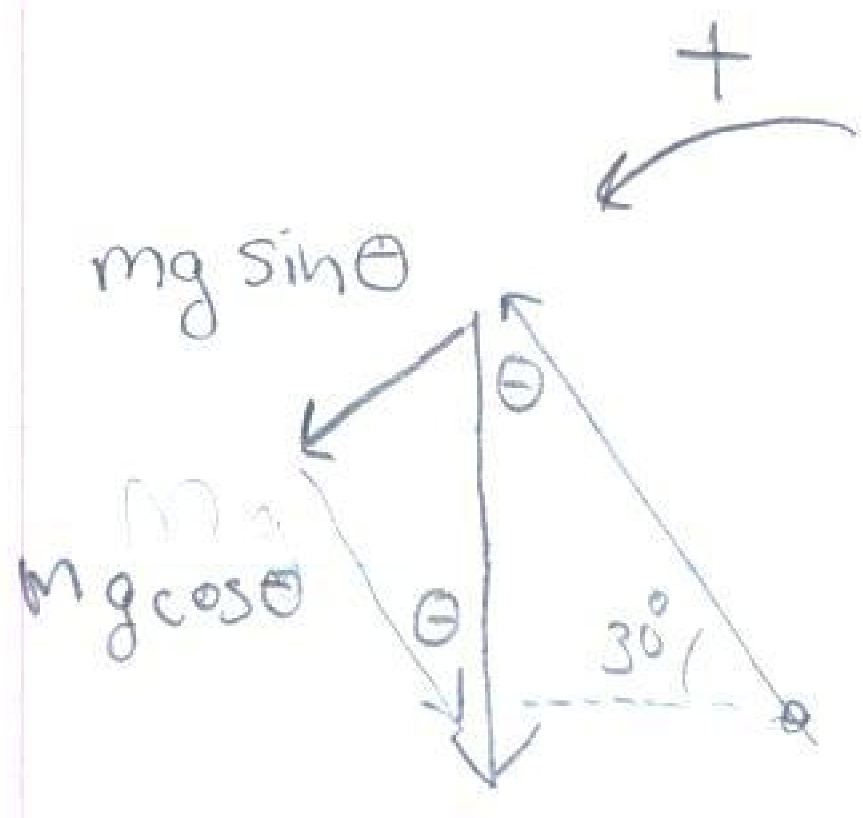
distance from pivot to point where fource acts

$$\tau_{gr} = +|Mg|\,|R|\,|\sin\theta|$$

90-30°

given by Right hand rule

See next page



→ Point your finger from pivot to force

→ Curl Them towards F

$$\tau_g = R\,F_\perp =$$

$$\tau_g = R\,(mg\sin\theta)$$

↳ because if forces makes you spin in counter clockwise -- out of page direction

$$\tau_g = \overbrace{\cancel{m}g\cos 30^\circ\,R}^{\tau_g} = \overbrace{\frac{3}{2}\cancel{M}R^2}^{I}\ \overbrace{\alpha_T}^{\alpha_r}$$

$$\frac{2}{3}\,\frac{g}{R}\cos 30^\circ = \alpha_r$$

$g = 9.8\,m/s^2$ $\quad R = 0.5\,m$ $\quad \alpha_r = 11.315 \cdot \frac{1}{s^2}$

$$\alpha = \alpha_r\ \text{rad} = +$$

Cross Produkt:

$$\vec{\tau} = \vec{R} \times \vec{F}$$

$$\vec{C} = \vec{A} \times \vec{B}$$

out of page by r.h.rule

$$|C| = |A||B||\sin\theta|$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\hat{i} \times \hat{i} = 0$$

$$\vec{A} \times \vec{A} \;\alpha\; \sin 0^\circ = 0$$

$$\hat{i} \times \hat{j} = \hat{k}$$ means out of page

$$\hat{j} \times \hat{i} = -\hat{k}$$ means into page

$$\vec{A} \times (\vec{B} + \vec{C}) = \underbrace{\vec{A} \times \vec{B} + \vec{A} \times \vec{C}}_{\text{Like multiplication}}$$

$$\vec{A} = (A_x \hat{i} + A_y \hat{j}) \qquad \vec{B} = (B_x \hat{i} + B_y \hat{j})$$

$$\vec{A} \times \vec{B} = (A_x \hat{i} + A_y \hat{j}) \times (B_x \hat{i} + B_y \hat{j})$$

$$= A_x B_x \cancel{\hat{i} \times \hat{i}}^{\,0} + A_y B_x \overbrace{\hat{j} \times \hat{i}}^{-\hat{k}} + A_x B_y \hat{i} \times \hat{j} + A_y B_y \cancel{\hat{j} \times \hat{j}}^{\,0}$$

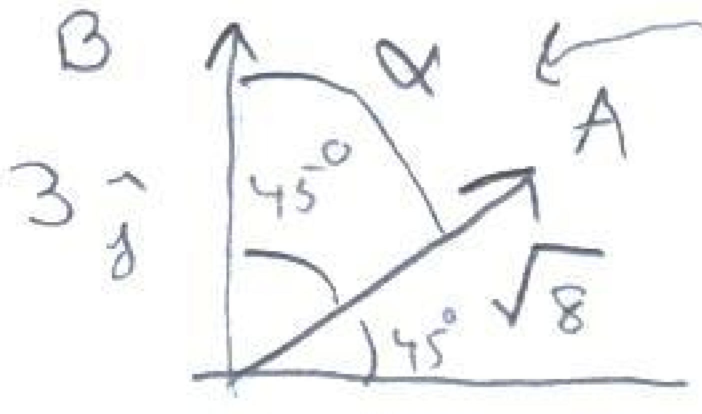
$$\vec{A} \times \vec{B} = (A_x B_y - A_y B_x)\hat{k}$$

Example

$\vec{A} = 2\hat{i} + 2\hat{j}$ $\qquad$ $\vec{B} = 3\hat{j}$

$|A| = \sqrt{2^2 + 2^2}$

Pictures: $= \sqrt{8}$

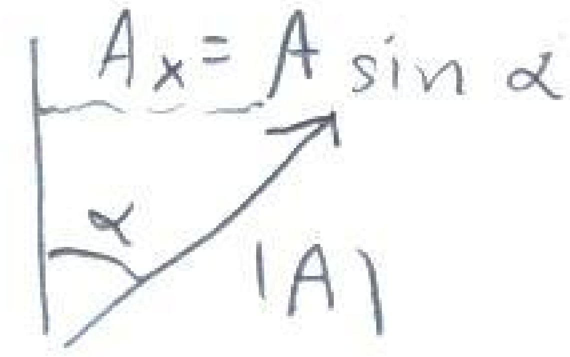


$\alpha = 45^\circ$

$\vec{A} \times \vec{B} = (\sqrt{8} \cdot 3 \cdot |\sin \alpha^\circ|)\ \odot$ out of page

$\vec{A} \times \vec{B} = (A_x B_y \ \hat{k})$

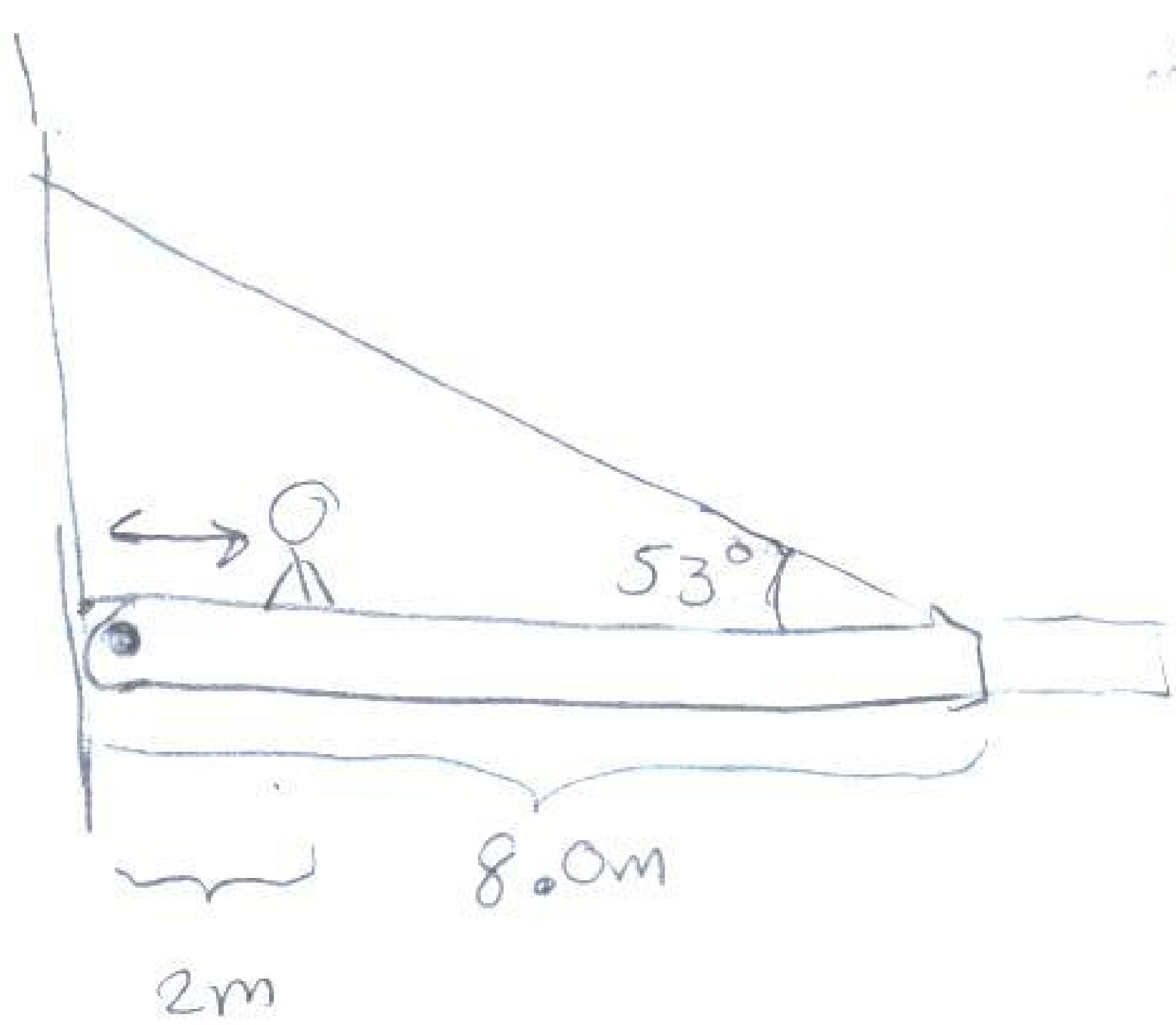
$= (\sqrt{8} \sin \alpha)(3) \ \hat{k}$



Static Equilibrium:

- Any Solid Body thats not moving

Example



beam 200 N

person 600N

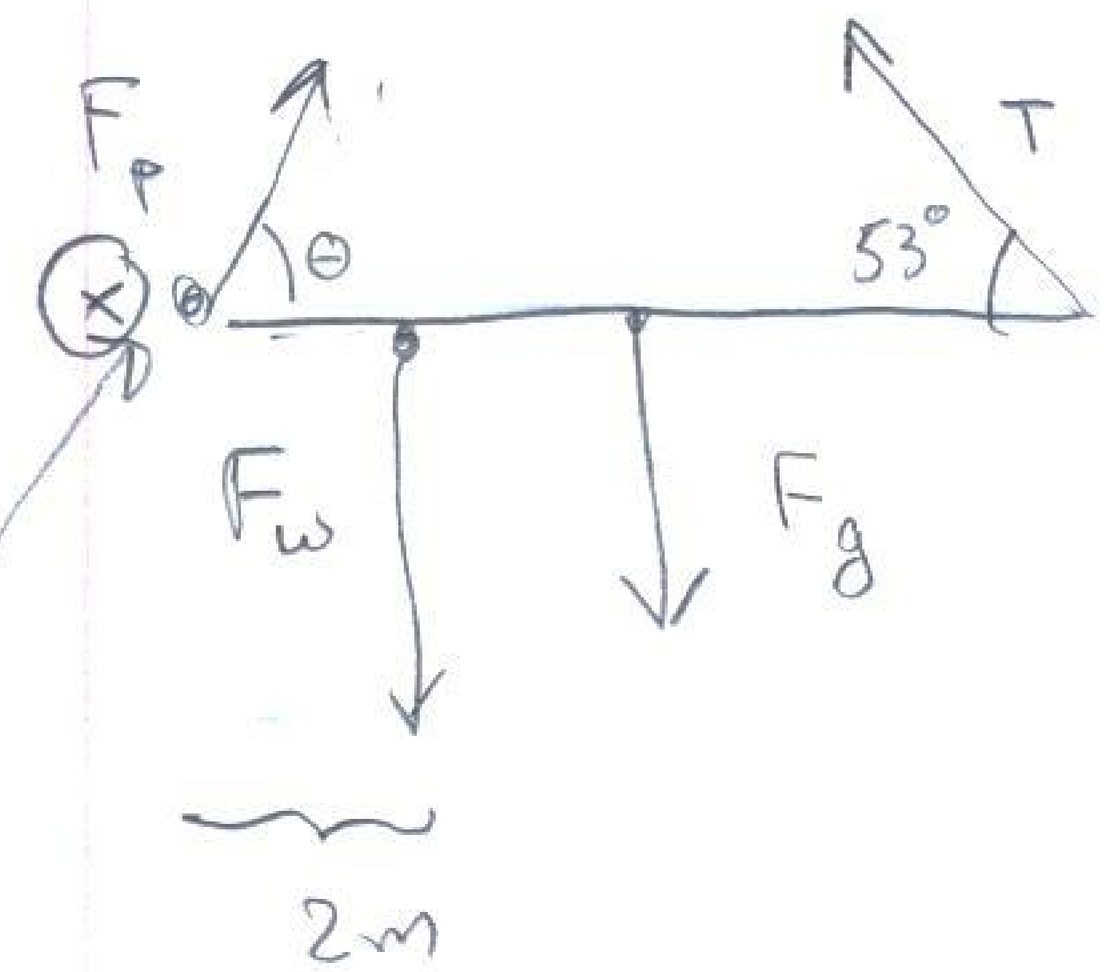
$$\Sigma \vec{F} = m\cancel{\vec{a}}^{\,0}_{cm} \qquad \Sigma \tau = I\cancel{\alpha}^{\,0}$$

Problem:

1. Determine the tension in the cable
2. Determine The Force on the pin
   - magnitude + direction

Solution:

① Draw a free body diagram:



② Pick a pivot point ← Advice: choose a place w/ lots of unknown forces

③ Write $\Sigma \vec{F}_{Net} = m\vec{a}$ & $\Sigma \tau_{Net} = I\alpha$

Break up forces into x, y
remember signs

Keep sign convention for counter clockwise (or +, or out of page) & clockwise (or -, or into page

(3) x]

$$F_p^x - T\cos 53^\circ = 0$$

(2) y] $$F_p^y + T\sin 53^\circ - 600N\,(2m) + 200N(4m) = 0$$

(1) $$\sum \tau = \cancel{\tau_p} + \tau_w + \tau_g + \tau_T = I\cancel{\alpha}$$

$\tau_p = 0 \leftarrow$ Distance From pivot to F = 0

$$\tau_w = RF_\perp = (2m)(600N) \times (-)$$

into page clockwise from RHR

$$\tau_g = RF_\perp = (4m)(200N) \times (-)$$

$$\tau_T = RF_\perp = (8m)\, T\sin 53^\circ$$

So 3 Eqs and 3 unknowns

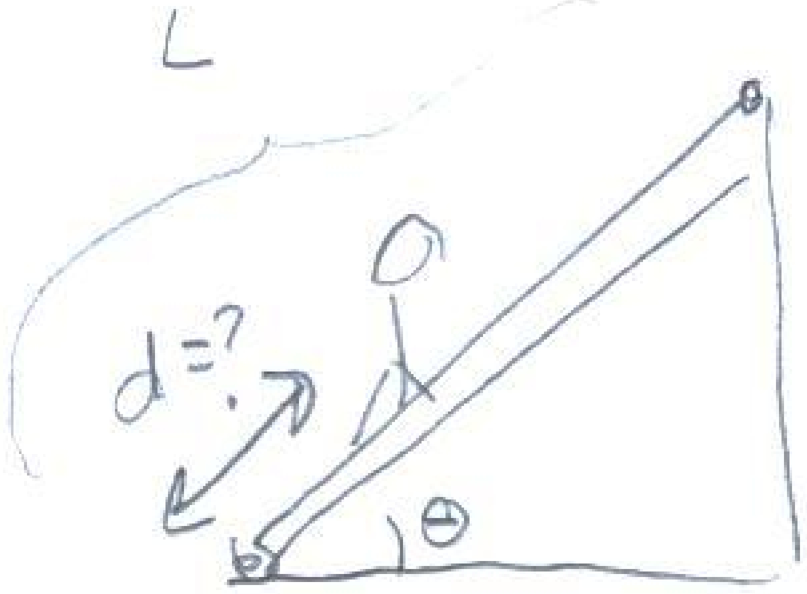
(1) $$-1200\,Nm - 800\,Nm + 8m\,T\sin 53^\circ = 0 \implies T = 313N$$

(3) $$F_p^x - T\cos 53^\circ = 0 \implies F_p^x = 188N$$

$$F_p^y + T\sin 53^\circ - 600N - 200N = 0 \implies F_p^y = 550N$$

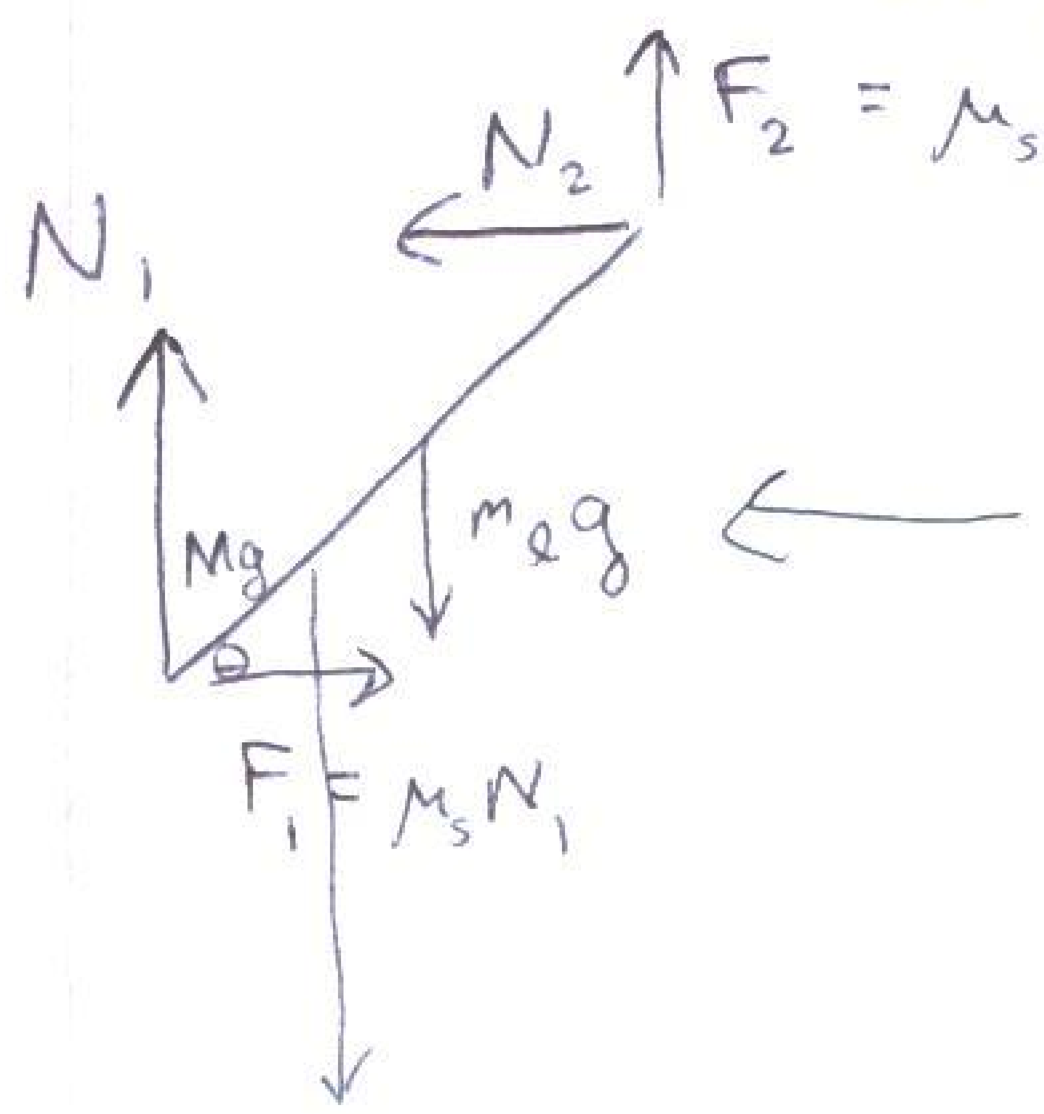
$$F_p = \sqrt{(F_p^x)^2 + (F_p^y)^2} = 580N \qquad \tan\theta = \frac{F_p^y}{F_p^x} \qquad \theta = 71.1^\circ$$

Classic Ladder problem:



The coefficient of static fric is $\mu_s = 0.4$
his mass is M, mass of Ladder is m_ℓ
How far up the ladder can he walk?

① Draw Free body:



← Make this problem slightly easier, set $m_\ell = 0$

② Pick a pivot, say the bottom of ladder

$$\sum F^x = 0$$

x] $\mu_s N_1 - N_2 = 0$

y] $N_1 - Mg - m_\ell g + \mu_s N_2 = 0$

τ] $\underbrace{\tau_{N_1} + \tau_{F_1}}_{0} + \tau_{Mg} + \tau_{m_\ell g} + \tau_{N_2} + \tau_{F_2} = 0$

Located at pivot

$\tau_{Mg} = R F_\perp = d\, Mg \cos\theta \times (-)$

$\tau_{m_\ell g} = R F_\perp = \frac{L}{2} m_\ell g \cos\theta \times (-)$

Clockwise torque

$\tau_{N_2} = R F_\perp = L N_2 \sin\theta \times (+)$

$\tau_{F_2} = R F_\perp = L \mu_s N_2 \cos\theta \times (+)$

$$- d Mg \cos\theta - \frac{L}{2} m_\ell g \cos\theta + L N_2 \sin\theta + L \mu_s N_2 \cos\theta = 0$$

<u>UnKnowns</u>: d, N_2, N_1 Equations = 3 ✓

So

$$\mu_s N_1 = N_2 \implies N_1 = \frac{N_2}{\mu_s}$$

$$\frac{N_2}{\mu_s} - (M+m_\ell)g + \mu_s N_2 = 0$$

$$\left(\mu_s + \frac{1}{\mu_s}\right) N_2 = (M+m_\ell)g$$

$$N_2 = \frac{(M+m_\ell)g}{\left(\mu_s + \frac{1}{\mu_s}\right)}$$

$$d\,Mg\cos\theta = -\frac{L}{2} m_\ell g \cos\theta + (L\sin\theta + L\mu_s\cos\theta)\, N_2$$

$$d\,M\cancel{g}\cos\theta = -\frac{L}{2} m_\ell \cancel{g} \cos\theta + (L\sin\theta + L\mu_s\cos\theta)\left(\frac{M+m_\ell}{\mu_s + \frac{1}{\mu_s}}\right)$$

$$d = L\left(\frac{-m_\ell/M\cos\theta + (\sin\theta + \mu_s\cos\theta)}{\cos\theta}\right)\frac{(1+m_\ell/M)}{\left(\mu_s + \frac{1}{\mu_s}\right)}$$

$$d = L\left[-m_\ell/M + (\tan\theta + \mu_s)\frac{\mu_s}{\mu_s^2+1}(1+m_\ell/M)\right]$$

$$d = L\left[-m_\ell/M + \frac{(\mu_s\tan\theta + \mu_s^2)}{(\mu_s^2+1)}(1+m_\ell/M)\right]$$

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$$\text{“}\ F = Ma$$