Last Time:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

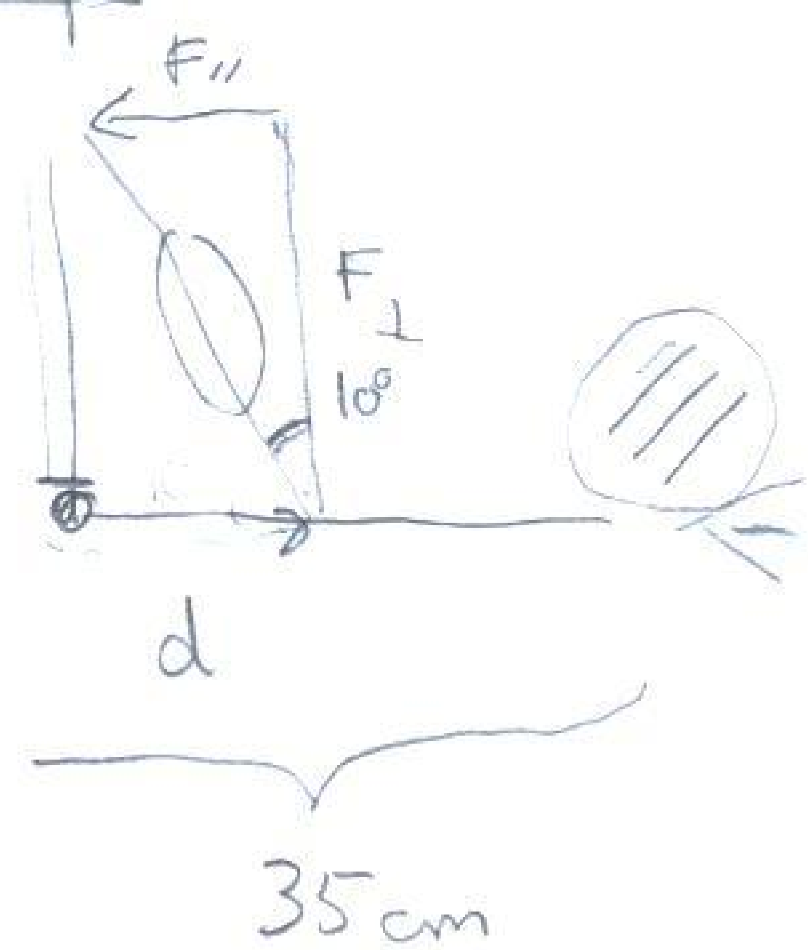
$$\vec{\tau} = |rF\sin\theta| \, (\pm\hat{k}) \qquad \text{(2d only)}$$

either into or out of page, depending on r.h.rule

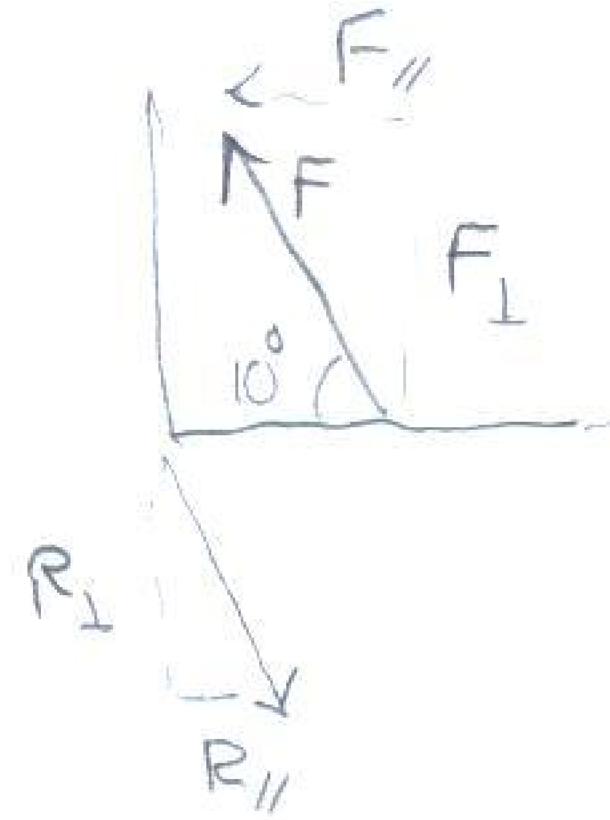
$$\vec{\tau} = |rF_\perp| \, (\pm\hat{k})$$

<u>Example</u>

<u>Arm</u>



$$W_g = 50N$$



$$\sum \vec{\tau} = I \cancel{\vec{\alpha}}^{\,0}$$

$$\sum F^y = F_\perp - R_\perp - W_g =$$

$$F_\perp d\hat{k} - W_g l \hat{k} = 0$$

$$\sum F^x = -F_\parallel + R_\parallel$$

$$F_\perp = 583N$$

$$F\cos 10° = 583N \Rightarrow 592N \qquad F_\parallel = 103N \qquad R_\perp = 533N$$

Example

Find the acceleration of the spool

Solution

$$\sum F^y = m\, a^y_{cm}$$

$$T - mg = -m|a_{cm}|$$

$$\sum \tau = I_{cm}\, \alpha$$

$\tau_g = 0$

$$\tau_g + \tau_T = I_{cm}\, \alpha$$

$\tau_T = |T R \sin\theta| \cdot \text{sign}$

$\tau_T = TR(-1)$

α negative means spool moves down

$$|\alpha| = \frac{|a|}{R}$$

$$TR(-1) = I_{cm} \, -|\alpha|$$

So we have

$$+TR = I_{cm}\frac{a}{R} \qquad T - mg = -ma$$

$$T = \frac{I_{cm}}{R^2}a \qquad \frac{I_{cm}}{R^2}a - mg = -ma$$

$$\left(\frac{I_{cm}}{mR^2} + 1\right)ma = mg$$

$$a = \frac{g}{1 + \frac{I_{cm}}{mR^2}}$$

Now $I_{cm} = \frac{1}{2}MR^2$ so

$$a = \frac{g}{1+\frac{1}{2}} = \boxed{\frac{2}{3}g = a}$$

Next, if the spool falls a distance h, what is its speed

$$v^2 = \cancel{v_0^2} + 2a\Delta x$$

$$v_f^2 = 2 \cdot \frac{2}{3}gh$$

$$v_f = \sqrt{\frac{4}{3}gh}$$

One can also solve this using E-conserve

$$\cancelto{0}{W_{ext}} = \Delta KE + \Delta KE$$

$$\cancel{KE_i} + PE_i = KE_f + \cancelto{0}{PE_f}$$

$$Mgh = \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} M v_{cm}^2$$

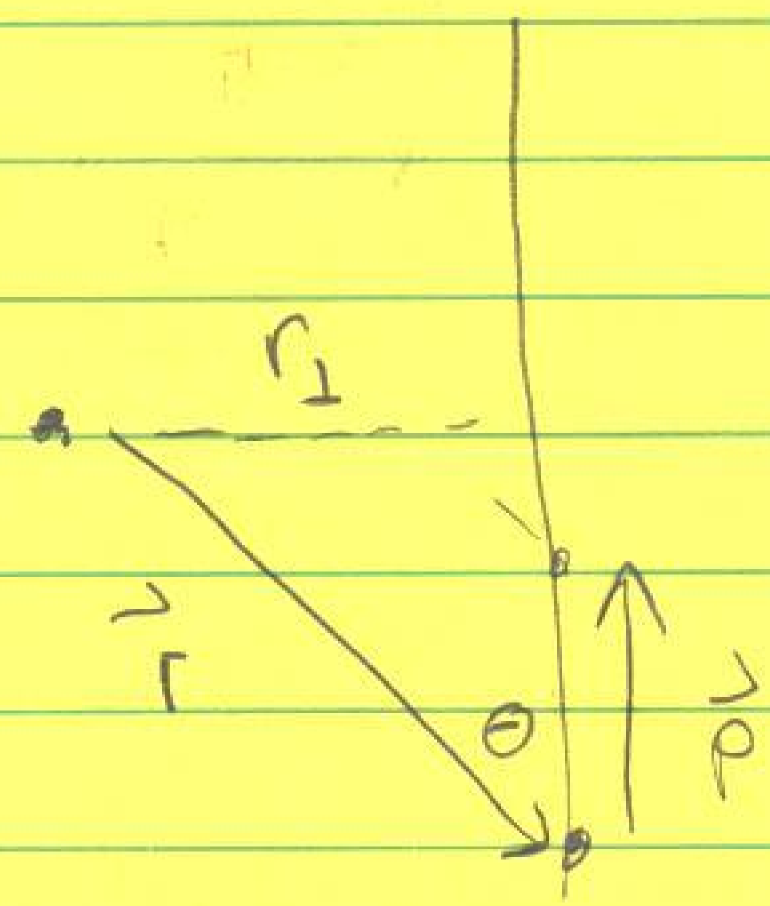
$$= \frac{1}{2} I_{cm} \frac{v^2}{R^2} + \frac{1}{2} M v^2$$

$$2gh = \left(\frac{I_{cm}}{MR^2} + 1 \right) v^2$$

$$\sqrt{\frac{2gh}{1 + \frac{I_{cm}}{MR^2}}} = v$$

Angular Momentum

(1) Angular momentum of a particle



$$\vec{l} \equiv \vec{r} \times \vec{p}$$

$$|l| = r p \sin\theta = r_\perp p$$

$$\vec{l} = r_\perp p \cdot \hat{k}$$

out of page given by r.h.r.

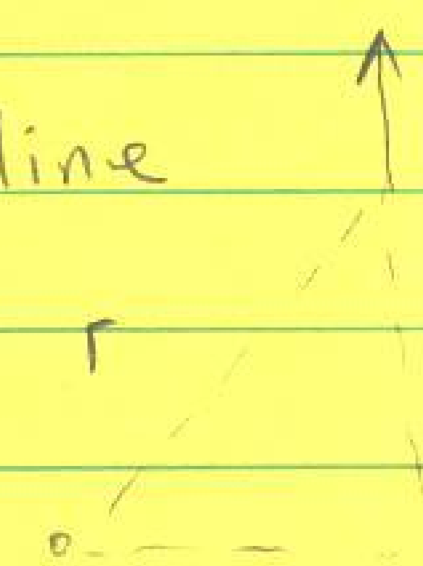
Problem

Show that $\vec{l}$ is constant for a particle moving in a straight line

(1)

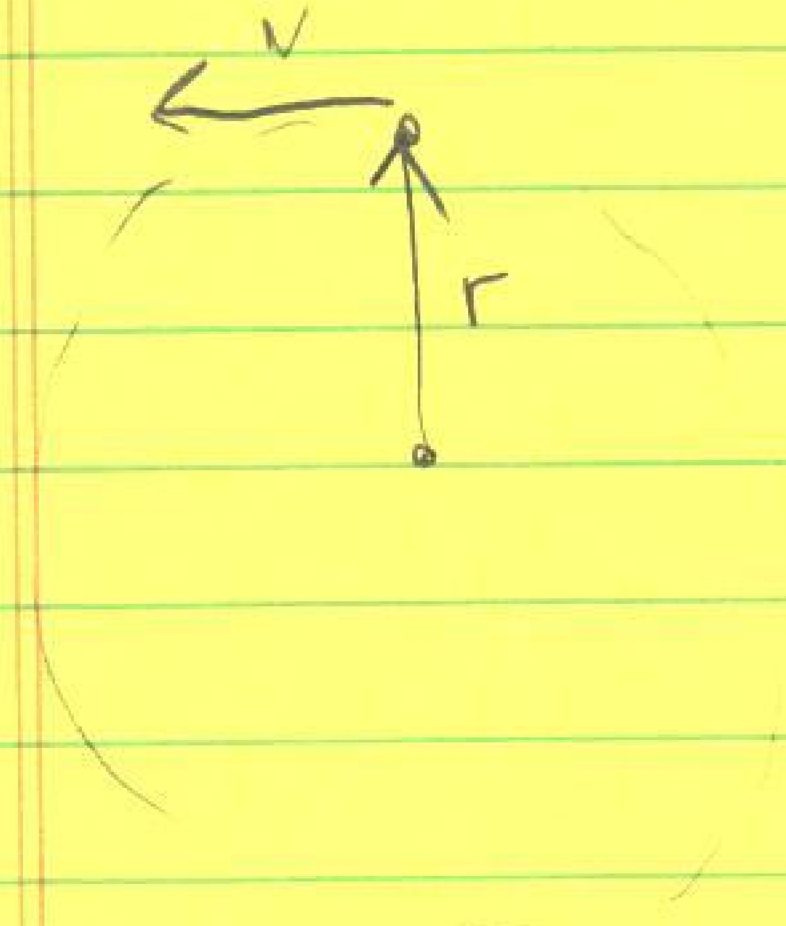
(2)

(3)



$|l| = r_\perp p$ $\quad |\vec{l}| = r_\perp m v$ $\quad |\vec{l}| = r_\perp m v$

For a particle moving in a circle



$$\vec{l} = \vec{r} \times \vec{p}$$

$$|l| = r_i m v \sin\theta$$

$$\vec{l} = r m v \sin 90^\circ \text{ out of page}$$

$$\vec{l} = r m v$$

$$l = r m \omega r$$

$$l = \underbrace{(m r^2)}_{I} \omega$$

For a system of particles

$$L = \sum \vec{r}_i \times p_i$$

$$L = \underbrace{\sum m_i r_i^2}\ \omega$$

$$L = I\omega$$

Why do we care about angular momentum

$$\vec{F} = m\vec{a}$$

$$\vec{F} = m\frac{\Delta \vec{V}}{\Delta t}$$

$$\vec{r} \times \vec{F} = \frac{\Delta(r \times \vec{p})}{\Delta t} = \frac{\Delta \vec{L}}{\Delta t}$$

Particle $\tau_{ext} = \frac{\Delta \vec{L}}{\Delta t}$

Body $\tau_{ext} = \frac{\Delta (I\omega)}{\Delta t}$

$\tau_{ext} = I\frac{\Delta\omega}{\Delta t} = I\alpha$

for I constant

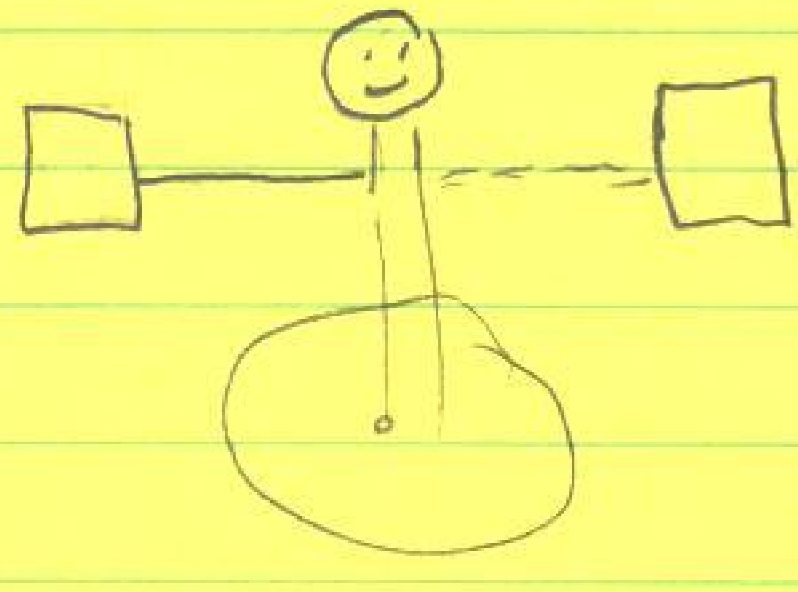
External torques change the angular momentum of a body

Suppose, $\tau_{ext} = 0$ then angular momentum is conserved

$$I_i\,\omega_i = I_f\,\omega_f$$

Problem:

A man on a platform rotates at 0.5 rev/s and holds two 4 kg blocks, Take the moment of inertia of the man and platform to be $I = 4 \text{kg m}^2$. Take the length of his arms to be 1m



① What is this rotational speed after he pulls in his arms

Solution: we want to use

$$I_i \omega_i = I_f \omega_f$$

$$\frac{I_i}{I_f} \omega_i = \omega_f$$

Qualitative Discussion:

$I_f < I_i$ since $I \propto \langle r^2 \rangle_M$

and $\langle r^2 \rangle_M$ is less with his arms in.

$$I_i = \underbrace{4\,kg\,m^2}_{I_{man}} + \underbrace{m_1 r_i^2 + m r_1^2}_{I_{weights}} = 12\,kg\,m^2$$

$$I_{final} = 4\,kg\,m^2 + \underbrace{m\cancel{r}^2 + m\cancel{r}^2}_{} = 4\,kg\,m^2$$

These are zero because he brings them close to his b

So $\omega_i = 0.5\frac{rev}{s} = 0.5 \times \frac{2\pi\,rad}{s} = \pi\frac{rad}{s}$

$$\omega_f = \frac{I_i}{I_f} = \frac{12}{4} \cdot \frac{kg\,m^2}{kg\,m^2}\,\omega_i = 3\pi\,\frac{rad}{s}$$

(2) Find the change in KE

$$KE_f - KE_i = \frac{1}{2} I_f \omega_{rf}^2 - \frac{1}{2} I_i \omega_{ri}^2$$

$$= \frac{1}{2}\left(4\,kg\,m^2\right)\left(\frac{3\pi}{s}\right)^2 - \frac{1}{2}\left(12\,kg\,m^2\right)\left(\frac{\pi}{s}\right)^2$$

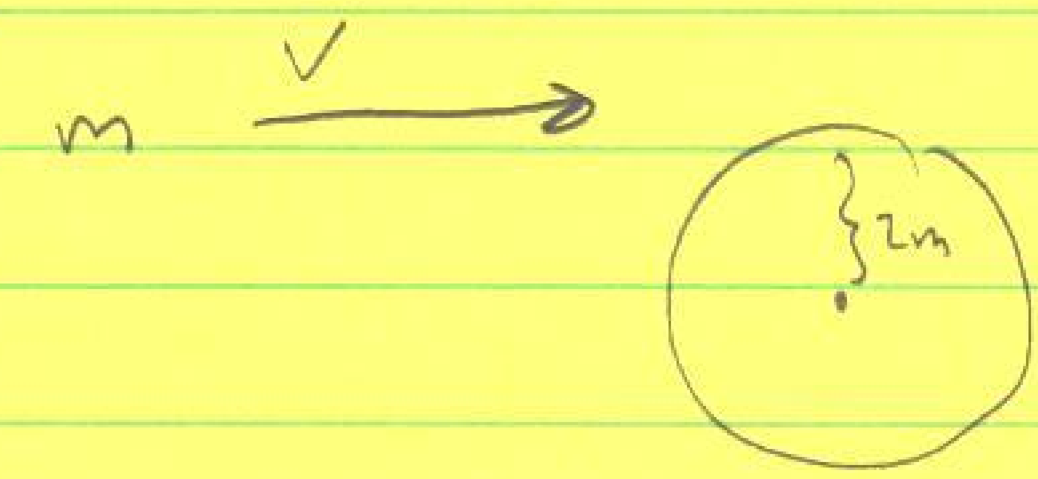
$$\Delta KE = \overbrace{18\pi^2\,J}^{} - 6\pi^2\,J$$

$$\Delta KE = 12\pi^2\,J$$

← The mans muscles do this work

Example 2

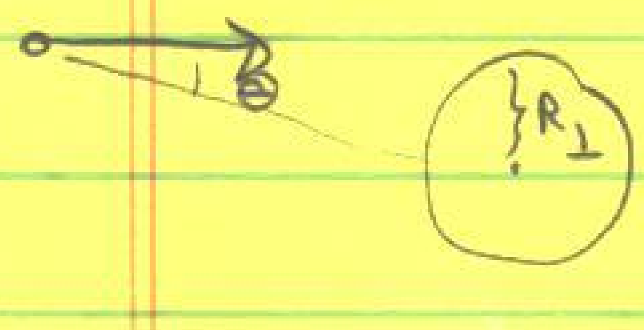
A boy is running and jumps on a merry-go-ro
Merry-go-round mass $M = 160\,kg$, Boy mass $= 80\,kg$
$v_{Boy} = 4\,m/s$, Radius of merry-go-round $= 2\,m$



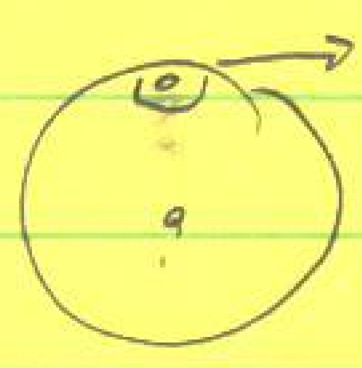
Find the angular speed just after the boy lands

$$\vec{l}_{before} = \vec{l}_{after}$$

Before



After



$l_{before} = r m v \sin\theta$

$= r_\perp m v$

$l_{before} = R m v$

$l_{after} = I\omega$

$l_{after} = (I_{wheel} + I_{boy})\,\omega$

$= \left(\frac{1}{2} M R^2 + m_{boy} R^2\right)\omega$

$$\frac{m_{boy} v}{\left(\frac{M}{2} + m_{boy}\right) R} = \omega_1$$

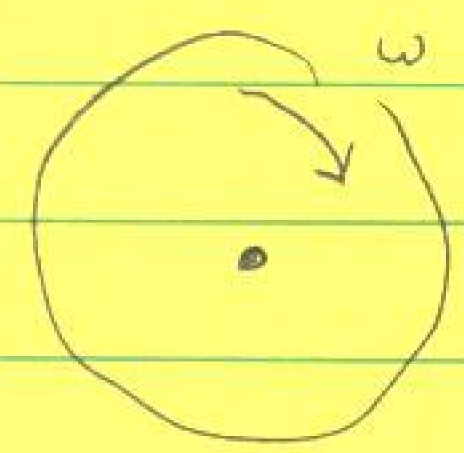
$$1 \text{ rad/s} = \omega_1$$

b) Then the boy walks into the center

$$L_{before} = L_{after}$$

Before

After



$$r_{boy}^2 = 0$$

$$l_{befor} = \left(\frac{1}{2}MR^2 + m_{boy}R^2\right)\omega_1 = l_{after} = \left(\frac{1}{2}MR^2 + 0\right)\omega$$

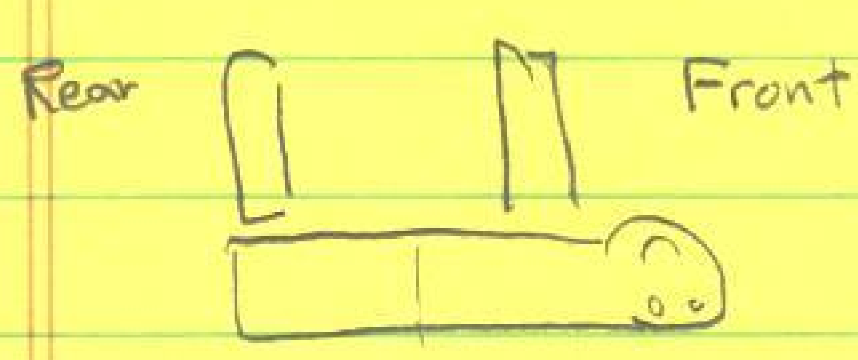
So

$$\frac{\left(\frac{1}{2}MR^2 + m_{boy}R^2\right)\omega_1}{\left(\frac{1}{2}MR^2\right)} = \omega_f$$

$$\left(1 + \frac{2m_{boy}}{M}\right)\omega_1 = \omega_f$$

$$\left(1 + 2\,\frac{80\text{ kg}}{160\text{ kg}}\right)\omega_1 = \omega_f = 2\text{ rad/s}$$

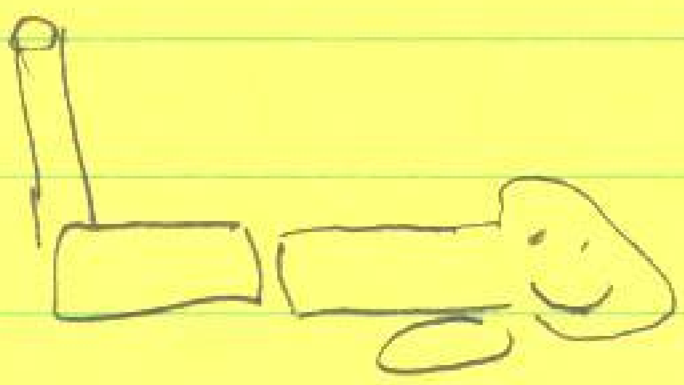
Cat Falling

(1) 

$I_{Rear} \approx I_{Front}$

(2) 

$I_R > I_F$

(3) 

Turn front, since $I_R > I_f$ the large
180° front turn means small rear tu

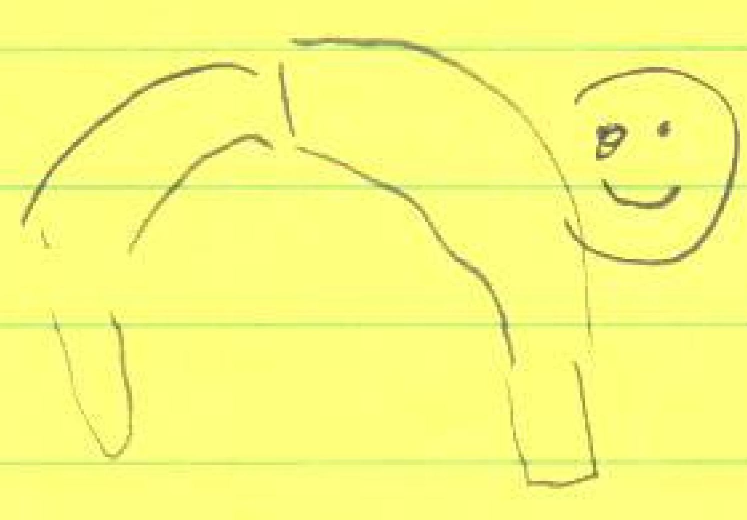
$$I_R \omega_R = I_F \omega_F$$

(4) 

$I_R < I_F$

(5) 

Turn rear, since $I_R > I_F$
Large rear turn 180° mea
small front turn

(6) 

Cat stretches legs to
prepare for landing

Last Time:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

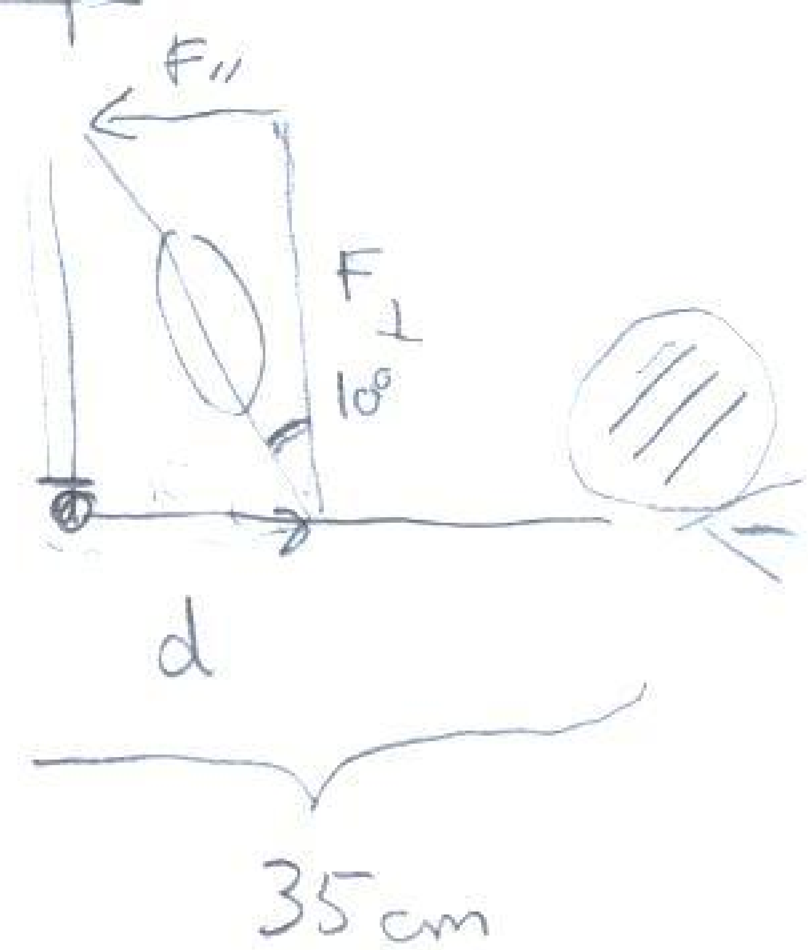
$$\vec{\tau} = |rF\sin\theta|\ (\pm\hat{k}) \qquad \text{(2d only)}$$

either into or out of page, depending on r.h.rule

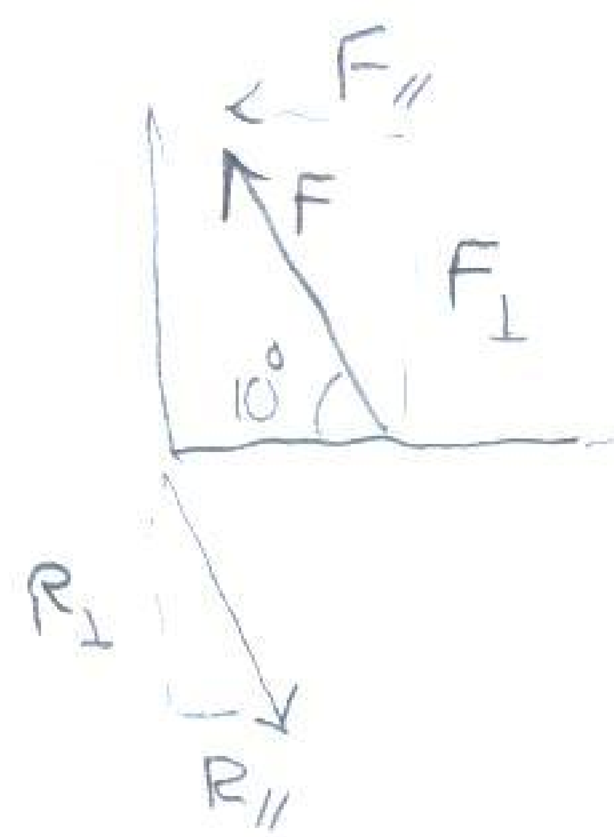
$$\vec{\tau} = |rF_\perp|\ (\pm\hat{k})$$

Example

Arm



$$W_g = 50\,N$$



$$\sum \vec{\tau} = I\cancel{\vec{\alpha}}^{\,0}$$

$$\sum F^y = F_\perp - R_\perp - W_g =$$

$$F_\perp d\hat{k} - W_g l\hat{k} = 0$$

$$\sum F^x = -F_{//} + R_{//}$$

$$F_\perp = 583\,N$$

$$F\cos 10° = 583\,N \Rightarrow 592\,N$$

$$F_{//} = 103\,N \qquad R_\perp = 533\,N$$