

Angular and Linear Quantities

$$\theta = \theta_r \cdot \text{rad} \quad \theta_r = \frac{\Delta x_t}{R} \quad (1)$$

$$\omega = \omega_r \cdot \text{rad} \quad \omega_r = \frac{v_t}{R} \quad (2)$$

$$\alpha = \alpha_r \cdot \text{rad} \quad \alpha_r = \frac{a_t}{R} \quad (3)$$

Const angular acceleration

$$\omega = \omega_o + \alpha t \quad (4)$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2 \quad (5)$$

$$\omega = \omega_o^2 + 2\alpha \Delta\theta \quad (6)$$

Centripetal and Linear acceleration

$$a_{\text{tot}} = \sqrt{a_t^2 + a_c^2} \quad (7)$$

$$a_c = \frac{v^2}{r} = \omega^2 r \quad (8)$$

Moment of Inertia

$$I = M \langle r_{\perp}^2 \rangle_M \quad (9)$$

$$= \sum_i m_i (r_{\perp}^2)_i \quad (10)$$

$$I_A = I_{cm} + Mr_{\perp,cm}^2 \quad (11)$$

$$I_{\text{tot}} = I_1 + I_2 + \dots \quad (12)$$

Torque

$$\vec{\tau} = \vec{r} \times \vec{F} \quad (13)$$

$$|\vec{\tau}| = |rF \sin(\theta)| = |rF_{\perp}| \quad (14)$$

In two dimensions we have

$$\vec{\tau} = |rF \sin(\theta)| \pm \hat{\mathbf{k}} \quad (15)$$

$$= |rF_{\perp}| \pm \hat{\mathbf{k}} \quad (16)$$

$$\sum \vec{\tau} = I\vec{\alpha}_r \quad (17)$$

Rotational Energy

$$KE = \frac{1}{2} I_A \omega_r^2 \quad (18)$$

$$KE = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm} \omega_r^2 \quad (19)$$

$$v_{cm} = \omega_r R \quad (20)$$

$$(\Delta PE_g) = Mg y_{cm} \quad (21)$$

$$W_{ext} = |\tau \Delta\theta| \cdot (\text{sign}) \quad (22)$$

where the sign is positive (negative) if the torque and angle are in the same (opposite) direction (counter clockwise vs. clockwise).

$$\vec{\ell} = \vec{r} \times \vec{p} = \vec{r} \times (m\vec{v}) \quad (23)$$

$$= |r_{\perp} m v| \pm \hat{\mathbf{k}} \quad \text{2D Only} \quad (24)$$

Angular Momentum

$$\vec{\ell} = I\vec{\omega}_r \quad (25)$$

$$\sum \vec{\tau}_{\text{ext}} = \frac{d\vec{\ell}}{dt} \quad (26)$$

$$\sum \vec{\ell}_{\text{before}} = \sum \vec{\ell}_{\text{after}} \quad (\vec{\tau}_{\text{ext}} = 0) \quad (27)$$

$$I_i \vec{\omega}_i = I_f \vec{\omega}_f \quad (\vec{\tau}_{\text{ext}} = 0) \quad (28)$$