(1) Problem Lab #2 (40/40)

Quadratic Formula :

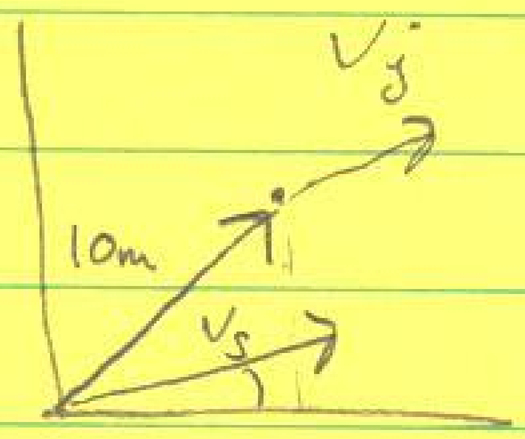
4/4 $t = -2.14$ and $t = 0.14$

$t = 0.102$ and $t = 6.02$

(1) Basic 2D (15/15)

joes init pos $= \vec{r}_{oj} = 10m \cos 45^\circ \hat{i} + 10m \sin 45 \hat{j}$

$r_{oj} = (7.07 \hat{i} + 7.07 \hat{j})\,m$



$\vec{v}_{oj}$ = Joes velocity

$\vec{v}_{oj} = 10 m/s \cos 30^\circ \hat{i} + 10 m/s \sin 30^\circ \hat{j}$
$= (8.66 \hat{i} + 5 \hat{j})\, m/s$

$\vec{V}_{os}$ = Sues velocity

$\vec{V}_{os} = 20 m/s \cos 15^\circ \hat{i} + 20 m/s \sin 15^\circ \hat{j}$
$= 19.3 m/s\ \hat{i} + 5.17\ m/s\ \hat{j}$

$\vec{r}_j(t) = \vec{r}_o + \vec{v}_{oj}\, t$

$$\begin{pmatrix} x_j(t) \\ y_j(t) \end{pmatrix} = \begin{pmatrix} 7.07m \\ 7.07m \end{pmatrix} + \begin{pmatrix} 8.66\ m/s \\ 5.00\ m/s \end{pmatrix} \cdot t$$

2/2 $$\begin{pmatrix} x_j(2) \\ y_j(2) \end{pmatrix} = \begin{pmatrix} 7.07m + 8.66 \cdot m/s \cdot 2s \\ 7.07m + 5.00\, m/s \cdot 2s \end{pmatrix} = \begin{pmatrix} 24.4m \\ 17.0m \end{pmatrix}$$

②

Sue

$$\vec{r}_s(t) = \vec{v}_{os}\, t$$

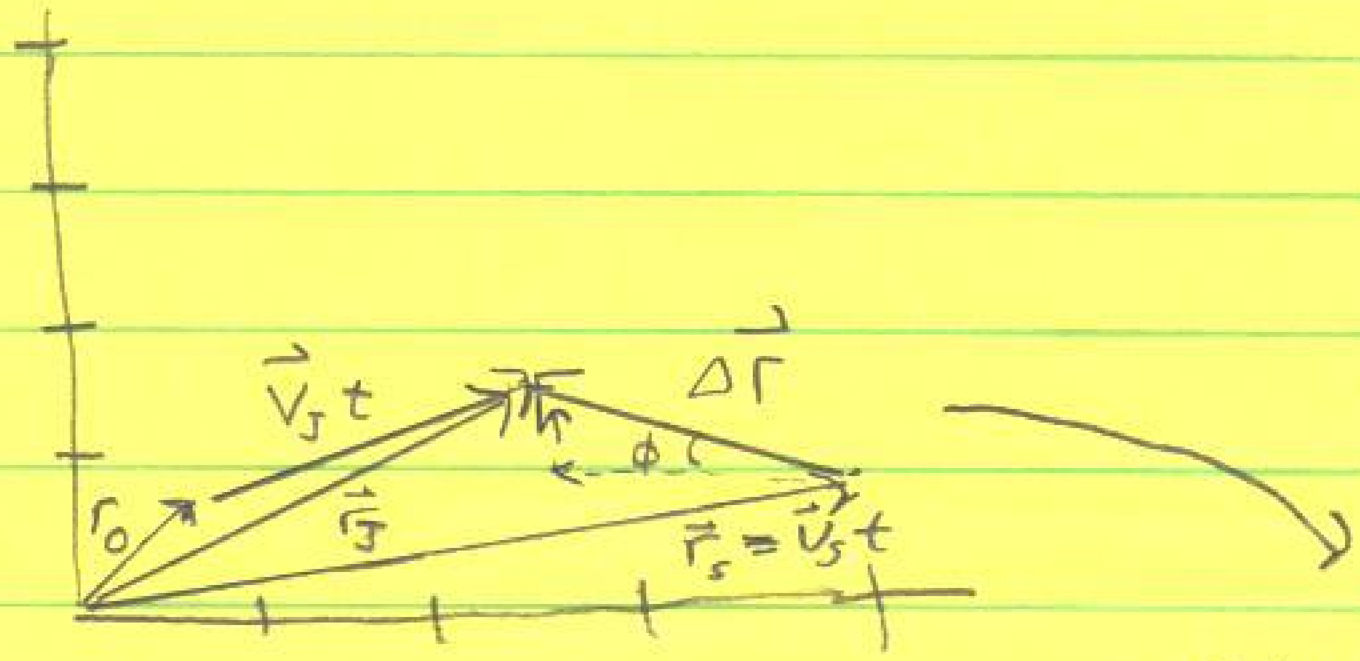
$$\begin{pmatrix} x_s(t) \\ y_s(t) \end{pmatrix} = \begin{pmatrix} 19.3\,m/s \\ 5.17\,m/s \end{pmatrix} (2s)$$

3/3

$$\begin{pmatrix} x_s(2) \\ y_s(2) \end{pmatrix} = \begin{pmatrix} 38.6\ m \\ 10.34\ m \end{pmatrix}$$

Sketch

5/5



Notice it points -14 un[its] in $\hat{i}$ direction and 6.7u[nits] up

So

$$\vec{r}_j = \vec{r}_s + \Delta\vec{r}$$

$$\Delta\vec{r} = \vec{r}_j - \vec{r}_s$$

3/3

$$\Delta\vec{r} = \begin{pmatrix} 24.4m \\ 17.0m \end{pmatrix} - \begin{pmatrix} 38.6m \\ 10.3m \end{pmatrix} = \begin{pmatrix} -14.2m\,\hat{i} \\ 6.7m\,\hat{j} \end{pmatrix}$$

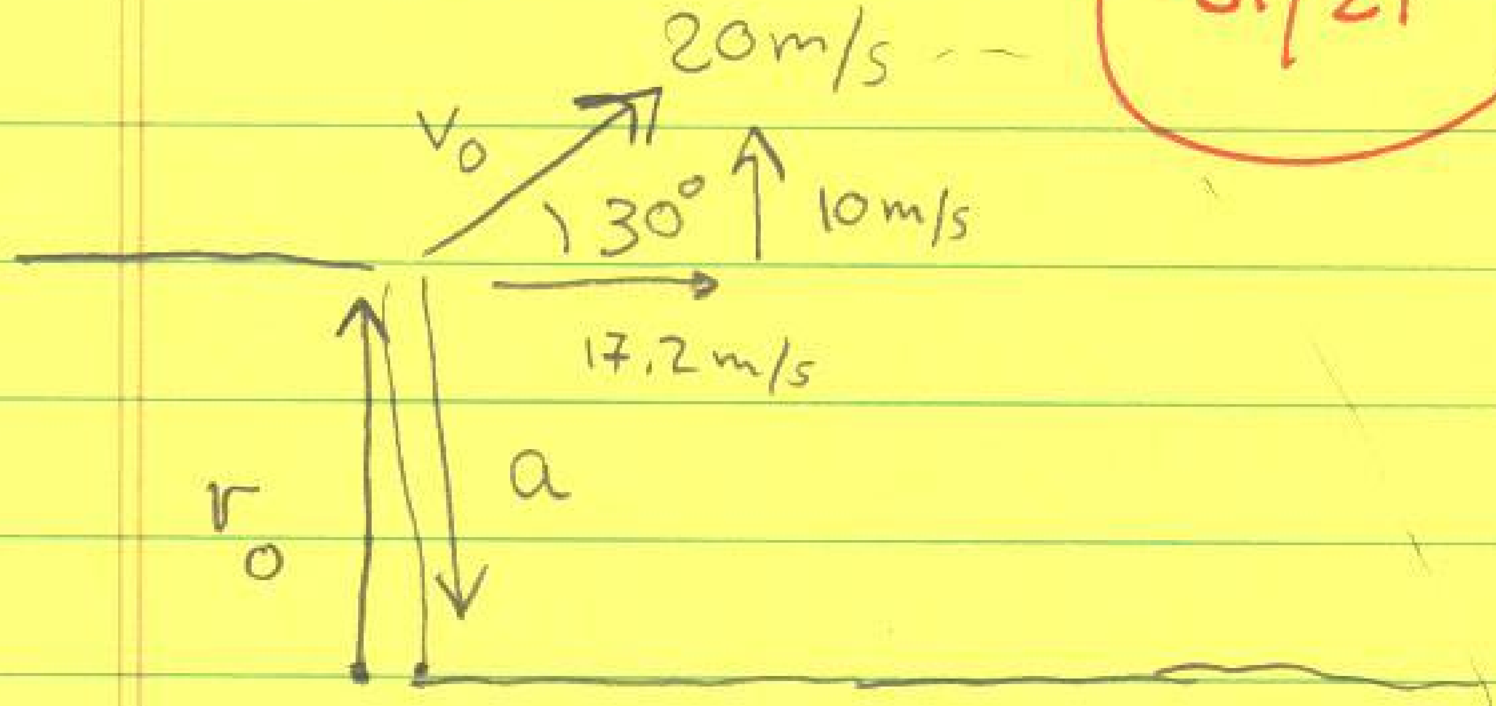
$$|\Delta\vec{r}| = \sqrt{(14.2)^2 + (10.3)^2} = 15.7\ m$$ = "The distance between them"

2/2

$$\phi = \text{Arctan}\left(\frac{6.7}{14.2}\right) \Rightarrow \phi = 25^\circ$$ = "see picture"

(3)

21/21



$\vec{V}_0 = 20\,m/s \cos 30\,\hat{i} + 20m/s \sin 30°\,\hat{j}$

$\vec{V}_0 = 17.2m/s\,\hat{i} + 10m/s\,\hat{j}$

↖ This is the initial velocity

$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

Note

$\vec{r}_0 = 45m\,\hat{j}$ $\vec{a} = -9.8m/s^2\,\hat{j}$

The initial position vec = 45m up

accel = 9.8 m/s² down

So

$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 45m \end{pmatrix} + \begin{pmatrix} 17.2m/s \\ 10\,m/s \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -9.8m/s^2 \end{pmatrix} t^2$$

4

$$\vec{V}(t) = \vec{V}_0 + \vec{a}\,t$$

$$\begin{pmatrix} V_x(t) \\ V_y(t) \end{pmatrix} = \begin{pmatrix} 17.2\,m/s \\ 10\,m/s \end{pmatrix} + \begin{pmatrix} 0 \\ -9.8\,m/s^2 \end{pmatrix} t$$

Note

$v_x(t) = 17.2\ m/s$ is constant

To find the time when rock hits the botto[m]
we look for a t_* such that $y(t_*) = 0$

$$y(t_*) = 45m + 10 m/s\ t_* - \frac{1}{2}(9.8\,m/s^2)\,t_*^2 = 0$$

This equation is the same as asking how long it takes for a rock thrown up with 10 m/s to land on the ground.

$$y(t) = y_0 + V_{0y}\,t + \frac{1}{2} a t^2$$

3/3

$$y(t) = 45m + (10\,m/s)t - \frac{1}{2}(9.8\,\tfrac{m}{s^2})\,t^2$$

↖ upward velocity

~~$t_* = -2.17s$~~ **$t_* = 4.2s$**

(5)

From before

$x(t) = X_0 + V_{0x} t$

$x(t) = 0 + (17.2 m/s)(t)$

$y(t) = 45m + 10m/s \; t - \frac{1}{2}(9.8 m/s^2) t^2$

$x\left(\frac{4.2}{2}\right) = (17.2 \frac{m}{s})\left(\frac{4.2s}{2}\right) = 36.12 m$

$y\left(\frac{4.2}{2}\right) = 45m + 10\frac{m}{s}(2.1s) - \frac{1}{2}\left(9.8\frac{m}{s^2}\right)(2$

3/3

$= 44.4 \; m$

What is the velocity when the rock hits the ground

$\vec{V}(t) = \vec{V}_0 + \vec{a} t$

$$\begin{pmatrix} V_x(t) \\ V_y(t) \end{pmatrix} = \begin{pmatrix} 17.2 m/s \\ 10 m/s \end{pmatrix} + \begin{pmatrix} 0 \\ -9.8 m/s^2 \end{pmatrix} t$$

$V_x(t = 4.21) =$ "x velocity when hits"

$V_y(t = 4.21) =$ " y velocity when hits"

(6)

$V_x = 17.2\,m/s$

$V_y = 10\,m/s - 9.8\,m/s^2\,(4.21s)$

$V_y = -31.258\,m/s$

3/3 the rock is going down when it hits

To find the velocity at half 4.21s:

$V_x\left(t = \frac{4.21}{2}\right) = 17.2\,m/s$

3/3 $V_y\left(t = \frac{4.21}{2}\right) = 10\,m/s - 9.8\frac{m}{s^2}\left(\frac{4.21}{2}s\right)$

$= -10.58\,m/s$

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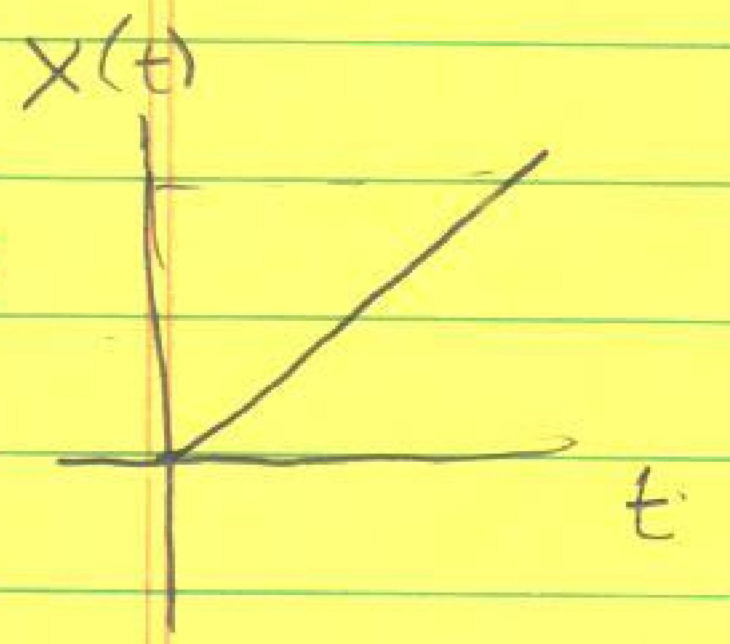

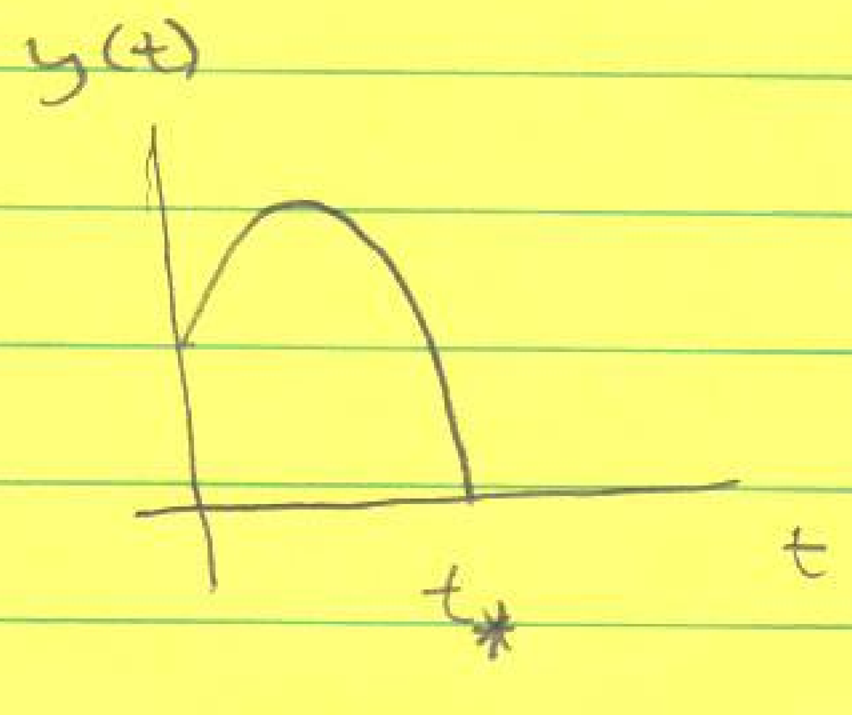

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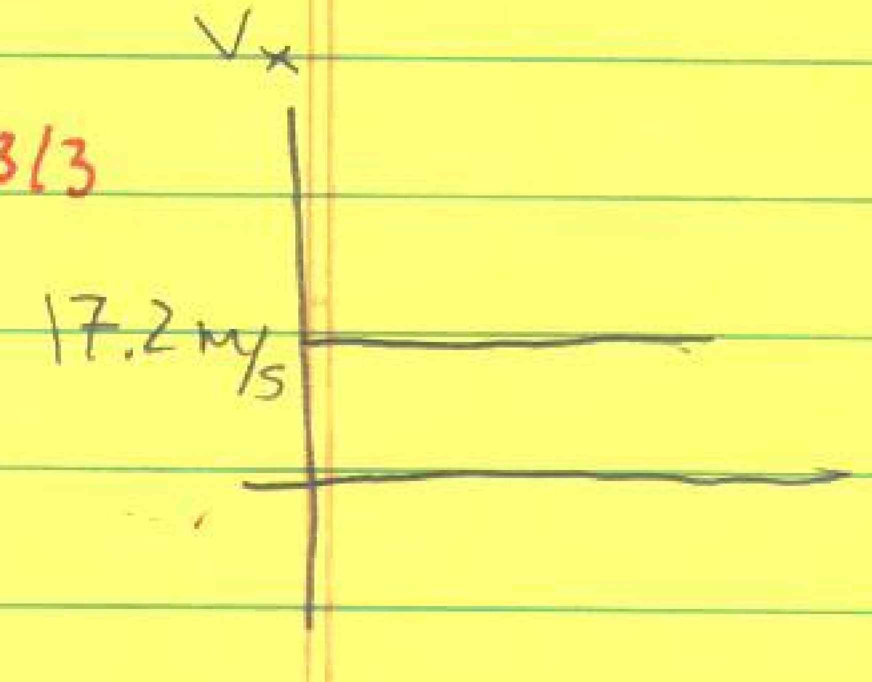

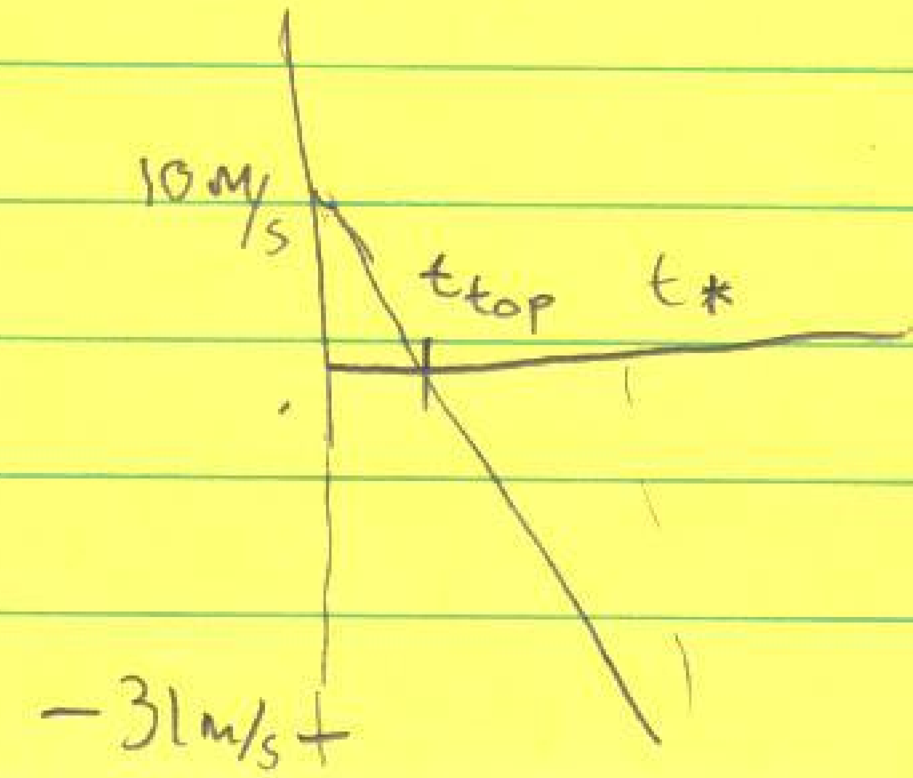

Then find the time when a rock reaches the top.

$V_y(t_{top}) = 0$ = "At top rock is not moving up and down"

3/3

$$V_y(t_{top}) = 10\,m/s - 9.8\,m/s^2\, t_{top} = 0$$

$$t_{top} = \frac{10\,m/s}{9.8\,m/s^2} = 1.02s$$

At the top the rock is only moving in x-direction, which is constant:

$\vec{V} = 17.2\hat{i}$ m/s $\quad |\vec{V}| = 17.2m/s$

More formally

$$\vec{V}(t_{top}) = V_0 + \vec{a}\, t_{top}$$

$$\vec{V}(t_{top}) = \begin{pmatrix} 17.2m/s \\ 10\,m/s \end{pmatrix} + \begin{pmatrix} 0 \\ -9.8\,m/s^2 \end{pmatrix} t_{top}$$

$$V(t_{top}) = \begin{pmatrix} 17.2\,m/s \\ 0 \end{pmatrix}$$

↖ we chose t_{top} so this is zer