In Physics, like every other experimental science, the numbers we "know" and the ones we measure have always some degree of uncertainty. In reporting the results of an experiment, it is as essential to give the uncertainty, as it is to give the best-measured value. Thus it is necessary to learn both the meaning or definition of uncertainty, and the techniques for estimating this uncertainty. Although there are powerful formal tools for this, simple methods will suffice for us. To large extent, we emphasize a "common sense" approach based on asking ourselves just how much any measured quantity in our experiments could be in error.

The experimental error is NOT the difference between your measurement and the accepted "official" value. Error means *your experimental estimate of the range of values within which the "true experimental value" of your measurement is likely to lie.* This range is determined from what you know, or can figure out experimentally, about your lab instruments and methods. It is conventional to choose the error range as that which would comprise about 68% of the results, if you were to repeat the measurement a very large number of times. In fact, we seldom make the many repeated measurements, so the error is usually an estimate of this range. Note that the error range is defined so as to include most of the likely outcomes, but not all. You might think of the process as a wager: pick the range so that if you bet on the outcome being within your error range, you will be right about 2/3 of the time. If you underestimate the error, you will lose money in your betting; if you overestimate it, no one will take your bet!

Error: If we denote quantities that are measured in an experiment by, say, *X, Y* and *Z*, then their corresponding errors would be denoted by *ΔX, ΔY* and *ΔZ*. So if *L* represents the length of a book measured with a meter stick, then you might say the length *L* = 25.1 ± 0.1 cm, where the central value (usually the most probable value) for the length is 25.1 cm and the error, *ΔL* is 0.1 cm. *Both central value and error of measurements must be quoted in your lab writeups.* Note that in this example, the central value is given with just three significant figures. Do not write significant figures (*e.g. L* = 25.08533 ± 0.1 cm) beyond the first digit of the error on the quantity. Failure to round off to 25.1 suggests that you assign additional precision to your number, which is misleading. Since the additional numbers are irrelevant, the reader will wonder, and perhaps misinterpret, why you are providing them.

An error such as that quoted above for the book length is called the absolute error; it has the same units as the quantity itself (cm in the example). You will also encounter relative error, defined as the ratio of the error to the central value of the quantity. The relative error of the book length is *ΔL/L* = (0.1/25.1) = 0.004. Relative error is dimensionless; it should be quoted with as many significant figures as are known for the absolute error. To write *ΔL/L* = (0.1/25.1) = 0.003984 would also be misleading!

Random Error: Random error occurs because of small random variations in the measurement process. Measuring the time of a pendulum's period with a stop watch will

give different results in repeated trials due to small differences in your reaction time in hitting the stop button as the pendulum reaches the end point of its swing. If this error is random, the period T averaged over the individual measurements would get closer to the true value as the number of trials is increased. The correct reported result would be the average for our central value and the error (usually taken as the standard deviation of the measurements). In practice, we seldom take the trouble to make a very large number of measurements of a quantity in this lab; a simple approximation is to take a few (3- 5) measurements and to estimate the range required to encompass about 2/3 of the results. We would then quote one half of this total range for the error, since the error is given for 'plus' or 'minus' variations. In the case that we only have one measurement, even this simple procedure won't work; in this cue you must guess the likely variation from the character of your measuring equipment. For example in the book length measurement with a meter stick marked off in millimeters, you might guess that the error would be about the size of the smallest division on the meter stick (0.1 cm). Note that in the measurement of the pendulum period, you can improve your accuracy by measuring not the time T of one swing, but the time NT of N swings, and divide by N. The error $\_T$ is about the same whether you do one swing or N, so the error in T (as will be shown later on) is $\_T/N$, smaller by N.

Systematic Error: Some sources of uncertainty are not random. For example, if the meter stick that you used to measure the book was warped or stretched, you would never get a good value with that instrument. More subtly, the length of your meter stick might vary with temperature and thus be good at the temperature for which it was calibrated, but not others. When using electronic instruments such voltmeters and ammeters, you obviously rely on the proper calibration of these devices. But if the student before you dropped the meter, there could well be a. systematic error. Estimating possible errors due to such systematic effects really depends on your understanding of your apparatus and the skill you have developed for thinking about possible problems. For example if you suspect a meter might be mis-calibrated, you could compare your instrument with a 'standard' meter - but of course you have to think of this possibility yourself and take the trouble to do the comparison. In this course, you should at least consider such systematic effects, but for the most part you will simply make the assumption that the systematic errors are small. However, if you get a value for some quantity that seems rather far off what you expect, you should think about such possible sources more carefully.

Propagation or Errors: Often in the lab, you need to combine two or more measured quantities, each of which has an error, to get a derived quantity. For example, if you wanted to know the perimeter of a rectangular field and measured the length l and width w with a tape measure, you would have then to calculate the perimeter,
$p = 2 x (l + w)$, and would need to get the error on p from the errors you estimated on l and w, Δl and Δw. Similarly, if you wanted to calculate the area of the field, $A = l x w$, you would need to know how to do this using Δl and Δw.
There are simple rules for calculating errors of such combined, or derived, quantities. Suppose that you have made primary measurements of quantities A and B, and want to get the best value and error for some derived quantity S. For addition or subtraction of measured quantities:

If $S = A + B$, then $\Delta S = \Delta A + \Delta B$.

If $S = A - B$, then $\Delta S = \Delta A + \Delta B$ (also).

Actually, these rules are simplifications, since it is possible for random errors (equally likely to be positive or negative) to partly cancel each other in the error ΔS. A more refined rule that requires a little more calculation for either $S = A \pm B$ is:

$$\Delta S = \sqrt{(\Delta A)^2 + (\Delta B)^2}.$$

This square root or the sum of squares is called "*addition in quadrature*".

For multiplication or division of measured quantities:

If $S = A \times B$ or $S = A/B$, then the simple rule is $\Delta S/S = (\Delta A/A) + (\Delta B/B)$.

Note that for multiplication or division, it is the *relative* errors in A and B which are added. The refined rule which accounts for the tendency for errors in A and B to partly compensate is:

$$\Delta S/S = \sqrt{(\Delta A/A)^2 + (\Delta B/B)^2}.$$

For the case that $S = A^2$ (or A^n), the rule reduces to $\Delta S/S = 2(\Delta A/A)$ (or $\Delta S/S = n(\Delta A/A)$). As an example, suppose you measure quantities A, B and C and estimate their errors ΔA, ΔB and ΔC and calculate a quantity $S = C + \pi A^2/B$. You should be able to derive from the above rules that:

$\Delta S = (\pi A^2/B)[2(\Delta A/A) + (\Delta B/B)] + \Delta C$

(Hint: π is a known constant with no error, and it may be helpful to decompose S as $S = P + C$, with $P = \pi A^2/B$.) As before, the more refined calculation replaces the additions in this formula with addition in quadrature.

Obtaining values from graphs: Often you will be asked to graph results obtained in the lab and to find certain quantities from the slope of the graph. An example would occur for a measurement of the distance l traveled by an air glider on a track in time t; from the primary measurement of l and t, you might plot $2l$ (as y coordinate) vs. t^2 (as x coordinate). In constructing the graph, plot a point at the central x-y value for each of your measurements, Small horizontal bars should be drawn whose length is the absolute error in x (here t^2), and vertical bars with length equal to the error in y (here $2l$). The full lengths of the bars are twice the errors since the measurement may be off in a positive or negative direction. The slope of the graph will be the glider acceleration (remember, $l=(1/2)at^2$). You can determine this slope by drawing the straight line which passes as close to possible to the center of your points; such a line is not however expected to pass through all the points the best you can expect is that it passes through the error bars of most (about 2/3) of the points, In getting the numerical value of the slope of your line, you must of course take account of the scale (and units) of your x- and y-axes.

You can get the error on the slope (acceleration) from the graph as well. Besides the line you drew to best represent your points, draw straight lines which pass near the upper or lower ends of most of your error bars (again, on average), These maximum and minimum slopes will bracket your best slope. Take the difference between maximum and minimum slopes as twice the error on your slope (acceleration) value.