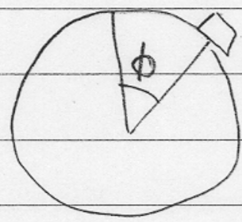
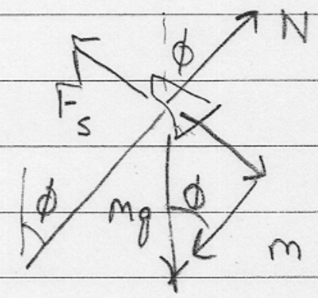
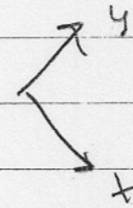
5.87




FBD




So

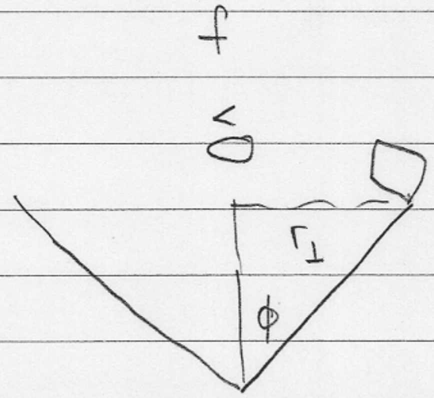
y) $N - mg\cos\phi = m\cancel{a}_y^{\,0}$ x) $mg\sin\theta - \mu_s N = m\cancel{a}_x^{\,0}$

$$mg\sin\theta - \mu_s mg\cos\theta = 0$$

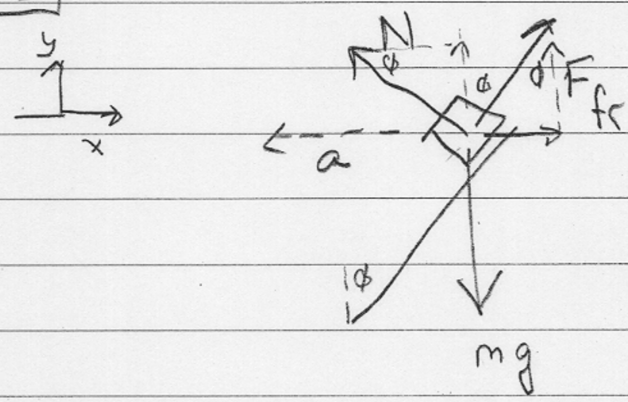
$$\tan\theta = \mu_s$$

$$\phi = \tan^{-1}\mu_s$$ (Too Easy)

5.98




FBD




$v = 2\pi r_\perp f$

$a = \dfrac{v^2}{r_\perp} = (2\pi)^2 f^2 \, r_\perp$

and poi

x|

$-N\cos\phi + \mu_s N \sin\phi = m a_x = -m\dfrac{v^2}{r_\perp}$ ← points in neg x-direc

$-N\cos\phi + \mu_s N\sin\phi = -m(2\pi)^2 f^2 r_\perp$

y|

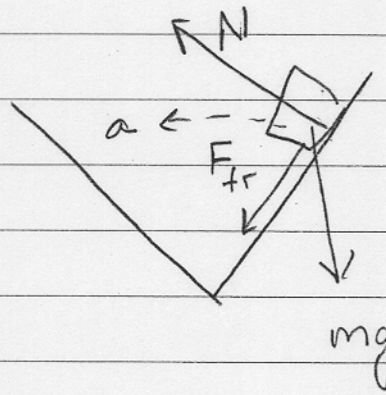
$N\sin\phi + \mu_s N\cos\phi - mg = m\cancel{a_y}^{\,0}$ object not movin vert

$N = \dfrac{mg}{\sin\phi + \mu_s\cos\phi}$

$$+N(\cancel{\mu}\cos\phi - \mu_s \sin\phi) = m(2\pi)^2 f^2 r_\perp$$

$$\frac{g}{(2\pi)^2 f^2}\left(\frac{\cos\phi - \mu_s \sin\phi}{\mu_s \cos\phi + \sin\phi}\right) = r_\perp^{min}$$

Now this is the min $-$ $r_\perp$ to find the max $r_\perp$ we must consider the force of friction holding the object on the cone

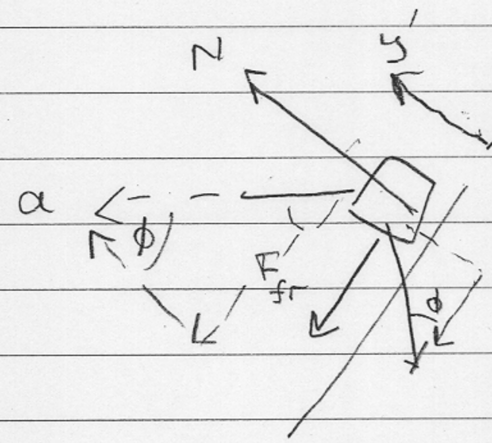



The simplest way to do this is just to change the sign of $F_{fr}$, i.e send $\mu_s \to -\mu_s$

$$\frac{g}{(2\pi)^2 f^2}\left(\frac{\cos\phi + \mu_s \sin\phi}{\sin\phi - \mu_s \cos\phi}\right) = r_\perp^{max}$$

$\to$ See next page

This is pretty slick to illustrate a different approach, Lets use a different coordinates system $x', y'$




$\underline{x'}$ $-\mu_s N - mg\cos\phi = m a_{x'}$

$-\mu_s N - mg\cos\phi = -m\frac{v^2}{r}\sin\phi$ $\qquad \frac{v^2}{r_\perp} = (2\pi)^2 f^2 r_\perp$

$\underline{y'}$ $N - mg\sin\phi = m\frac{v^2}{r}\cos\phi$

$N = mg\sin\phi + m(2\pi)^2 f^2 r_\perp \cos\phi$

So

$-\mu_s(mg\sin\phi + m(2\pi)^2 f^2 r_\perp\cos\phi) = -m(2\pi)^2 f^2 r_\perp \sin\phi - mg\cos\phi$

Or

$\mu_s g\sin\phi + g\cos\phi + \mu_s(2\pi)^2 f^2 r_\perp\cos\phi = (2\pi)^2 f^2 r_\perp \sin\phi$

$\frac{g}{(2\pi)^2 f^2}\left(\frac{\cos\phi + \mu_s\sin\phi}{\sin\phi - \mu_s\cos\phi}\right) = r_\perp$ (Agrees w $\mu_s \to -\mu_r$)

$$\vec{r} = 2\cos(3t)\,\hat{i} + 2\sin 3t\,\hat{j} \qquad (1)$$

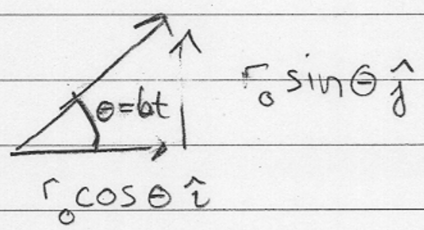
In symbols :

$$\vec{r} = r_0 \cos(bt)\,\hat{i} + r_0 \sin bt\,\hat{j} \qquad r_0 \equiv 2m \quad b = 3\,rad/s \qquad (1^*)$$

$$\vec{v} = \frac{d\vec{r}}{dt} = -r_0 \sin(bt)\, b\,\hat{i} + r_0 \cos(bt)\, b\,\hat{j} \qquad (2)$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = -r_0 \cos(bt)\, b^2\,\hat{i} - r_0 \sin(bt)\, b^2\,\hat{j} \qquad (3)$$

So show that Eq $(1^*)$ represents a circle




(A)

We show that this is a circle we show $|\vec{r}| = const$

$$|\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{r_0^2 \cos^2 bt + r_0^2 \sin^2 bt}$$

$$= r_0 \sqrt{\cos^2 + \sin^2} = \boxed{r_0 = 2m = |\vec{r}|}$$

Thus the radius is constant and indep of time

(B)

$$\vec{V} = -\overbrace{r_0 b}^{6\,m/s} \sin(bt)\,\hat{i} + r_0 b \cos(bt)\,\hat{j}$$

$$\vec{V} = -6\,m/s \sin(3t)\,\hat{i} + 6\,m/s \cos(3t)\,\hat{j}$$

(C)

$$\vec{a} = -\overbrace{(r_0 b^2)}^{18\,m/s^2} \sin bt\,\hat{i} - (r_0 b^2) \cos bt\,\hat{j}$$

$$\vec{a} = -18\,m/s^2 \sin(3\tfrac{1}{s}t) - (18\,m/s^2) \cos 3\tfrac{1}{s}t\,\hat{j}$$

(D)

$$|\vec{V}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(-6\sin)^2 + (6\cos)^2}$$

$$= 6\sqrt{s^2 + c^2} = 6\,m/s = r_0 b = |\vec{V}|$$

(E)

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2} = \sqrt{(-r_0 b^2)^2 \sin^2 + (-r_0 b^2)^2 \cos^2}$$

$$= r_0 b^2 \sqrt{s^2 + c^2} = \boxed{r_0 b^2 = |\vec{a}|}$$

$$\vec{a} = (2m)\cdot\left(3\tfrac{1}{s}\right)^2 = 18\,m/s^2$$

(F)

$$|\vec{a}| = r_0 b^2 \qquad \frac{|\vec{V}|^2}{|\vec{r}|} = \frac{(r_0 b)^2}{r_0} = r_0 b^2$$

So $|\vec{a}| = \dfrac{|\vec{V}|^2}{|\vec{r}|}$

For part G

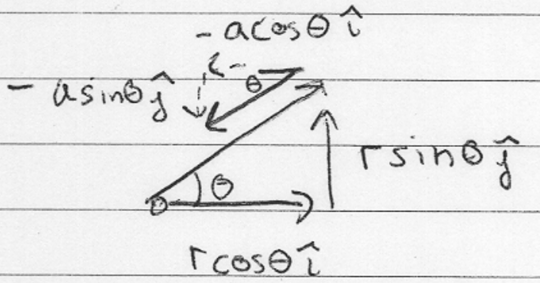
$$\vec{a} = a\left(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right)$$

Compare

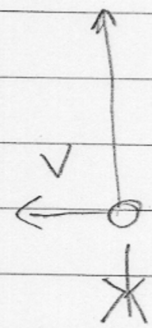
$a = r_0 b^2$ $\qquad \theta = bt \qquad b = 3\,\frac{\text{rad}}{\text{s}}\ t$

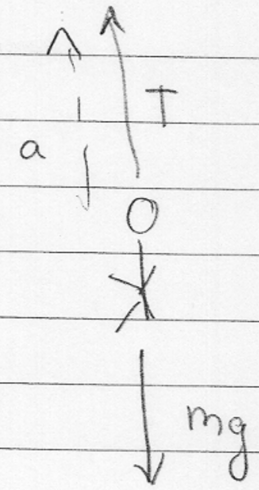
$$\vec{r} = r\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right) = r\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$

The two vectors are opposite in direction



5.55




$$T - mg = \frac{mv^2}{R}$$

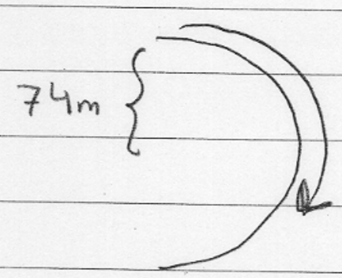
$$F - mg = \frac{mv^2}{l}$$

$$\sqrt{l\left(\frac{F - mg}{m}\right)} = v$$

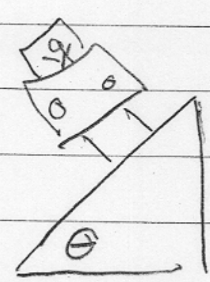
$m = 73\,kg$ $\quad F = 1500\,N$

$l = 5.5\,m$

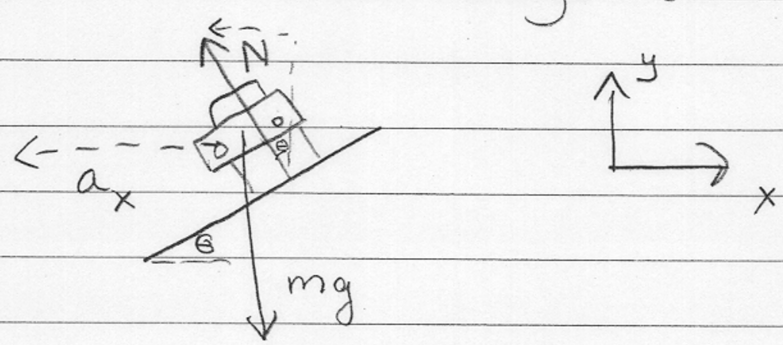
5.59




$V = 75\,km$



The angle $\theta$ is chosen so that w/out friction the car stays on the road




Then writing N Laws

$x\rfloor \quad -N\sin\theta = ma_x$ $\qquad$ $y\rfloor \quad N\cos\theta - mg = m\cancel{a_y}^{\,0}$

$-N\sin\theta = -m\dfrac{v^2}{R}$ $\qquad$ $N = \dfrac{mg}{\cos\theta}$

$-\cancel{m}g\dfrac{\sin\theta}{\cos\theta} = -\cancel{m}\dfrac{v^2}{R}$

$\tan\theta = \dfrac{v^2}{gR} = 0.5983$ $\qquad$ $V_0 = \underline{75\ km/h} \simeq 20.83\ m/s$

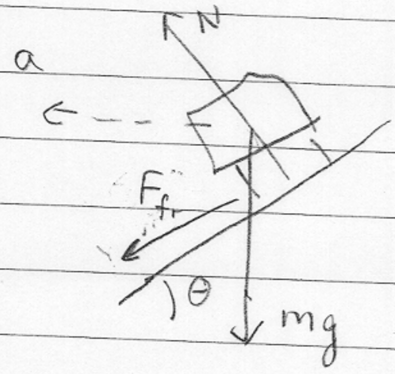
$\simeq 46$ mph

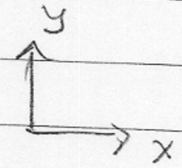
$R = 74m$

$\tan\theta = 0.5983$ $\qquad$ $g = 9.8m/s^2$

$\theta = 30.89^\circ$

Now to find $v_{max}$

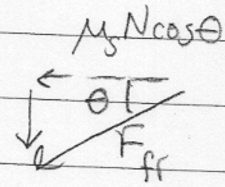


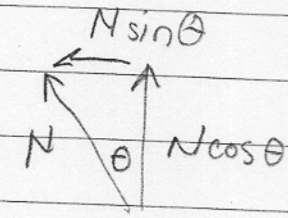




a points in

x|

$$-\mu_s N\cos\theta - N\sin\theta = -m\frac{v^2}{R}$$




y|

$$N\cos\theta - \mu_s N\sin\theta - mg = m\cancel{a_y}^{\,0}$$

$$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}$$

Now

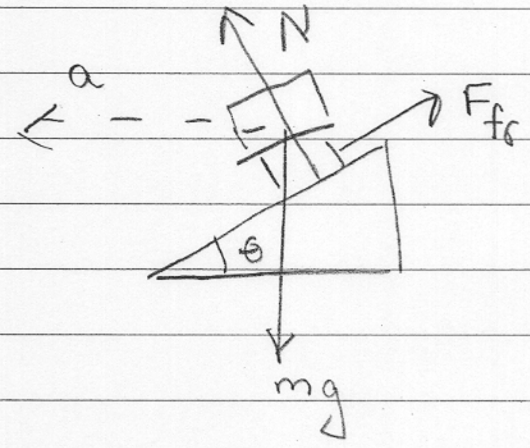
$$N(\mu_s\cos\theta + \sin\theta) = m\frac{v^2}{R}$$

$$\cancel{m}g\,\frac{(\mu_s\cos\theta + \sin\theta)}{\cos\theta - \mu_s\sin\theta} = \cancel{m}\frac{v_{max}^2}{R}$$

$$Rg\left(\frac{\mu_s + \tan\theta}{1 - \mu_s\tan\theta}\right) = v_{max}^2$$

$$U^2_{max} = Rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s \tan\theta}\right)$$

again for min speed, the sign of friction is reversed




(see picture)

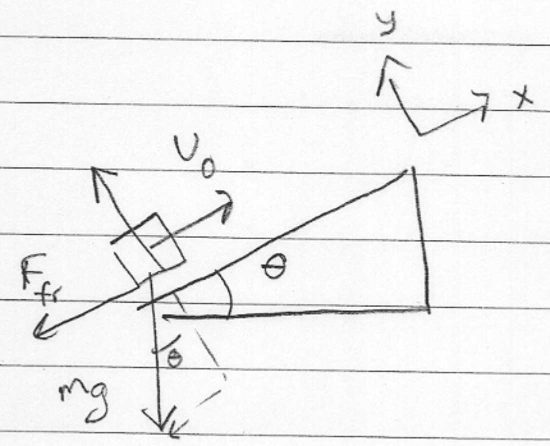
So we can either redo the steps or simply make the replacement $\mu_s \to \mu_s$

$$V^2_{min} = Rg\left(\frac{\tan\theta - \mu_s}{1 + \mu_s \tan\theta}\right)$$

Note $\tan\theta = v_0^2/Rg = 0.5983$ $\mu_s = 0.28$ $R = 75$ km

$v_{min} = 14.061$ m/s $= 50.62$ km/h $\simeq 51$ km/h

$v_{max} = 27.661$ m/s $= 99.58$ km/h $= 100$ km/h




$$N - mg\cos\theta = m\cancel{a_y}^{\,0} \qquad -\mu_k N - mg\sin\theta = ma_x$$

$$N = mg\cos\theta \qquad -(\mu_k \cancel{m}g\cos\theta + mg\sin\theta) = \cancel{m}a$$

$$-g(\mu_k\cos\theta + \sin\theta) = a_{up}$$

$$\cancel{V_f^2}^{\,0} = V_0^2 + 2a\Delta x$$

$$-\frac{V_0^2}{2a} = L$$

$$\frac{V_0^2}{2g(\mu_k\cos\theta + \sin\theta)} = L = 0.8027\,\text{m}$$

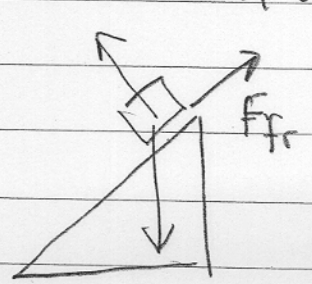
$\theta = 28^\circ$

$= 0.4887$ rad

$V_0 = 3.1$ m/s

$\mu_k = 0.16$

The Downhill:




$$N - mg\cos\theta = m\cancel{a_y}^{\,0} \qquad +\mu_k N - mg\sin\theta = ma_x$$

$$a_x = -g(\sin\theta - \mu_k\cos\theta)$$

To find the time

$$\frac{1}{2} a_{up} t_{up}^2 + v_0 t_{up} = L$$

$$\frac{1}{2} a_{dn} t_{dn}^2 = \Delta x = x_f - x_i = -L$$

$$t_{dn} = \left( -\frac{2L}{a_{dn}} \right)^{1/2} = \left[ \frac{2L}{g(\sin\theta - \mu_k \cos\theta)} \right]^{1/2}$$

$$t_{dn} = \left[ \frac{2 v_0^2}{2g^2 (\sin\theta + \mu_k \cos\theta)(\sin\theta - \mu_k \cos\theta)} \right]^{1/2}$$

$$t_{dn} = \frac{v_0}{g} \frac{1}{(\sin^2\theta - \mu_k^2 \cos^2\theta)^{1/2}} = 0.706528 \text{ s}$$

$$v_0 + a t_{up} = v = 0$$

$$t_{up} = \frac{-v_0}{a} = \frac{v_0}{g(\mu_k \cos\theta + \sin\theta)} = 0.5179 \text{ s}$$

$$t_{up} + t_{down} = 1.22 \text{ s}$$