Yesterday - Gravity

$\vec{F}_{ba}$

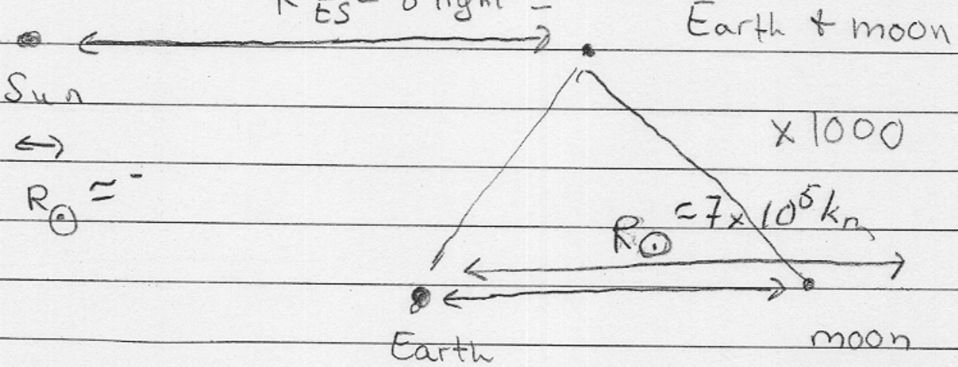
r_{ba}

$$\vec{F}_{Ba} = \frac{G m_a m_b}{r^2}\, -\hat{r}_{ba}$$

$$g = \frac{G m_E}{R_E^2} = 9.8\,m/s^2$$

Solar System:

$R_{ES} \simeq 8$ light min $\simeq 1.5\times10^8$ km



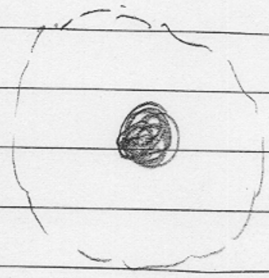
$R_{Em} \sim 1$ light sec $\sim 4\times10^5$ km $\sim 100\, R_E$

Kepler Law

$$\frac{r_1^3}{T_1^2} = \frac{r_2^3}{T_2^2}$$

Then: Consider a Sattelite orbitting the earth at $2R_E$

What is the force of gravity



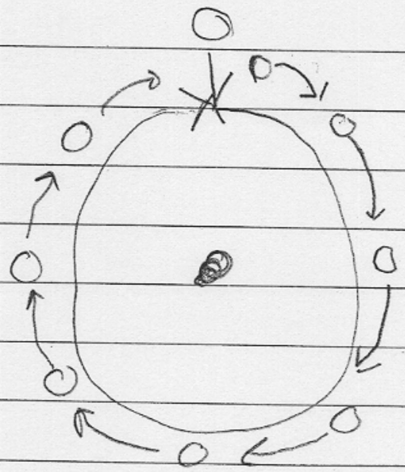
$$F_g = \frac{G M_E m}{(2R_E)^2}$$

not zero!

$$= \frac{1}{4} \underbrace{\frac{G M_E}{R_E^2}}\, m =$$

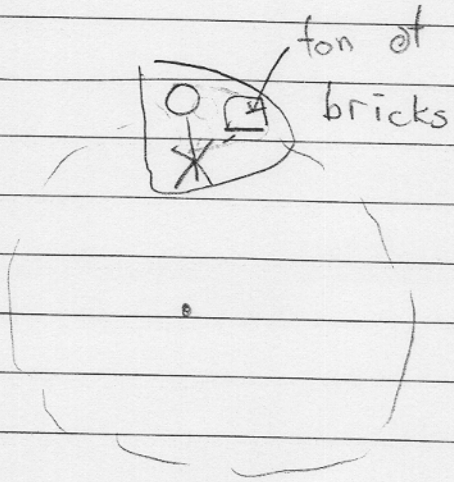
$$F_g = \frac{1}{4} \, g \, m$$

So why are we weightless in space?



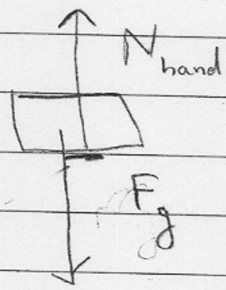
So when we throw a ball in orbit it "falls" around the earth

Derivation



$$F_g = \frac{mv^2}{r}$$

Free body diagram:

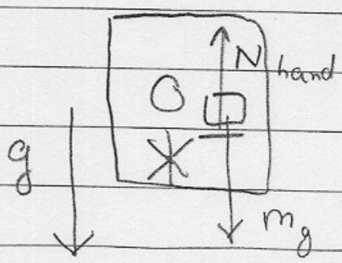


$$N_{hand} - F_g = \frac{mv^2}{r}$$

So

$$N_{hand} = 0$$

Also Consider a person holding up a ton of bricks in a free falling elevator

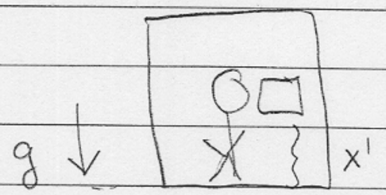


$$N_{hand} - mg = ma$$

$$N_{hand} - \cancel{mg} = -\cancel{mg}$$

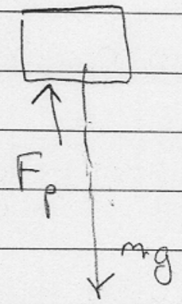
$$N_{hand} = 0$$

Summary: In an accelerating "appropriate" free falling frame the effect of gravity is null.



$$x = x' - \overbrace{\frac{1}{2}gt^2}^{\text{height of elevator floor}}$$

$$a = \ddot{x} = \ddot{x}' - g = a' - g$$



$$F_p - mg = ma$$

$$F_p - mg = m(a' - g)$$

$$F_p - \cancel{mg} = ma' - \cancel{mg}$$

$$F_p = ma' \leftarrow \text{Newtons Law w/out gravity}$$

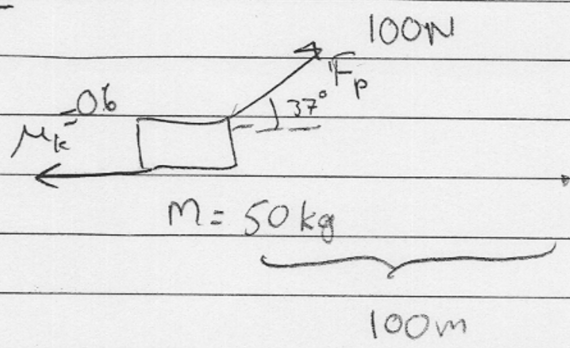
Summary there is an free-falling coordinate system where the Force of gravity can be neglected (locally)

↑ The equivalence principle, the start of General relativity

Work - Constant Force

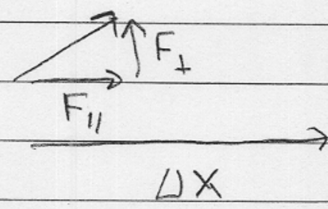
$W = F_{\parallel}\,\Delta x = F\,\Delta x\cos\theta$ ← Force parallel to displacement

Ex 1



What is the work done

$W_p = F_{\parallel}\,\Delta x$



$W_p = F_p\cos\theta\,\Delta x$

$W_p = (100N)\cos 37° \cdot 100m$

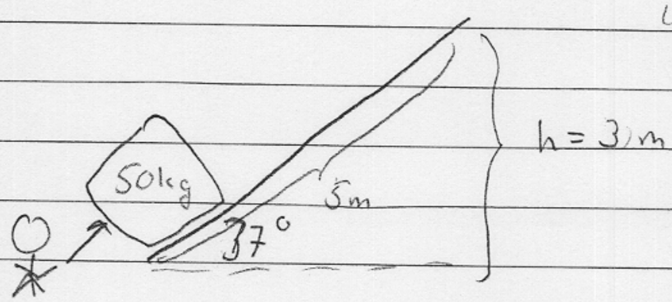
$W_p = 8000\ \underbrace{N\cdot m}$

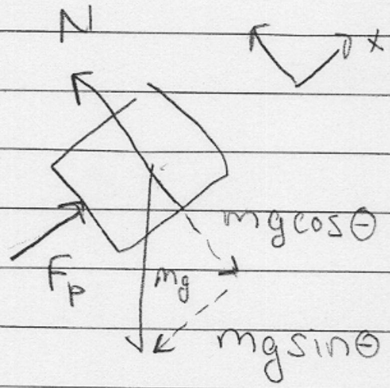
$1N\cdot m = 1J$

Other units

$1\,erg = \underbrace{(1g)}_{m}\,\underbrace{\frac{cm}{s^2}}_{a}\cdot\underbrace{cm} = 10^{-7}\,J$

Ex2: A fearless graduate student, pushes a 50 kg crate up a slope with constant (slow) speed





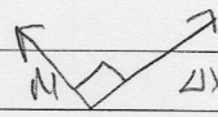
$N - mg\cos\theta = m\cancel{a_y}^{\,0}$ $\llcorner y$

$F_p - mg\sin\theta = m\cancel{a_x}^{\,0}$

So $N = mg\cos\theta$ $F_p = mg\sin\theta$

Three forces each one does a certain amount of work:

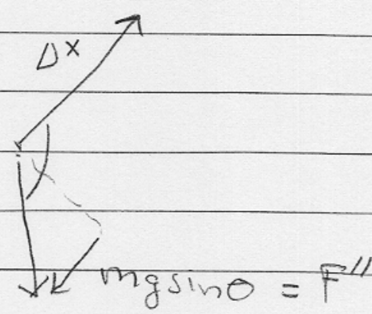
$W_N = F_{//}\Delta x = N\,\Delta x\,\cos\theta$



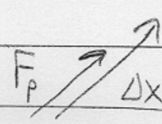
$= N\,\Delta x\,\cos 90° = 0$

$W_g = F_g^{//}\,\Delta x = -\underbrace{mg\sin\theta\cdot L}_{h}$

$W_g = -mgh \simeq -1500\,J$



$W_p = F_{p\,//}\,\Delta x = F_p\,L = \underbrace{mg\sin\theta\,L}_{h}$



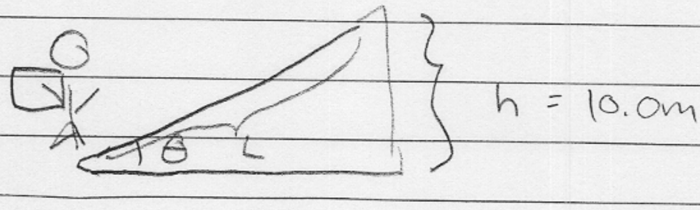
$W_p = +mgh \simeq +1500\,J$

So total work

$$W_{TOT} = \cancel{W_N}^{0} + W_g + W_f$$

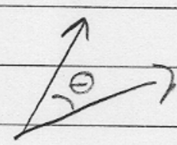
$$W_{TOT} = -1500 + 1500 = 0$$

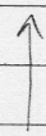
Ex2



Scalar Product or Dot Product

$\vec{A} \cdot \vec{B} = A B \cos\theta$





= "(mag A) x (mag B) x cos angle between"

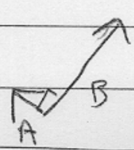
$W = \vec{F} \cdot \Delta\vec{x}$ ← Force dotted with displaceme

The Dot prod enjoys:

(1) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ Comuttatie

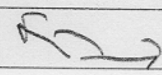
(2) $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{C}$ Associative

(3) Whenever two vectors are orthogonal



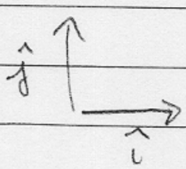
$\vec{A} \cdot \vec{B} = |A||B| \cos 90^\circ = 0$

(4) Whenever they oppose each other



then $\theta > 90^\circ$ and $\vec{A} \cdot \vec{B} < 0$

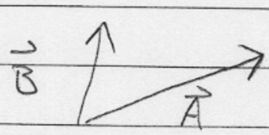
$$\begin{cases} \hat{i}\cdot\hat{i} = (1)(1)\cos 0^\circ = 1 \\ \hat{j}\cdot\hat{j} = 1 \\ \hat{k}\cdot\hat{k} = 1 \end{cases}$$



$$\begin{cases} \hat{i}\cdot\hat{j} = 1\cdot 1 \cos 90^\circ = 0 \\ \hat{i}\cdot\hat{k} = 0 \\ \hat{j}\cdot\hat{k} = 0 \end{cases}$$

Now consider:

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$



$$\vec{B} = B_x\hat{i} + B_y\hat{j}$$

① $\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j})\cdot(B_x\hat{i} + B_y\hat{j})$

$$= A_x B_x \overbrace{\hat{i}\cdot\hat{i}}^{1} + A_y B_x \cancel{\hat{j}\cdot\hat{i}} + A_x B_y \cancelto{0}{\hat{i}\cdot\hat{j}} + B_y A_y \overbrace{\hat{j}\cdot\hat{j}}^{1}$$

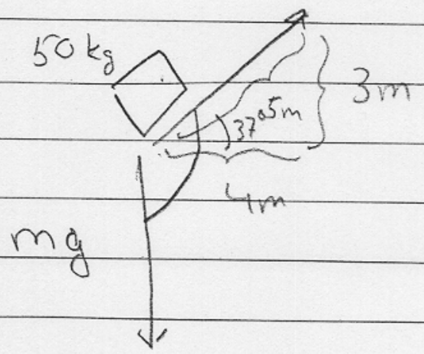
$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y$$

② $\vec{A}\cdot\hat{i} = A_x$

$\vec{A}\cdot\hat{j} = A_y$

③ $\vec{A}\cdot\vec{A} = A_x^2 + A_y^2 = |\vec{A}|^2$

Example:



$\vec{\Delta x} = 4m\,\hat{i} + 3m\,\hat{j}$

$\vec{\Delta x} \cdot \vec{\Delta x} = 4m \cdot 4m + 3m \cdot 3m$

$\sqrt{\Delta x \cdot \Delta x} = \sqrt{4^2 + 3^2} = 5$

$\vec{F} = -mg\,\hat{j} = (-50kg)(9.8)\,\hat{j}$

$\vec{F} \simeq -500N\hat{j}$

So

$|\vec{F}| = 500N$

$W = \vec{F} \cdot \vec{\Delta x} = (-500\hat{j})(4m\hat{i} + 3m\hat{j})$

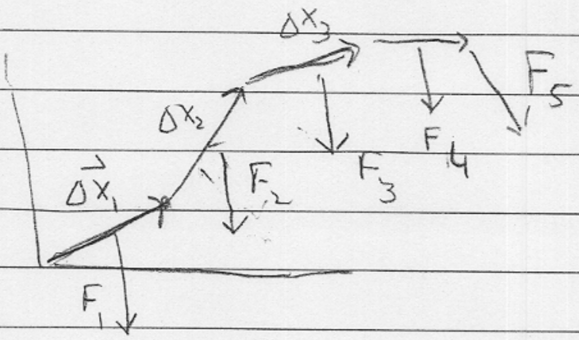
$= -500 \cdot 3 \; Nm = -1500\,Nm$

$W = F\,\Delta x \cos\theta = (500N)(5m)\cos(90^\circ + 37^\circ)$

$= (500N)(5m)\,[\underbrace{-\sin 37^\circ}_{3/5}]$

$= -500N \cdot 3m = -1500J$

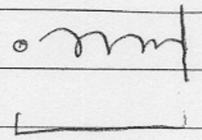
Non-constant Force:

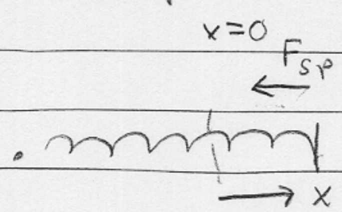


$$W_{TOT} = \sum_i \vec{F}_i \cdot \vec{\Delta x}_i = \sum F_i \cos\theta_i \, \Delta x_i$$

$$W_{TOT} = \int_a^b \vec{F} \cdot d\vec{x} = \int_a^b \overbrace{F \cos\theta}^{\text{a function of } x} \, dx$$

Example: Work done by a person stretching a spring

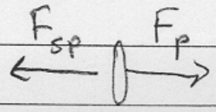




Force is a fcn of x

$$F_{sp} = -kx$$

Newton Law



$$-kx + F_p = m\cancel{a}^{\,0} \quad \text{so} \quad F_p = kx \quad \text{(duh!)}$$

So the work done stretch the

$$F_p = \int [F_p(x)\,\hat{i}] \cdot [dx\,\hat{i}]$$

$$W_p = \int_0^{x_f} \vec{F}_p \cdot d\vec{x} \qquad \xrightarrow[x_f]{}$$

$$= \int_0^{x_f} kx \; dx$$

$$W_p = \frac{1}{2}kx^2 \Big|_0^{x_f} = \frac{1}{2}kx_f^2$$