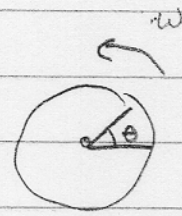
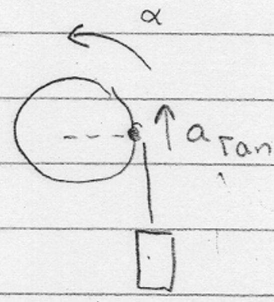
Last Time

(1) $\omega = \dfrac{d\theta}{dt}$ = "rad per sec"



(2) $\alpha = \dfrac{d\omega}{dt}$ = "rad per sec per sec"



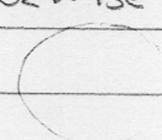
$\alpha = \dfrac{d^2\theta}{dt^2}$ = how fast the spinning is speeding up

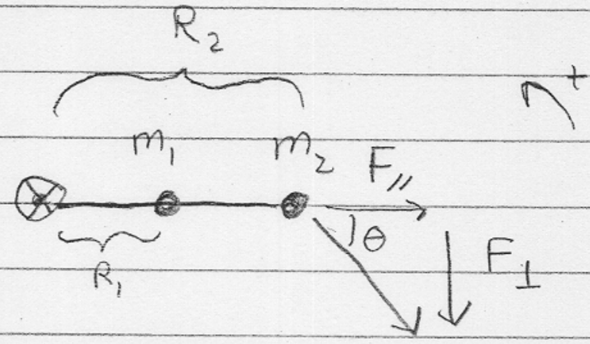
$\alpha = \dfrac{a_{Tan}}{R}$ $\qquad a_{Tan} = \alpha R$

(3) What causes accel?

+ if causes counter clockwise rotati

$\tau = R F_{\perp} \cdot \{\pm\} = RF\sin\theta \cdot \{\pm\}$





$\tau = -R_2 F \sin\theta$

④ How much accel?

$$\sum \tau_{ext} = I\alpha \qquad \text{analog of } F=ma$$

$$I = \sum_i m_i r_i^2 = m_1 R_1^2 + m_2 R_2^2 \quad \text{becomes} \quad I = \int dm\, R^2$$

- Holds for rotation about a fixed pivot point
- Or around the CM for a accelerating abject.

Examples of Moment of Inertia

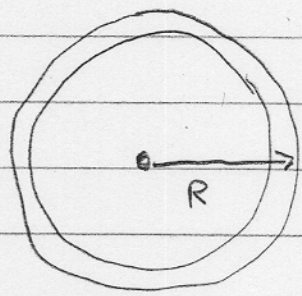
$$I = \sum_i m_i r_{i\perp}^2 = \sum m_i \frac{\sum m_i r_{i\perp}^2}{\sum m_i} \qquad \langle r^2 \rangle_m = \frac{\sum m_i r_i^2}{M_{TOT}}$$

$$= M_{TOT} \langle r_\perp^2 \rangle_m \qquad \langle r^2 \rangle \Rightarrow \frac{m_1 r_1^2 + m_2 r_2^2}{m_1 + m_2}$$

- So the moment of inertial is the total mass times the "mass weighted average" of perpendicular radius squared

$$I = M_{TOT} \langle x^2 + y^2 \rangle_m$$

Ex 1



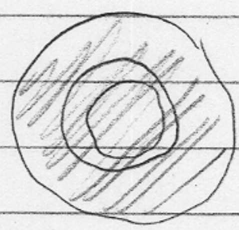
$$I = M \langle r^2 \rangle_m$$

$$I = MR^2$$

more formally:

$$I = \int dm\, r^2 = \int dm\, R^2 = R^2 \int dm = MR^2$$

Ex 2



$$I = M \langle r^2 \rangle_m < MR^2$$

Ans: $I = \frac{1}{2} MR^2$

Computation:

$$\sigma = \text{Density} = \frac{M}{A} = \frac{M}{\pi R^2}$$

$$I = \sum_{\text{rings}} dm\, r^2$$

$$I = \sum_{\text{rings}} \sigma \underbrace{dA}_{\substack{\text{area} \\ \text{of a ring} \\ = 2\pi r^2 dr}} r^2 = \int_0^R \sigma\, 2\pi r\, dr \cdot r^2 = \sigma \int_0^R 2\pi r^3\, dr$$

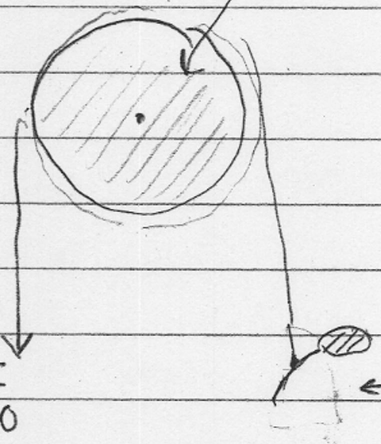
So

$$I = \sigma \frac{\pi r^4}{2}\Bigg|_0^R = \sigma \frac{\pi R^4}{2}$$

Or $$I = \frac{M}{\pi R^2}\,\frac{\pi R^4}{2} = \frac{1}{2} M R^2 = I$$

What is moment of Inertial Good For?

10 tons of concrete $= M$ $\quad R = 0.5\,cm$



- Consider two cases:
  - → Hollow Cylinder $I = MR^2$
  - → Solid Cylinder $I = \frac{1}{2} M R^2$

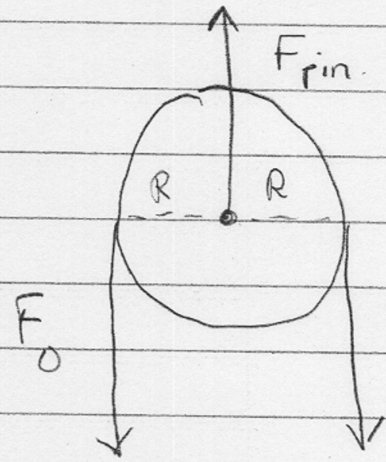
- Want to determine the acceleration of the feather

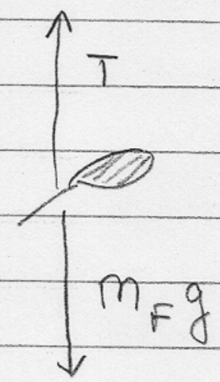
- Tension clearly not the same on both sides

- Most of the force will be used to acceerlerate the wheel

Solution

① Draw a free body diagram for each moving or rotating piece





② write down $F_{net} = ma$ and $\tau_{net} = I\alpha$

→ $T - m_F g = m_F a$

→ $\tau_{F_0} + \tau_{pin} + \tau_T = I\alpha$

$I = CMR^2$

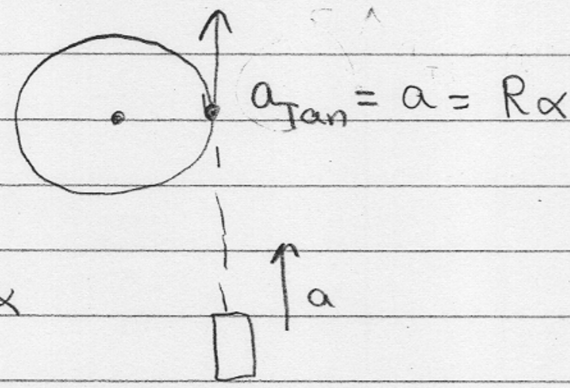
$C = \frac{1}{2}$ for solid

$= 1$ for hollow

$F_0 R + \cancel{F_{pin}} \cdot 0 + TR \cdot (-) = CMR^2 \alpha$

no moment arm

③ Relate the angular and tangential accelerations and Solve



① $T - m_F g = m_F a$

$F_0 R - TR = CMR^2 \alpha$

$a = R\alpha$

② $F_0 R - TR = CMR\,a$

Unknowns T, a

$T = m_F g + m_F a$

So

$$F_0 \cancel{R} - (m_F g + m_F a)\cancel{R} = CM\cancel{R}a$$

$$F_0 - m_F g = Cma + m_F a$$

$$F_0 - m_F g = (Cm + m_F)a$$

$$\boxed{\frac{F_0 - m_F g}{Cm + m_F} = a}$$

Comments

→ For $M = 10$ tons your acceleration will be small unless your force is very large

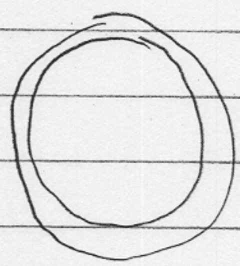
$$a = \frac{F_0 - m_F g}{CM + m_F} \simeq \frac{F_0}{CM}$$

$m_F \ll M$

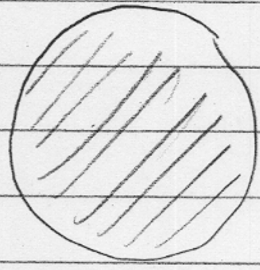
$F_0 \gg m_F g$

Your force is accelerating the wheel

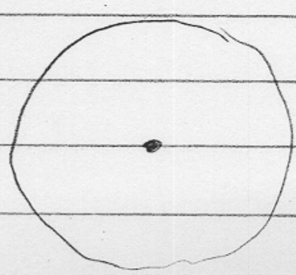
→ In order to get a large acceration want C = smal



$C=1.$ $I = MR^2$ Smaller accelerat



$C = \frac{1}{2}$ $I = \frac{1}{2} MR^2$ ← Larger aceleratio



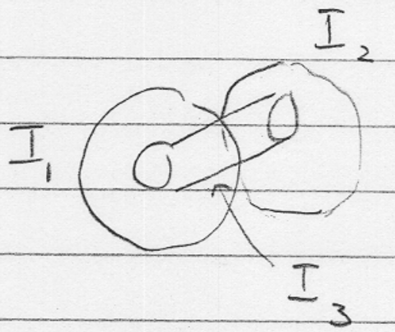
Optimal Case ↔ put mass in middle

$\langle r^2 \rangle \sim$ small

$I \sim$ small

In this case as the wheel turns the mass only moves a little bit.

Moment of Inertia Tricks



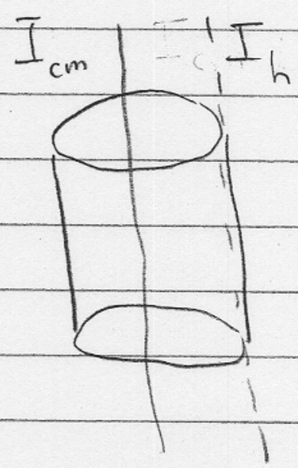
- Two wheels M_w, R_w
- A cylinder M_c, R_c

①

$$I_{TOT} = I_1 + I_2 + I_3$$ ← moment of inertia is a sum

$$= \frac{1}{2}M_w R_w^2 + \frac{1}{2}M_w R_w^2 + \frac{1}{2}M_c R_c^2$$

② Parallel Axis Theorem



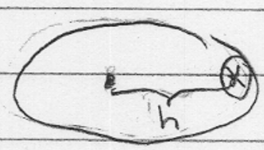
You know I_{cm}

moment inertia arou a "parallel axis"

$I_{cm} = \frac{1}{2}MR^2$ and want to know I_h

Then $I_h = I_{cm} + Mh^2$ (see Book)

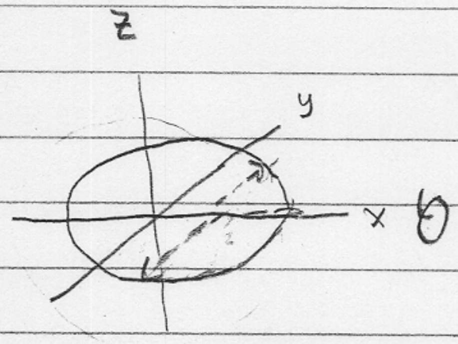
So for instance in this case



$$I_h = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

③ Perpendicular Axis Theorem

- For flat objects



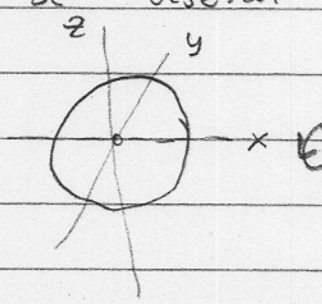
$I_z = M\langle x^2 + y^2\rangle$

$I_x = M\langle y^2\rangle$ $\qquad I_y = M\langle x^2\rangle$

Then: $I_z = M\langle x^2\rangle + M\langle y^2\rangle$

$I_z = I_y + I_x$

Can be useful



$I_z = \frac{1}{2}MR^2 = I_x + I_y$

$\frac{1}{2}MR^2 = 2I_x$

$\frac{1}{4}MR^2 = I_x$