Last Time

- Acceleration

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

- Basic Kinematic equations for constant acceleration

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \qquad (x \text{ vs. } t)$$

$$v = v_0 + at \qquad (v \text{ vs. } t)$$

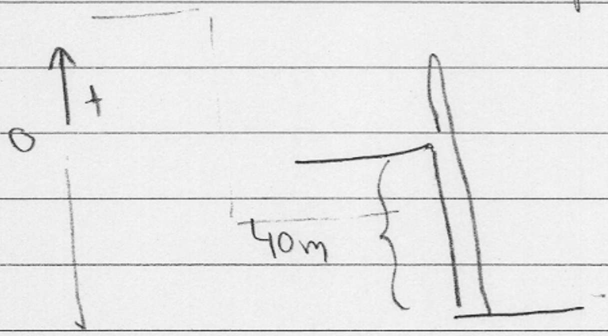
$$v^2 = v_0^2 + 2a(x - x_0) \qquad (v \text{ vs. } x)$$

Gravity

- Things fall, velocity changes
- Detailed measurements show
  - The acceleration is constant and downward $g = 9.8\,m/s^2$
  - Independent of velocity
  - Independent of mass

Problem: A random physics professor (you know who) throws a baseball straight up, while standing at the top of the physics building. Estimate when it hits the ground

Sol: The height of the building ~ 40m.
A typical person can throw at a speed of ~ 30 mph ~ 15 m/s



· Since the problem asks when and where)

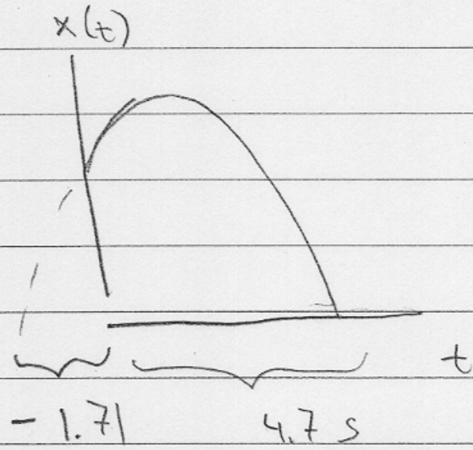
$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \qquad (t, \text{ vs } x)$$

$$(-40m) = 0 + (15 m/s)t - \frac{1}{2} 9.8 m/s^2 \, t^2$$

$$0 = (-4.9 m/s^2)t^2 + (15 m/s)t + 40m$$

Sol: $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad b = 15 m/s \quad a = -4.9 m/s^2 \quad c = 40m$

$t = -1.71$ and $t = 4.7 s$

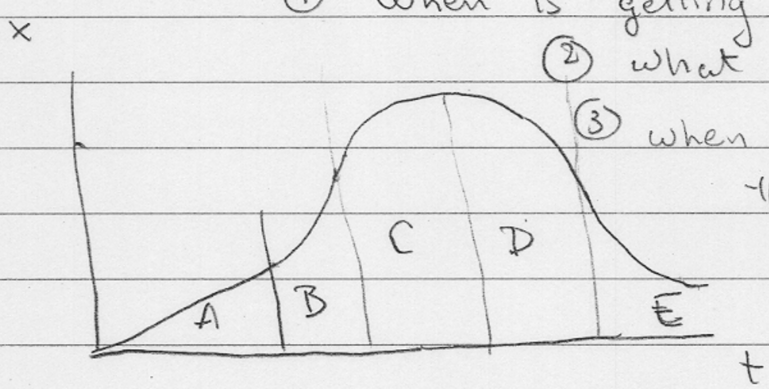


Last Problem: Describe whats going on.

① When is it getting faster or slower,

② what is the sign of the accel

③ when is it moving to the right and to the left



Ⓐ moving the constant velocity $v > 0$ $a = 0$

Ⓑ Then acceleration occurs $a > 0$, right moving

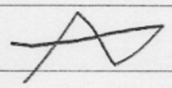
Ⓒ Right moving, Slowing down $a < 0$ $v > 0$

ⓓ Left moving, speeding up $a < 0$ $v < 0$

ⓔ Left moving, slowin gown

Vectors

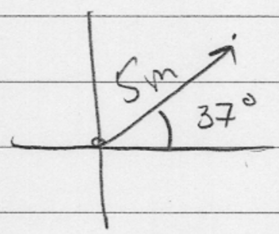
- We usually move in more than one dimensio



Need a formalism to express this

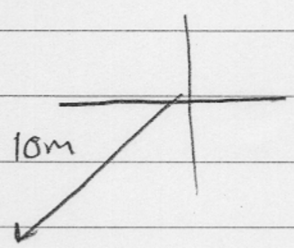
- A vector is something with a magnitude and direct

Canonical Example: A person walks 5m at an angle of 37° north of east

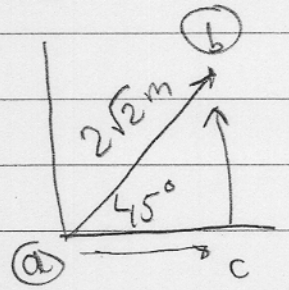


5m is the magnitud of the vector

Others



Adding Vectors

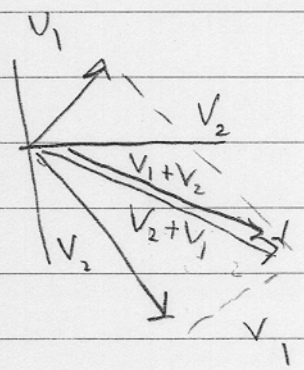


→ A person walking at 45° from a to b can go directly or go a → b and b → c

So this suggests how to add vectors

1. Draw the first vector to scale
2. Shift the second vector so that the vectors are "tail-to-tip"

Ex



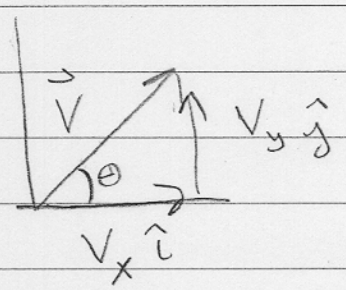
$$\vec{V}_1 + \vec{V}_2 = \vec{V}_2 + \vec{V}_1$$

"Vector addition commutes"

HW - give a graphical proof that

$$(A+B)+C = A + (B+C)$$

Components of Vectors



$\hat{i}$ = vector with unit magnitud in x-derection

$\hat{j}$ = vector ⓦ unit magnitude in y-direction

$$\vec{V} = V_x\,\hat{i} + V_y\,\hat{j}$$

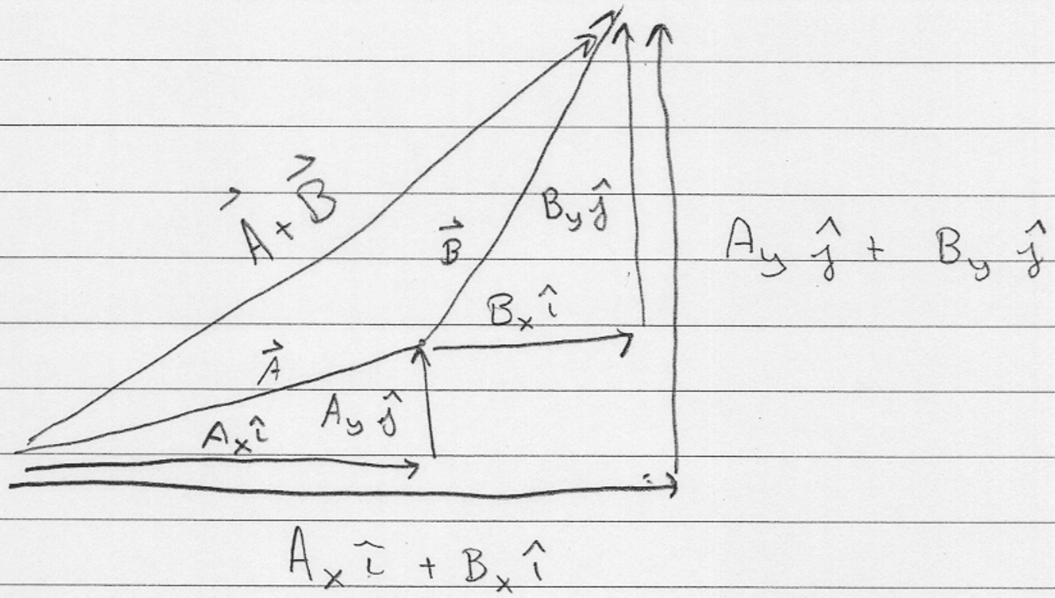
We will also use a "column vector" notation

$$\vec{V} = \begin{pmatrix} V_x \\ V_y \end{pmatrix} \equiv \text{same thing}$$

Simple Geometry:

- $V_x = V\cos\theta \qquad V_y = V\sin\theta \qquad \tan\theta = V_y / V_x$
- $V = |\vec{V}| \equiv \sqrt{V_x^2 + V_y^2}$ = The magnitude of $\vec{V}$

Addition of Vectors



So

$$\vec{S} = \vec{A} + \vec{B}$$

$$= (A_x \hat{i} + A_y \hat{j}) + (B_x \hat{i} + B_y \hat{j})$$

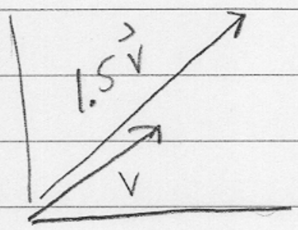
$$\vec{S} = (A_x + B_x)\,\hat{i} + (A_y + B_y)\,\hat{j}$$

see picture $\leftrightarrow$ components add

Column Vector

$$\begin{pmatrix} S_x \\ S_y \end{pmatrix} = \begin{pmatrix} A_x \\ A_y \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \end{pmatrix} = \begin{pmatrix} A_x + B_x \\ A_y + B_y \end{pmatrix}$$

Multiplication by a constant

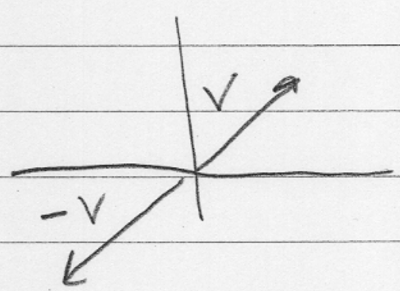


- increases magnitude of $\vec{V}$ but does not change direction

$$1.5\vec{V} = 1.5(V_x\hat{\imath} + V_y\hat{\jmath}) = (1.5V_x\hat{\imath}) + (1.5V_y\hat{\jmath})$$

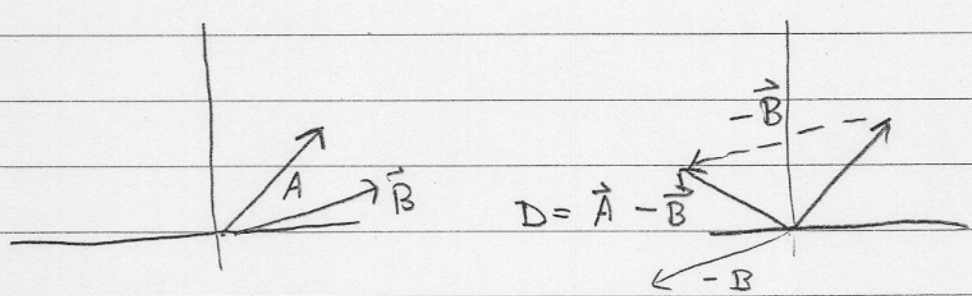
$$1.5\vec{V} = \begin{pmatrix} 1.5V_x \\ 1.5V_y \end{pmatrix}$$

Multiplication by -1



$$-\vec{V} = \begin{pmatrix} -V_x \\ -V_y \end{pmatrix} = (-V_x\hat{\imath} - V_y\hat{\jmath})$$

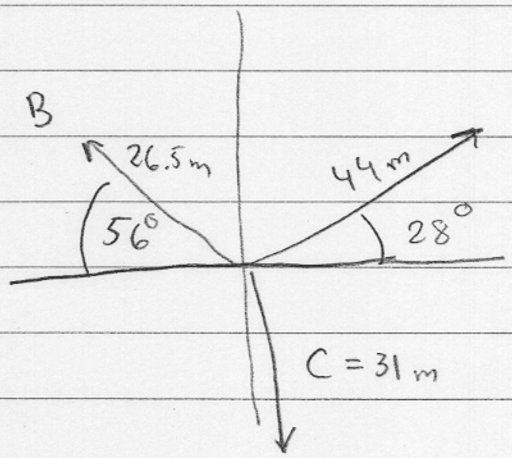
Difference $\qquad \vec{D} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$



$$\vec{D} = \vec{A} - \vec{B}$$

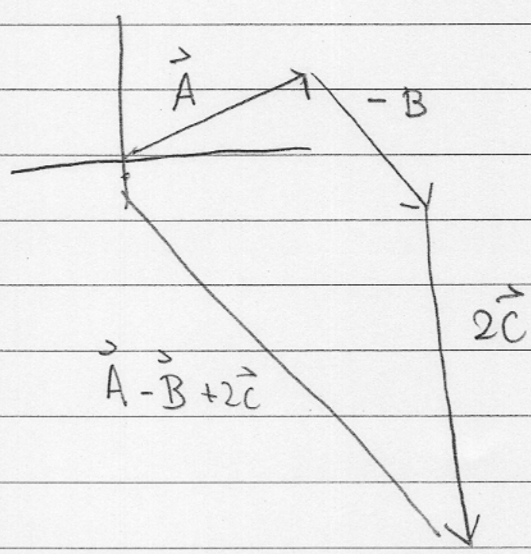
$$\vec{D} = \begin{pmatrix} A_x \\ A_y \end{pmatrix} + \begin{pmatrix} -B_x \\ -B_y \end{pmatrix} = \begin{pmatrix} A_x - B_x \\ A_y - B_y \end{pmatrix}$$

Example



Sketch:

$\vec{A} - \vec{B} + 2\vec{C}$



Formulae

$$\vec{A} = \begin{pmatrix} A\cos 28^\circ \\ A\sin 28^\circ \end{pmatrix} = \begin{pmatrix} 38.8\,m \\ 20.6\,m \end{pmatrix}$$

$$\vec{B} = \begin{pmatrix} -B\cos 56^\circ \\ B\sin 56^\circ \end{pmatrix} \Rightarrow \begin{pmatrix} -14.8\,m \\ 21.9\,m \end{pmatrix}$$

$$2\vec{C} = \begin{pmatrix} 0 \\ -62\,m \end{pmatrix} = \begin{pmatrix} 0 \\ -62\,m \end{pmatrix}$$

$$\vec{A} - \vec{B} + 2\vec{C} = \begin{pmatrix} 53.6\,m \\ -63.\,m \end{pmatrix}$$ ← Does kind of look like what we drew