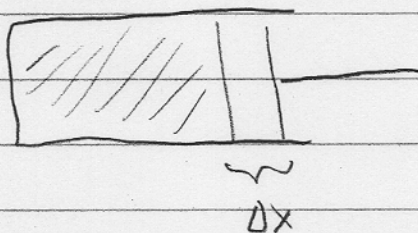


Work and Heat

- Heat is the transfer of the microscopic energy from one part to the other
- The energy in the gas can also be used to do macroscopic work



Now

$$\text{Force on piston} \rightarrow \text{Force} = P A \quad \leftarrow \text{area of piston}$$

$$dW = F dx = P \underbrace{A dx}_{dV}$$

So

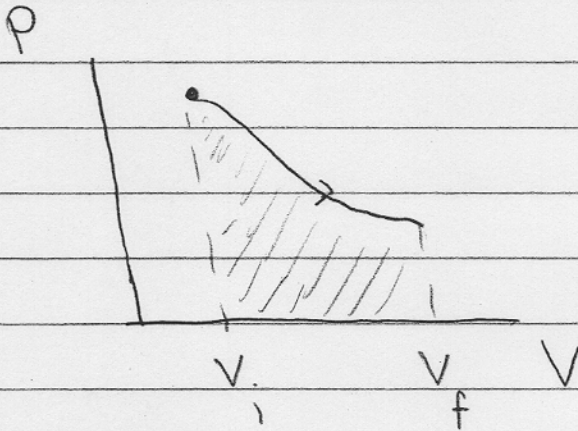
$$\boxed{dW = p dV}$$

More generally

$$W = \int_{V_i}^{V_f} p dV$$

← pressure is a function of volume which you need to know

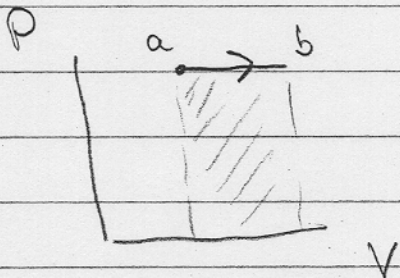
• Typically, the pressure decreases as it expands, unless heat is added



Work is the area under the PV curve

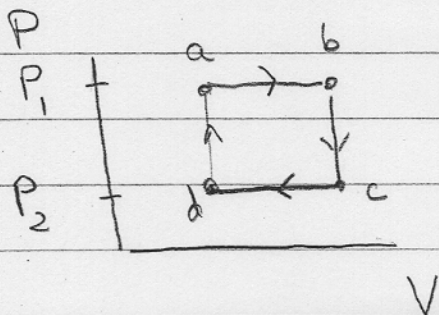
Consider two special cases

Isobaric - Constant Pressure



$$W = P(V_f - V_i)$$

Now consider a more elaborate trajectory



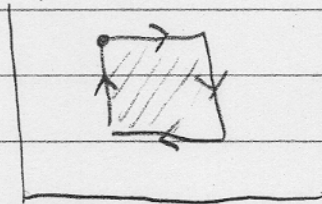
Then

$W_{ab} > 0$ gas does work

$W_{bc} = 0$ $dV = 0$ no work $V = \text{const}$

$W_{cd} < 0$ $\Delta V < 0$ we do work on gas
gas does neg work

$W_{da} = 0$

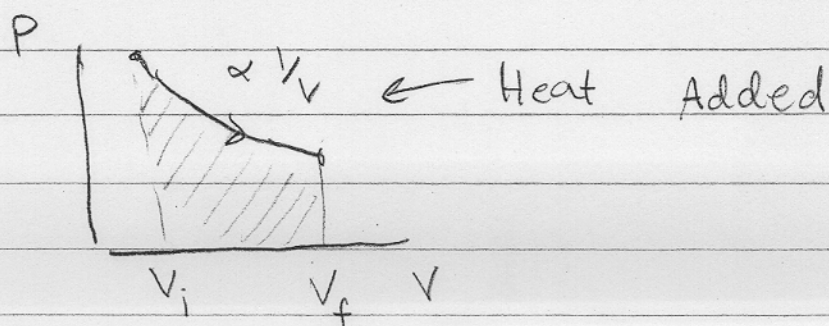


Net work = area of filled region

Isothermal Expansion - Constant Temp - Heat Added

- For simplicity consider an ideal gas

$$P = \frac{nRT}{V}$$



$$W = \int_{V_i}^{V_f} P dV = \int_{V_i}^{V_f} \frac{nRT}{V} dV$$

$$= nRT \int_{V_i}^{V_f} \frac{dV}{V} -$$

$$= nRT \ln \frac{V_f}{V_i}$$

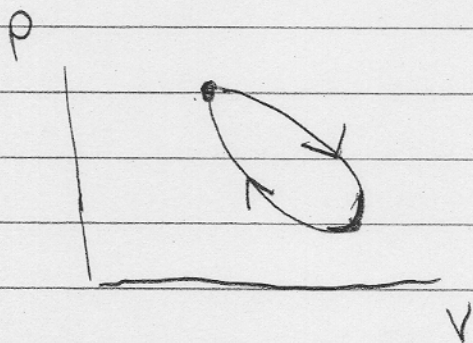
The first Law of Thermo

The internal energy ^{of a gas} can change due to an influx of heat, and macroscopic work

$$\Delta U = Q - W$$

Special Cases:

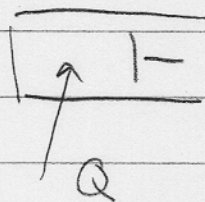
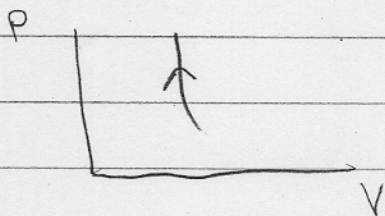
① A cyclic process $\Delta U = 0$



• since we start and stop in the initial state

$$\Delta U = Q - W \quad Q = W$$

② Const volume:

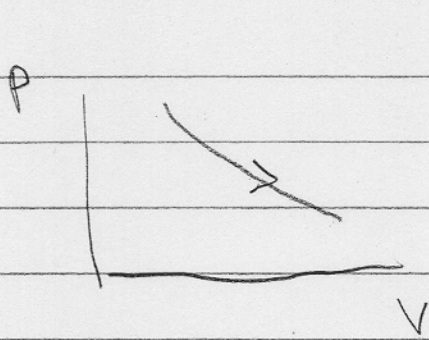


$$\Delta U = Q - W$$

$$\Delta U = Q$$

③ Adiabatic Processes:

no heat exchange



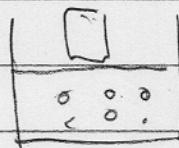
$$\Delta u = \overset{\circ}{Q} - W$$

$$\Delta u = -W$$

↑ internal Energy decreases
due to the work done

Adiabatic Expansion and Specific Heats :- Ideal Gasses

Generally:

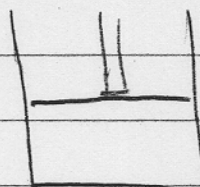


$$dQ_p = n C_p dT$$

constant P

specific heat at const P

Can also measure



$$dQ_v = n C_v dT$$

const V

specific heat at const volume

For const volume:

$$dQ = dU - dW$$

$$dU = dQ$$

$$dU = n C_v dT$$

← always true at const volume

For an ideal gas U indep of volume so

$$dU = n C_v dT$$

← always even if vol not const for Ideal Gas

Homework: Show

$$C_v = \frac{3}{2} R \text{ for mono-atomic}$$

$$C_v = \frac{5}{2} R \text{ for diatomic}$$

Homework

Now for an ideal-mono-atomic gas (He, Ne, Ar)

$$U = \frac{3}{2} n R T$$

$$dU = n \left(\frac{3}{2} R \right) dT$$

$$\equiv n C_V dT$$

So

$$\boxed{C_V = \frac{3}{2} R}$$

← Ideal Mono-atomic gas

For diatomic gas (O_2, N_2, \dots)

$$U = \frac{5}{2} n R T$$

And similarly

$$\boxed{C_V = \frac{5}{2} R}$$

← Ideal diatomic gas

Specific Heats C_p and C_v and $\gamma = C_p/C_v$ for IG

There is a relation between C_p and C_v for IG.
Consider press const :

$$dU = dQ_p - p dV$$

dQ_p = heat exchanged
at const p

$$dU = n C_v dT \quad \text{and} \quad p dV = n R dT$$

$$n C_v dT = dQ_p - n R dT$$

$$n C_v dT = n C_p dT - n R dT$$

So

$$\boxed{C_v = C_p - R}$$

For ideal gas mono-atomic-IG

$$C_p = C_v + R = \frac{3}{2}R + R = \frac{5}{2}R = C_p \quad \gamma = \frac{C_p}{C_v} = \frac{5/2}{3/2}$$

For ideal gas diatomic IG

≈ 1.7

$$C_p = C_v + R = \frac{5}{2}R + R = \frac{7}{2}R = C_p$$

and

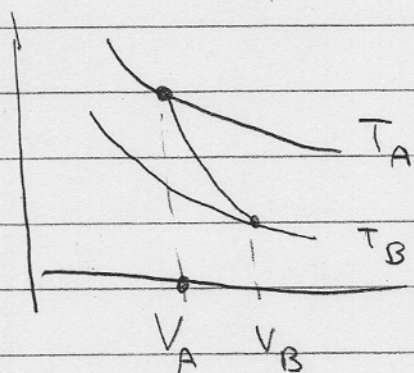
$$\gamma = \frac{C_p}{C_v} = \frac{7/2 R}{5/2 R} \approx 1.4 = \gamma$$

Adiabatic Expansion $Q = 0$

$$dU = dQ - dW$$

$$dU = -dW$$

← An analysis of this equation shows (see next pg.) that



① $PV^\gamma = \text{Constant}$

② The temperature decreases during the expansion

Calculate The Work Done

$$W = \int p dV \quad \text{and} \quad PV^\gamma = P_A V_A^\gamma \quad \text{or} \quad P = P_A \left(\frac{V_A}{V} \right)^\gamma$$

$$W = \int_{V_A}^{V_B} P_A \left(\frac{V_A}{V} \right)^\gamma dV$$

$$W = \int_{V_A}^{V_B} P_A V_A \left(\frac{V_A}{V} \right)^\gamma \frac{dV}{V_A} \quad \text{let } u = \frac{V}{V_A}$$

$$W = P_A V_A \int_1^{V_B/V_A} du \quad u^{-\gamma}$$

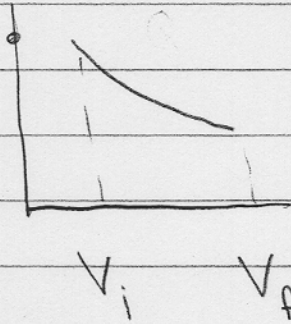
$$W = P_A V_A \frac{u^{-\gamma+1}}{-\gamma+1} \Big|_{V_B/V_A}^1$$

$$W = \frac{P_A V_A}{-\gamma+1} \left[\left(\frac{V_B}{V_A} \right)^{-\gamma+1} - 1 \right] \quad \gamma > 1$$

$$W = \frac{P_A V_A}{\gamma-1} \left[1 - \left(\frac{V_A}{V_B} \right)^{\gamma-1} \right]$$

Adiabatic Expansion $Q=0$ Proof: Skip if pressed for time!

$$dU = dQ - dW$$



$$nC_v dT = -p dV$$

From the gas law $PdV + VdP = nR dT$

Then with $dT = -\frac{pdV}{nC_v}$ we have

$$PdV + VdP = -pdV \frac{nR}{nC_v}$$

$$P dV \left(1 + \frac{R}{c_v}\right) + V dP = 0$$

$$\frac{dV}{V} \frac{(c_v + R)}{c_v} = -\frac{dP}{P}$$

$$\text{Now } \frac{c_v + R}{c_v} = \frac{c_p}{c_v} = \gamma$$

Or

$$\gamma \frac{dV}{V} = -\frac{dP}{P} \Rightarrow PV^\gamma = \text{const}$$

i.e. integrating

$$-\gamma \log V = \log P + C$$

$$V^{-\gamma} = CP$$

$$1 = CPV^{\gamma}$$

Or

$$PV^{\gamma} = \text{Const}$$