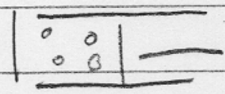
Last Time

① Work

work done by gas

$dW = p\,dV$



gas expands $dW > 0$

gas campressed $dW < 0$ work done on gas > 0

② The firs law:

$$\Delta U = Q - W$$

$$dU = dQ - p\,dV$$

③ For ideal gasses used the first law

To show:

$$C_p = C_V + R$$

$dQ_p = n C_p \Delta T$ ← at const press., temp changes when heat added

$dQ_V = n C_V \Delta T$

↖ const vol, temp changes when heat added

Used this to show for a mono-atomic ideal gas

MAIG:

definition ≡ "the adiabatic index"

$C_V = \frac{3}{2} R$ $\quad C_p = \frac{5}{2} R \quad \gamma \equiv \frac{C_p}{C_V} = \frac{5}{3}$

For diatomic IG

notice γ always > 1

DAIG

$C_V = \frac{5}{2} R \quad C_p = \frac{7}{2} R \quad \gamma = \frac{C_p}{C_V} = \frac{7}{5}$

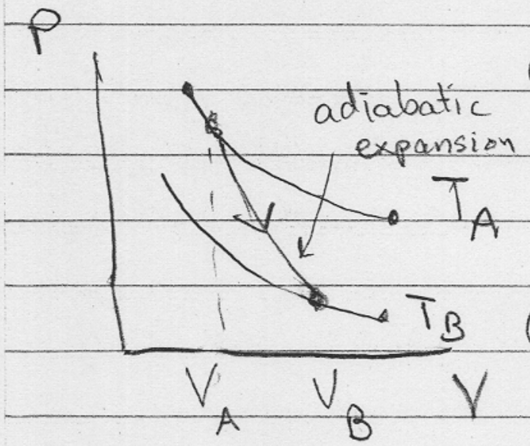
(4) Types of expansions ⇔ computed

a) isothermal $T = \text{Const}$
b) isobaric $P = \text{Const}$
c) const vol. $V = \text{Const}$
d) Adiabatic $Q = 0$

Adiabatic Expansion $Q = 0$ ← no heat influx

$$dU = \cancel{dQ}^{\,0} - dW$$

$$dU = -dW$$ ← the internal energy of the gas is used to do work



① In general if a system expands heat would be need to be added maintain the temperature.

② In an Adiabatic expansion, $Q = 0$ then the temperature decrease

③ An analysis of the first law, shows (see following pages)

$$PV^{\gamma} = C \qquad C = \text{constant}$$

Calculate the Work Done going from $V_A \rightarrow V_B$:

$$W = \int p\,dV \qquad PV^{\gamma} = P_A V_A^{\gamma} \Rightarrow P = P_A \left(\frac{V_A}{V}\right)^{\gamma}$$

$$W = \int P_A \left(\frac{V_A}{V}\right)^{\gamma} dV$$

So

$$W = \int_{V_A}^{V_B} P_A V_A \left(\frac{V_A}{V}\right)^{\gamma} \frac{dV}{V_A} \qquad \text{let} \quad u \equiv \frac{V}{V_A}$$

$$W = P_A V_A \int_{u=1}^{u=V_B/V_A} du \quad u^{-\gamma}$$

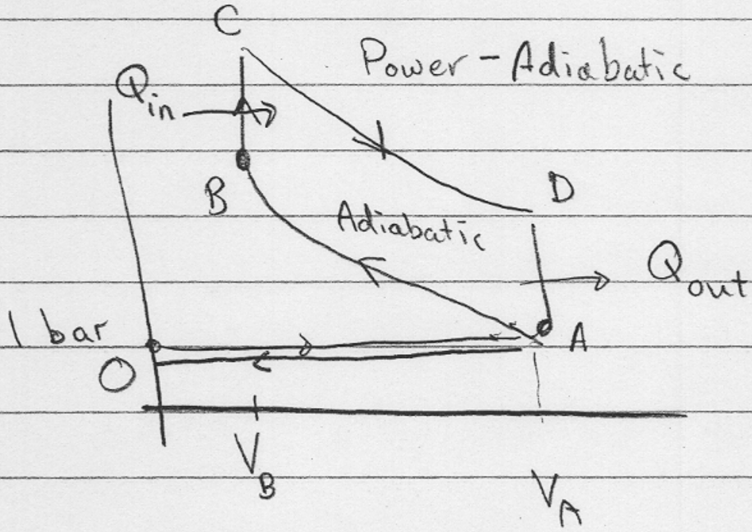
$$= P_A V_A \left. \frac{u^{-\gamma+1}}{-\gamma+1} \right|_{1}^{V_B/V_A}$$

$$W = \frac{P_A V_B}{-\gamma+1} \left[\left(\frac{V_B}{V_A}\right)^{-\gamma+1} - 1 \right]$$

notice $-\gamma + 1 < 0$ a "hidden minus"

$$W = \frac{P_A V_A}{(\gamma - 1)} \left[1 - \left(\frac{V_A}{V_B}\right)^{\gamma - 1} \right]$$

Otto Cycle:



$$\frac{V_A}{V_B} = \text{"The compression factor"} = r = 8$$

Treat as a diatomic gas $\gamma = 1.4$

The Otto - Cycle - We will anaylze w/ 4 Problems

- A four stroke engine - see video

① Pull in air-gas mixture (OA)
- gas brought in at $T \simeq 300°K$ and $P = 1atm$

② Compress the mixture (AB) - Adiabatic
- for arguement take $\frac{V_A}{V_B} \equiv$ the compression ratio $= 8$

③ Ignite the mixture (BC)
- Volume remains constant Q_{in} comes from burning gas

④ Power Stroke - Adiabatic (CD)

⑤ Exhaust Valve - openned (DA)
- Pressure down to atmospheric

⑥ Cylinder compressed AO

happens $\sim 1000 \frac{\text{times}}{\text{min}}$ $t \sim 0.06s$

Probl - estimate the volume of gas sucked in, estimate the number of moles

$V \sim 3.0L$

$pV = nRT \Rightarrow n = \frac{PV}{RT} = 0.13$

$P = 1\text{bar} = 10^5 N/m^2$

$V = 3 \times 10^{-3} m^3$

$R = 8.3 \quad T = 300°K$

Prob2 - estimate the rise in temp of the compression stroke

Sol: Since its adiabatic

$PV^\gamma = C$ now $PV = nRT$

$P = \frac{nRT}{V}$

$\frac{nRT}{V} V^\gamma = C$

a useful result

So

$\boxed{TV^{\gamma-1} = C'}$ ← another const

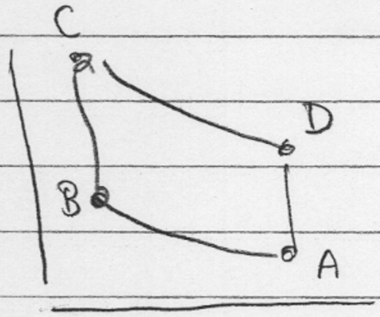
$T_B V_B^{\gamma-1} = T_A V_A^{\gamma-1}$

$\frac{T_B}{T_A} \simeq \left(\frac{V_A}{V_B}\right)^{\gamma-1} = r^{\gamma-1} = (8)^{0.4} \simeq 2.29$

So

$$T_B = 300^\circ K \cdot 2.29 = 689^\circ K \sim 750^\circ F$$

Notice: CD similar to BA



$$\frac{T_B}{T_A} = \left(\frac{V_A}{V_B}\right)^{\gamma-1} = r^{\gamma-1}$$

$$\frac{T_C}{T_D} = \left(\frac{V_C}{V_D}\right)^{\gamma-1} = r^{\gamma-1}$$

1

The same as (T_B/T_A)

Engine Construction

① Four Cylinders tied to a crank shaft.

② How do the valves know how to open and close?

- The cam is a long metal shaft - which by rotating opens and closes the valves

Problem 3

- The efficiency is defined as the work done per unit heat input

$$\varepsilon = \frac{W}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Since: $\cancel{\Delta U}^{0} = Q - W$ $\qquad Q = Q_{in} - Q_{out}$

$$Q_{in} - Q_{out} = W$$

Want to determine the efficiency.

(1) Show that:

$$e = 1 - \left(\frac{V_B}{V_A}\right)^{\gamma - 1} \qquad \frac{V_A}{V_B} \equiv \text{the compression ratio} \equiv r \sim 8$$

$$e = 1 - \frac{1}{r^{\gamma-1}}$$

Solution:

Analyzé the heat brought in

$$\Delta U = Q - \cancel{W}^{0}$$

$$n C_V dT = dQ$$

$$Q_H = nC_v\,(T_C - T_B)$$

Now Q_L

$$\Delta U = Q - \cancelto{0}{W}$$

heat flowing into gas

$$\Delta U = nC_v\,(T_A - T_D) = Q$$

$T_A < T_B$

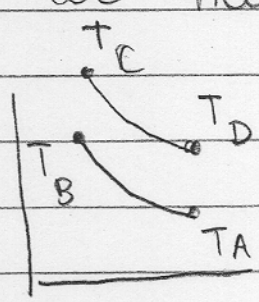
So

$$Q_{out} = -Q = nC_v\,(T_D - T_A)$$

Then

$$\varepsilon = \frac{W}{Q_{out}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \left[\frac{T_D - T_A}{T_C - T_B}\right]\frac{\cancel{nC_v}}{\cancel{nC_v}}$$

Now we need to relate



T_C and T_D and T_B and T_A

Using

$$T_C = T_D\, r^{\gamma-1} \qquad T_B = T_A\, r^{\gamma-1}$$

So

$$e = 1 - \left[\frac{T_D - T_A}{T_D\, r^{\gamma-1} - T_A\, r^{\gamma-1}}\right]$$

$$e = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{8^{0.4}} \approx 0.56$$

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