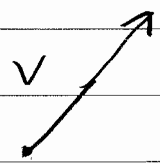
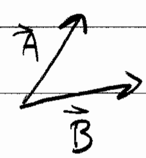
Last Time

Vectors

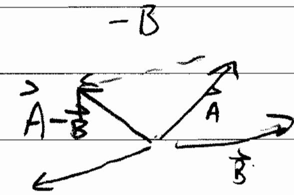


- Addition, Subtraction, multiplication by const, etc

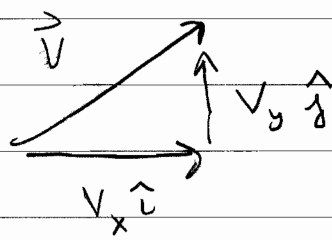
Ex:



then

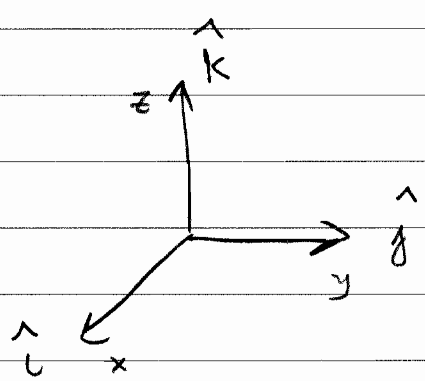


- Components



$$\vec{V} = V_x \hat{i} + V_y \hat{j} \qquad (2D)$$

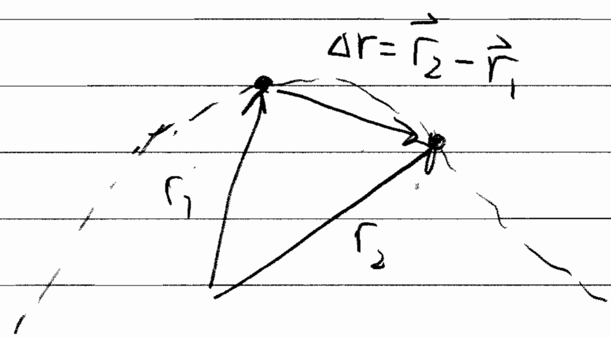
$$\vec{V} = V_x \hat{i} + V_y \hat{j} + V_z \hat{k} \qquad (3D)$$



Alternative notation

$$\vec{V} = \begin{pmatrix} V_x \\ V_y \end{pmatrix} \quad \text{or} \quad \text{in 3D} \quad \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

Physical Quantities



Suppose we have a particle moving along (e.g. Basket ball)

1) $\Delta\vec{r}$ = displacement

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$= (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (x_1$$

$$\Delta r = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} - \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{pmatrix}$$

More generally

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$$

$$\vec{r} = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$$

② Velocity

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\frac{d\vec{r}}{dt} = \frac{dx}{dt}\,\hat{i} + \frac{dy}{dt}\,\hat{j} + \frac{dz}{dt}\,\hat{k}$$

$$\equiv v_x\,\hat{i} + v_y\,\hat{j} + v_z\,\hat{k}$$

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \frac{d}{dt}\begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} dx/dt \\ dy/dt \\ dz/dt \end{pmatrix} = \vec{v}$$

③ Acceleration

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}\begin{pmatrix} V_x(t) \\ V_y(t) \\ V_z(t) \end{pmatrix} = \begin{pmatrix} \frac{dV_x}{dt} \\ \frac{dV_y}{dt} \\ \frac{dV_z}{dt} \end{pmatrix}$$

$$\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$$

Also

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$\vec{a} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

For Constant $\vec{a}(t) = \vec{a}$

$$\vec{V} = \vec{V}_0 + \vec{a} \cdot t$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

This means

$\Rightarrow$ $$\vec{V} = \vec{V}_0 + \vec{a}\,t$$

$$\begin{pmatrix} V_x(t) \\ V_y(t) \end{pmatrix} = \begin{pmatrix} V_{0x} \\ V_{0y} \end{pmatrix} + \begin{pmatrix} a_x \\ a_y \end{pmatrix} t$$

$$V_x = V_{0x} + a_x t$$

$$V_y = V_{0y} + a_y t$$

$\rightarrow$ $$\vec{r} = \vec{r}_0 + \vec{V}_0 t + \frac{1}{2}\vec{a}t^2$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} V_{0x} \\ V_{0y} \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} a_x \\ a_y \end{pmatrix} t^2$$

$$x(t) = x_0 + V_{0x} t + \frac{1}{2} a_x t^2$$

$$y(t) = y_0 + V_{0y} t + \frac{1}{2} a_y t^2$$

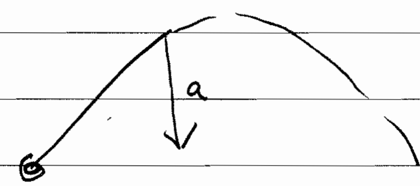
$\rightarrow$ Finally

$$V_x^2 = V_{0x}^2 + 2a_x (x - x_0)$$

$$V_y^2 = V_{0y}^2 + 2a_y (y - y_0)$$

Real Use of these equations - Projectile Motion

Acceleration is const



$a_x = 0$

$a_y = -g = -9.8\,m/s^2$

Eq

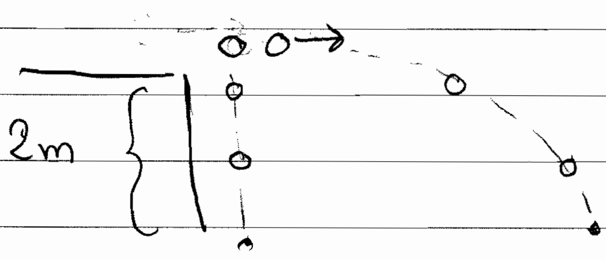
$V_x = V_{ox}$ $\qquad$ $V_y = V_{oy} - gt$

$x = x_o + V_{ox}t$ $\qquad$ $y = y_o + V_{oy}t - \frac{1}{2}gt^2$

$V_y^2 = V_{oy}^2 - 2g(y - y_o)$

Ex 1 - X + Y are totally independent

Two balls: One fired and one dropped

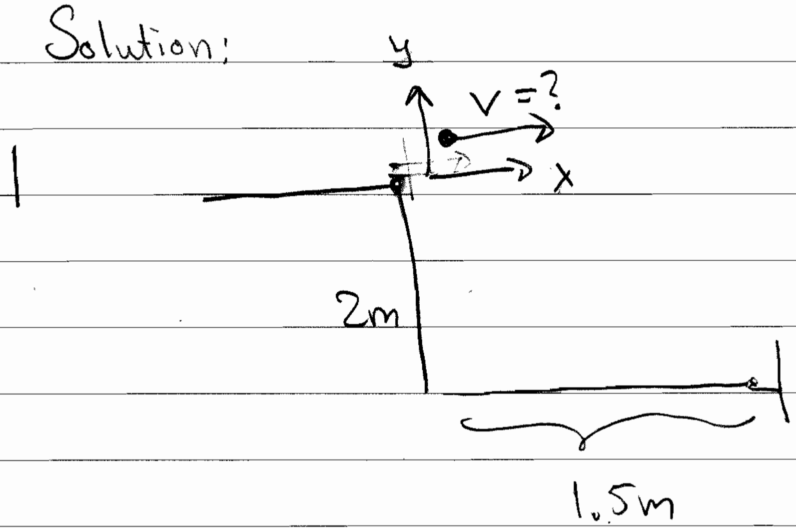


Q: Which hits the ground first?

A: Neither - they hit at same time

If the fired ball is to achieve a distance of 1.5m, how fast should it be fired

Solution:



If we knew how long its in the air that would help

$$y = \cancel{y_0}^{0} + \cancel{v_{0y}}^{0} t - \frac{1}{2} g t^2$$

$$\sqrt{\frac{-2y}{g}} = t$$

$$\left(\frac{-2 \cdot (-2m)}{9.8\,m/s^2}\right)^{1/2} = t$$

$t = 0.64s$

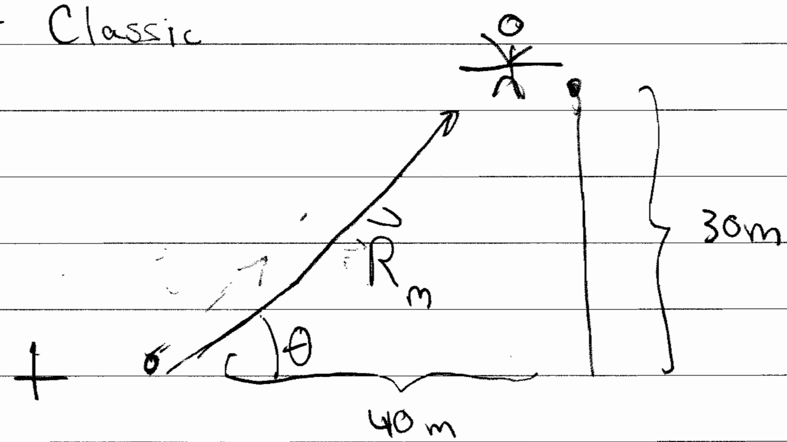
Then the x - distance

$$x = v_{ox}\, t + \cancel{x_o}^{\,0}$$

$$\frac{x}{t} = v_{ox}$$

$$\frac{1.5\,m}{0.64\,s} = v_{ox} = 2.34\ m/s$$

Ex 2 - Classic



A zookeeper fires a dart at a monkey. He knows that the moment the gun is fired the monkey will let go and start to fall. Which way should he aim? Does this depend on the speed of his gun?

Solution: First consider the position of monkey at $t=0$

$$\vec{R}_{om} = 40\,m\,\hat{i} + 30\,m\,\hat{j}$$

$$\vec{R}_{om} = \begin{pmatrix} 40m \\ 30m \end{pmatrix}$$

Then $\vec{V}_0$ unknown $\begin{pmatrix} V_{ox} \\ V_{oy} \end{pmatrix} = \begin{pmatrix} V_0 \cos\theta \\ V_0 \sin\theta \end{pmatrix}$

The monkey $\vec{a} = \begin{pmatrix} 0 \\ -g \end{pmatrix} = \vec{g}$

So the EOM for monkey

$$\vec{r}_m(t) = \vec{R}_{om} + \frac{1}{2}\vec{g}\,t^2$$

The EOM for dart

$$\vec{r}_D(t) = \cancel{\vec{r}_{oD}}^{\,0} + \vec{V}_0 t + \frac{1}{2}\vec{g}\,t^2$$

So Equating

$$\vec{R}_{om} + \cancel{\frac{1}{2}\vec{g}\,t^2} = \vec{V}_0 t + \cancel{\frac{1}{2}\vec{g}\,t^2}$$

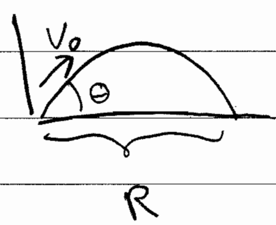
$\vec{R}_{om} = \vec{V}_0 t \leftarrow$ The answer $\vec{V}_0$ should point in the direc of monkey

$$\begin{pmatrix} 40m \\ 30m \end{pmatrix} = \begin{pmatrix} v_0 \cos\theta \\ v_0 \sin\theta \end{pmatrix} t$$

$$\frac{40m}{30m} = \frac{v_0 t \cos\theta}{v_0 t \sin\theta} \simeq \tan\theta$$

End of Lecture $\quad \theta \simeq 37^\circ$ and indep of v_0

Level Horizontal Range:

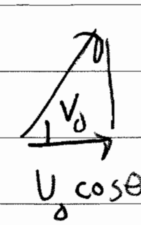


) Determine R as a function of θ, and v

(2) If napoleon's cannons had a muzzle speed of 60m/s at what angle should it be aimed to strike 320m away?

Sol: The time in the air is the time it takes to go up and down again

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a} t^2$$



$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \vec{0} \end{pmatrix} + \begin{pmatrix} v_0 \cos\theta \\ v_0 \sin\theta \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ -g \end{pmatrix} t^2$$