Forces & Masses

Typical Mass

coin ~ several grams

lead brick ~ 10 kg ~ 20 lbs

male person ~ 200 lbs ~ 100 kgs

car ~ 5000 kg

$m_e \sim 10^{-6} \times M_\odot$

$M_\odot \sim 2 \times 10^{30}$ kg

Typical Forces ~ ma

$1N = 1 kg\,m/s^2$

$\frac{1}{2} \cdot 10 m/s^2$

Sports car stopping ~ (1000 kg)($\frac{1}{2}g$) ~ 5000 N

Throwing a ball ~ (.1 kg)(1 g) ~ 1 N

Bullet launched ~ (2 grams)(1000 g) ~ 20 N

~~Newtons Laws & Forces~~

~~Q: What causes acceleration? How to compute acceleration?~~

~~A: Forces ← Any kind of push or pull on object~~

Newton First Law

(1) A body @ constant velocity remains with constant velocity until acted upon with an external force

Ex A careless motor biker runs into a parked car and goes flying off. what force makes him go flying?

Ans. None

Newtons Second Law:

(2) The acceleration of a body is proportional the net sum of all forces acting on a body

The proportionality constant is the mass

$$\sum_{\text{all forces}} \vec{F} = m\vec{a}$$

This begs the question: what are the forces we know about

1. gravity ← Now
2. Electro-magnetic ← Next semester
3. Strong + nuclear forces ← second year grad school

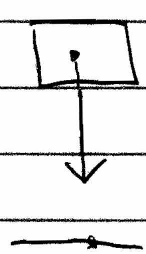
Other:

Friction, normal, tension ..

↳ These we will talk about. They are really electromagnetic forces between your molecules and the ground / air / rope / etc

For the moment consider gravity. The acceler[ation] is $g = 9.8\,m/s^2$

$\vec{F}_g$



$$\boxed{F_g = m\,g}$$ ← from newtons law

Aside -

- In this course mass is

$$m = 65\,kg \leftarrow \text{always the same}$$

- Then weight is

$$W = mg = (65\,kg)(9.8\,m/s^2) \approx 650\,\frac{kg\,m}{s^2}$$

$$\approx 650\,N$$

  - can be different on the moon

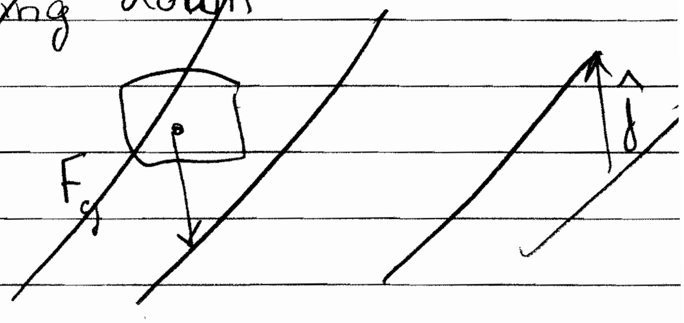
The weight is a vector pulling down

$$\vec{W} = -mg\,\hat{j}$$

~~$\vec{F}_g$ is a vector Pulling down~~

$$\cancel{\sum \vec{F} = m\vec{a}}$$

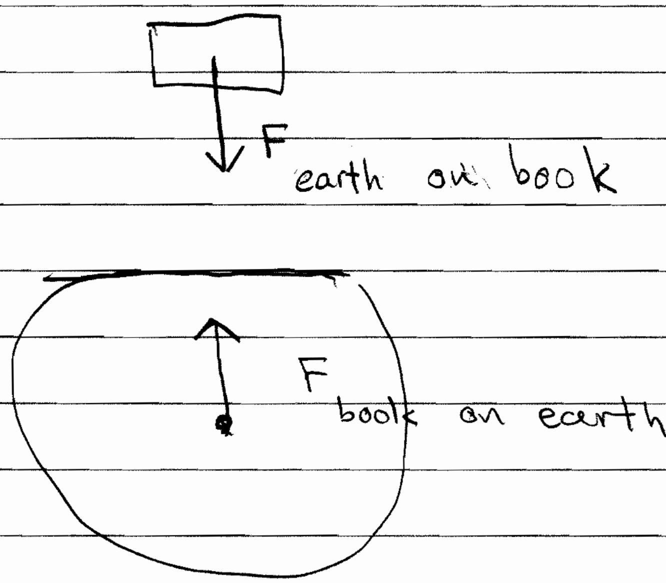
$$\cancel{\vec{F}_g = -mg\hat{y}}$$



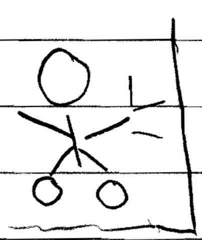
Newtons Third Law:

(3) For every force there is an equal and opposite force (Warning! This is hard)

Ex 1

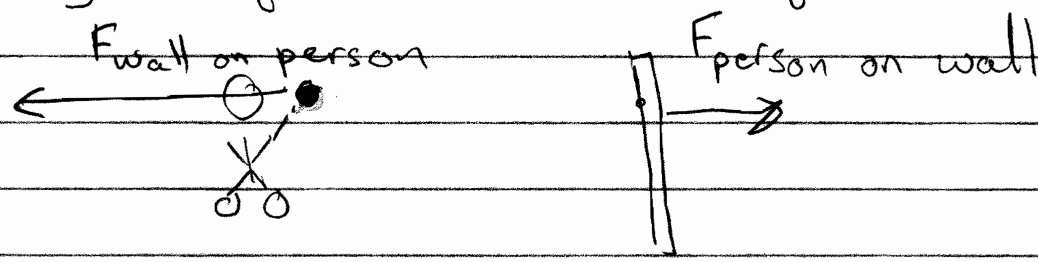


Ex 2



$$\vec{F}_{\text{person on wall}} = -\vec{F}_{\text{wall on person}}$$

Free body diagram: Separate objects

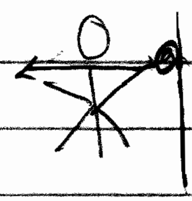


$\vec{F}_{wall\ on\ person}$ = example of normal force = $\vec{F}_N$

Normal Forces

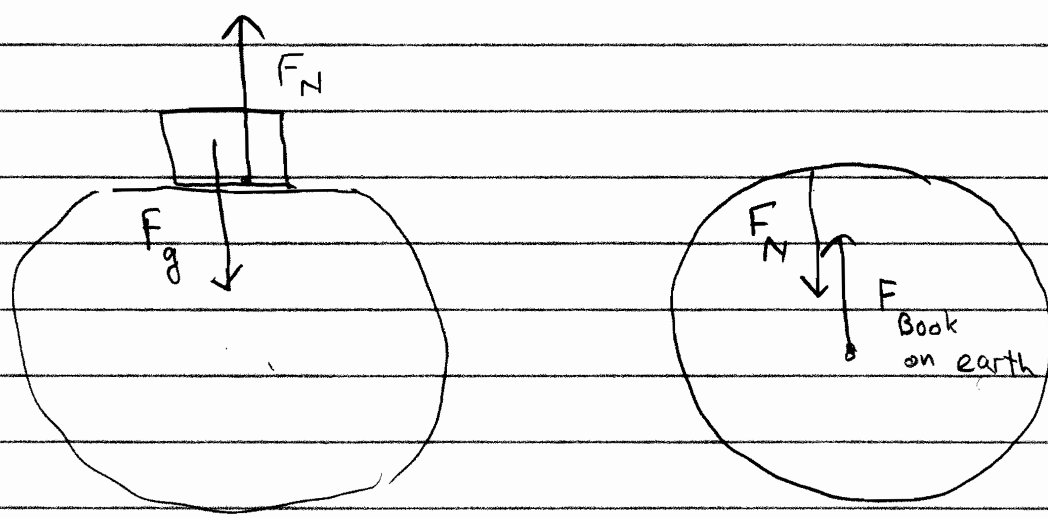
a) When you press on the wall you compress the molecules in the wall, they then act like a spring, pushing back on you.

b) Always perpendicular to the surface

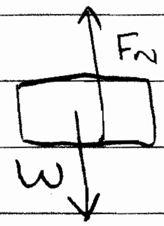


c) always equal in magnitude to the force on the wall

Ex 3 – Book on ground:



Now use newtons Laws, 1kg block



So $\vec{w} + \vec{F}_N = m\cancel{\vec{a}}^{\,0}$ ← Book not sinking into earth, $y_{Book} = \text{const}$, This is called an equation of constraint.

$$-\vec{W} = \vec{F}_N$$

$$-(-mg\hat{j}) = \vec{F}_N$$

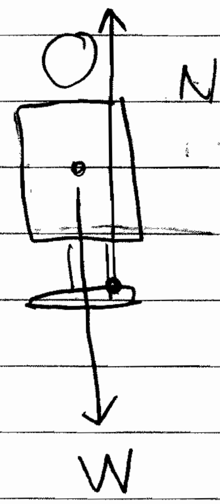
$$+mg\hat{j} = F_N$$

Normal Force supports the mass. The equation of constraint determines F_N.

The normal force is what is measured by a scale. It then divides by g to report an apparant mass

Ex4 A 65 kg woman descends in an elevator with an acceler ation of $0.2g$. She feels slightly weightless. If she is standing on a scale what does it read?

Sol ->



$$\sum \vec{F} = m\vec{a}$$

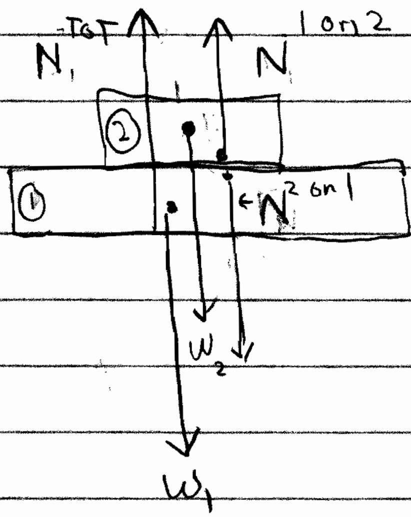
$$\vec{N} - mg\,\hat{j} = -m\,0.2g\,\hat{j}$$

$$\vec{N} = 0.8mg\,\hat{j}$$ The force registered by scall

So the scale would read an "apparant mass" of

$$(0.8mg)/g = 0.8m \simeq 52kg$$

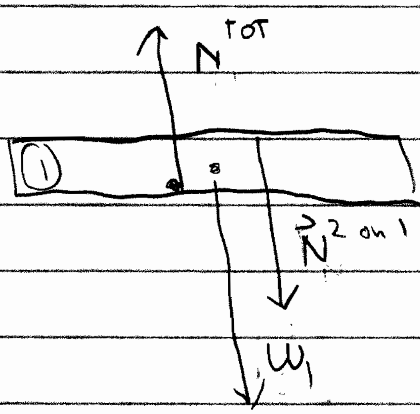
Ex4 Two Books

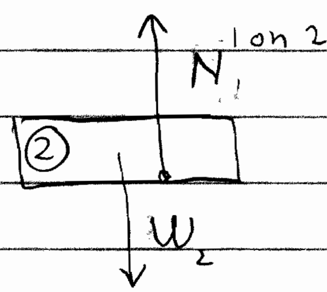


Clearly expect the normal force supports the total mass

$$N_1^{TOT} = (m_1 + m_2)\, g\, \hat{j}$$

Free body Diagrams:





This is a constaint y_2 = const

$$a_2 = \frac{d^2y}{dt^2} = 0$$

Work with Book 2

$$-m_2 g\,\hat{j} + \vec{N}^{1 on 2} = m_2 \cancel{\vec{a}_2}^{\,0}$$

book not accel,

$$\vec{N}^{1 on 2} = mg\,\hat{j} = -\vec{N}^{2 on 1}$$

Note the constaint y_2 = const allows you to determine the normal force $N^{1 on 2}$

So

$$\vec{W}_1 + \vec{N}^{TOT} + \vec{N}^{2 on 1} = m_1 \cancel{\vec{a}_1}$$

also a constraint y_1 = const

$$-m_1 g\,\hat{j} + \vec{N}_{tot} + -m_2 g\,\hat{j} = 0$$

i.e.

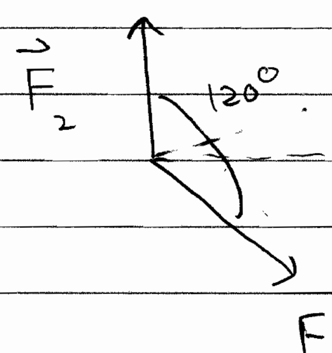
$$\vec{N}_{tot} = (m_1 + m_2)\, g\, \hat{j}$$

The constraints have allowed you to determine the normal forces

<u>Vector nature of Newtons. Law:</u>

• A 18.5 kg object (How heavy is this?) is sliding on a frictionless tabletop

At time $t=0$ it is at the origin, and is subjected to two forces $F_1 = 10.2\,N$, $F_2 = 16.\,N$



Where is the object after 0.5s give x,y coord

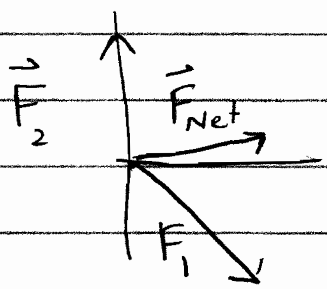
$$\vec{F}_1 + \vec{F}_2 = m\vec{a}$$

$$\vec{F}_1 = \begin{pmatrix} F_1 \cos 30^\circ \\ -F_1 \sin 30^\circ \end{pmatrix} \qquad \vec{F}_2 = \begin{pmatrix} 0 \\ F_2 \end{pmatrix}$$

$$\vec{F}_1 = (8.83\hat{\imath} - 5.1\hat{j}) \qquad F_2 = 5.2\,\hat{j}$$

So

$$\vec{F}_1 + \vec{F}_2 = \begin{pmatrix} 8.83\,N \\ -5.1\,N \end{pmatrix} + \begin{pmatrix} 0 \\ 5.2\,N \end{pmatrix} = \begin{pmatrix} 8.83\,N \\ +0.1\,N \end{pmatrix} = \vec{F}_{Net}$$



$$\vec{F}_{net} = m\vec{a}$$

$$\frac{\vec{F}_{net}}{m} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} = \frac{1}{18.5}\begin{pmatrix} 8.83\,N \\ 0.1\,N \end{pmatrix}$$

$$\begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} 0.47\ m/s^2 \\ 0.005\ m/s^2 \end{pmatrix}$$

So

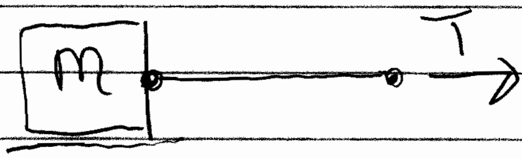
$$\vec{r} = \cancel{\vec{r}_0}^{\,0} + \cancel{\vec{v}_0 t}^{\,0} + \frac{1}{2}\vec{a}\,t^2$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0.47\,m/s^2 \\ 0.005\,m/s^2 \end{pmatrix}(1s^2)$$

$$x = 0.235\ m$$

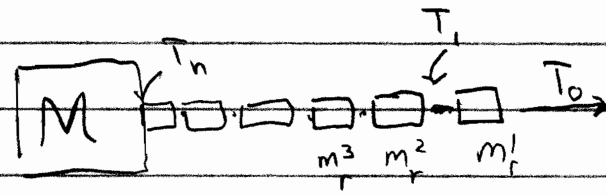
$$y = 0.0025\,m$$

Ropes:

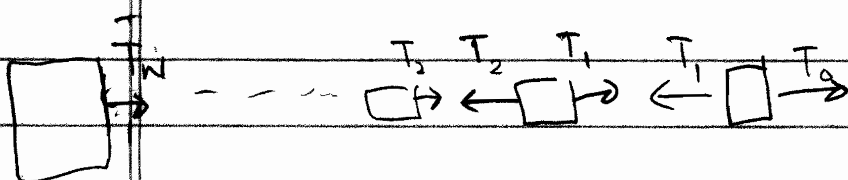


$$a = \frac{T}{M}$$

Physicist view of a rope



Then they would draw a free body diagram for each segment.



$$T_0 - T_1 = \overset{0}{\cancel{m_r^1}}\, a_r^1$$

$$T_1 - T_2 = \overset{0}{\cancel{m_r^2}}\, a_r^2$$

$$\vdots$$

$$T_N - T_{N-1} = \overset{0}{\cancel{m_r^n}}\, a_r^n$$

now if the rope is $\underbrace{\text{massless}}_{\text{very light}}$ then

we conclude that

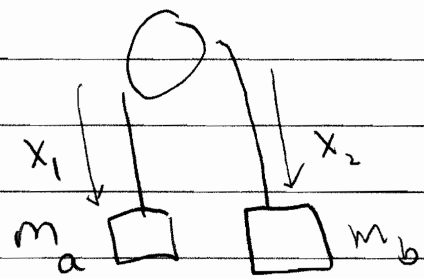
$$T_0 = T_1 = T_2 = T_3 \ldots = T_N$$

The tension is constant throughout for a massless rope

Atwood Machine:

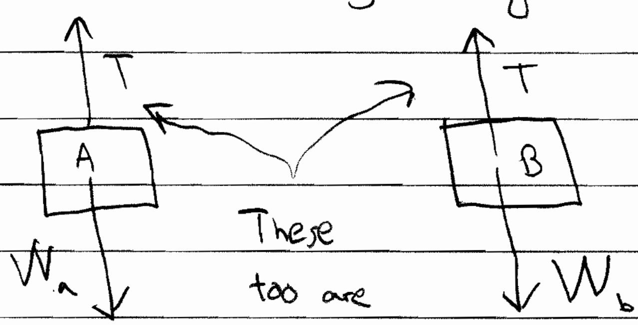
- Ropes provide a constraint, rope is a fixed Length, Tension is a force

Problem: Determine the acceleration of the system?



$m_b > 0$

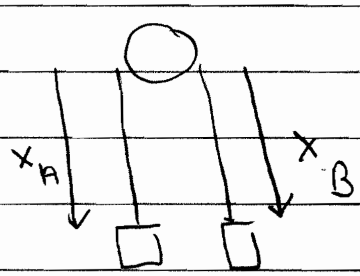
① Draw a free body diagram for each body



These two are equal and opposite forces

You may wish to turn the next page to item #3 to write down Newtons Laws F=ma at this point.

② Write down the equation of constraint



$x_A + x_B$ = Length of rope = const

From the constaint equation we differentiate

$$\frac{dx_A}{dt} + \frac{dx_B}{dt} = 0$$

$$\frac{d^2x_A}{dt^2} + \frac{d^2x_B}{dt^2} = 0$$

If one goes up, the other goes down

$$a_A = -a_B \qquad (1)$$

③ Now write newtons Law for each body body A and body B

$$T - W_A = m\ a_A \qquad (2)$$

$$T - W_B = m\ a_B \qquad (3)$$

Unknowns T, a_A, a_B, you should have enough equations to solve for the unknowns.

Extra tension in the problem is determined by constaint

Working $T - m_a g = m_A a_A$

$$T - m_B g = -m_B a_A$$

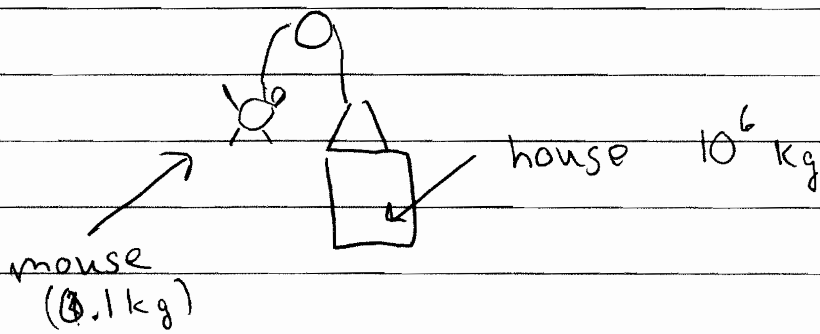
Subtracting

$$-m_A g + m_B g = (m_A + m_B)\, a$$

Leading to:

$$a_A = g\,\frac{(m_B - m_A)}{(m_B + m_A)}$$

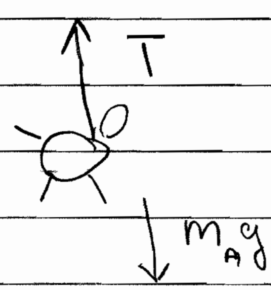
Check:



Clearly house is going to fall with accelerati
g, sending $m_B \gg m_a$ (house is much heavier than the mouse!!!)

$$a_{mouse} = g\left(\frac{10^6\,kg - 0.1\,kg}{10^6\,kg + 0.1\,kg}\right) \simeq g\,\frac{\cancel{10^6\,kg}}{\cancel{10^6\,kg}}$$

Clearly in this limit

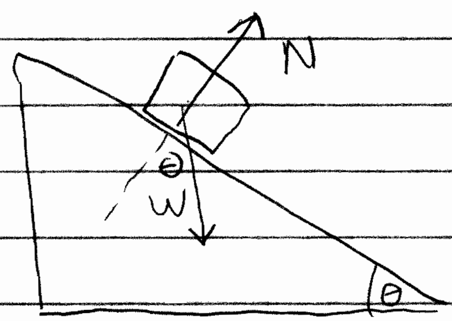


acceleration of mouse

$$T - m_a g \simeq m_a g$$

$$T = 2 m_a g$$

Inclined Plane

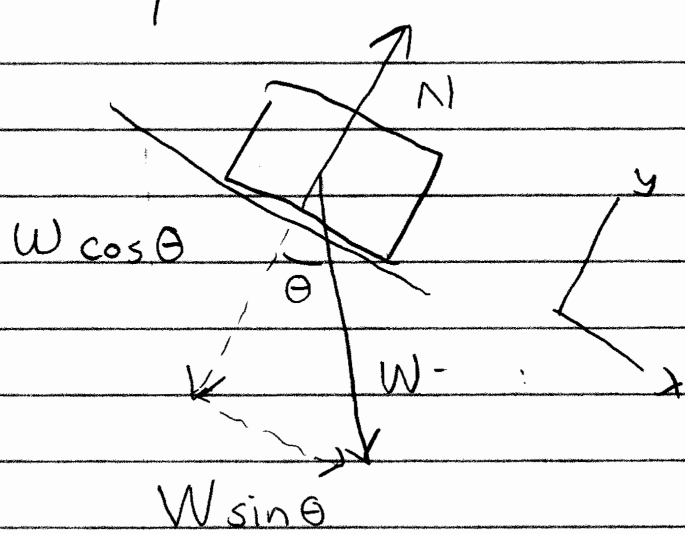


- A 1kg brick is allowed to slide on a frictionless plane with angle θ. Determine the acceleration

- This problem is normally solved by turning your head to look along the incline - see example 4-16

- We will [redacted] do this and also not to show the general method

(1) Draw a free body diagram for each body and write down Newton's Laws in all possible directions. Break up vectors into components



In x direction

In y-direction

(1) $W \sin\theta = ma_x$

$-W\cos\theta + N = m\cancel{a_y}^{0}$ (2

(see constraint

(2) Then apply the constaints y = constraints

y = Constant

y = Constant

$$\frac{dy}{dt} = 0$$

$$a_y = \frac{d^2 y}{dt^2} = 0$$

(3) Count, unknowns N, a_x and two constrain

Now work

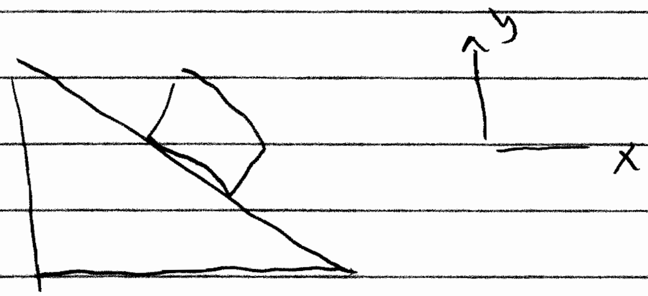
(1) $\Rightarrow$ $\cancel{m}g \sin\theta = \cancel{m}a_x \Rightarrow \boxed{a_x = g\sin\theta}$

(2) $\Rightarrow$ $N = mg\cos\theta$

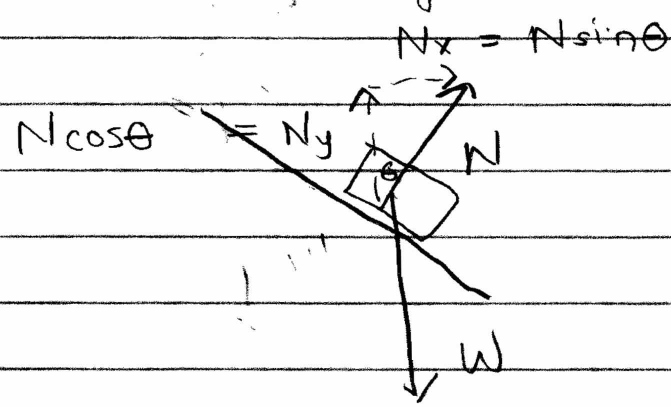
Note the normal force is less than the full weight

Now we will use the regular x and y axes.

Of course there is no particular reason you need to rotate your head



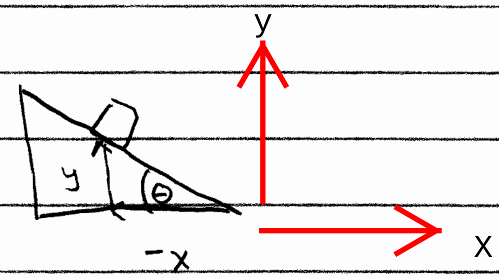
① Free body diagram



(1) $-W + N\cos\theta = ma_y$ (2) $N\sin\theta = ma_x$

② Constraint

$$\frac{y}{-x} = \tan\theta$$



Note that x is negative

$$a_y = \frac{d^2y}{dt^2} = -\tan\theta \frac{d^2x}{dt^2} = -\tan\theta \, a_x$$

This makes sense: for a step $dx > 0$, then $dy < 0$

So

$$-W + N\cos\theta = -m\tan\theta\, a_x \quad \text{and} \quad N\sin\theta = m a_x$$

$$-mg + N\cos\theta = m\tan\theta\, a_x \quad \text{and} \quad N = \frac{m a_x}{\sin\theta}$$

So,

$$-\cancel{m}g + \cancel{m}a_x \frac{\cos\theta}{\sin\theta} = \cancel{m}\tan\theta\, a_x$$

Multiply by cos

$$-g\sin\theta + a_x\cos\theta = -\frac{\sin^2\theta}{\cos\theta} a_x$$

Now multiply by sin

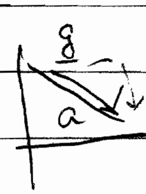
$$-g\sin\theta\cos\theta + a_x\cos^2\theta = -\sin^2\theta\, a_x$$

$$-g\cos\theta\sin\theta = -a_x\,(\cos^2 + \sin^2$$

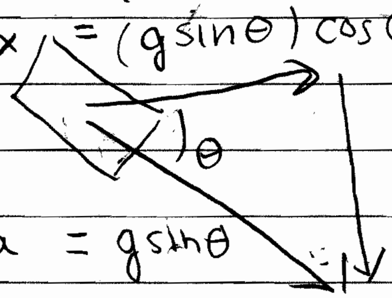
with cos^2 + sin^2 = 1

$$g\cos\theta\sin\theta = a_x \quad \checkmark$$

This is right because
a = gsin(theta)
So ax is
given by
the following
picutre



$$a_x = \overbrace{(g\sin\theta)}^{a}\cos\theta$$



$$a = g\sin\theta$$

$$a_y = -\underbrace{(g\sin\theta)}_{a}\sin\theta$$

Also note that ay = -tan(theta) ax = - g sin(t) sin(t)