Last Times

Newton Laws

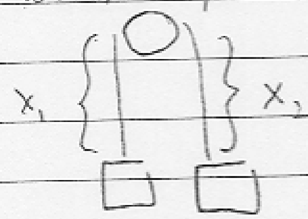
① For every object which can move (a free body) Draw all forces; Be sure to identify all equal and opposite forces; Write down $\vec{F}_{net} = m\vec{a}$ in all directions

② Often there are constraints amongst the coordinates which relates the accelerations of different bodies

- One constaint per tension or normal force

equations of constraint

Ex.



Length of rope

$$x_1 + x_2 = L$$

$$\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} = 0$$

$$a_1 + a_2 = 0 \Rightarrow a_1 = -a_2$$

if one moves up the other moves down

③ If you have written all Newton Laws and all equations of constraint, you should be able to determine all accelerations and all forces of constaint (i.e. tension & normal forces)

Friction

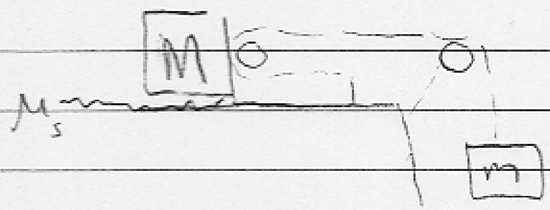
Static Friction: $|\vec{F}_s| < |\mu_s \vec{N}|$ and opposes the motion

Kinetic Friction:

$$|\vec{F}_k| = |\mu_k \vec{N}|$$ and opposes the motion

Today two more challenging probs

Problem



$M = 20kg$

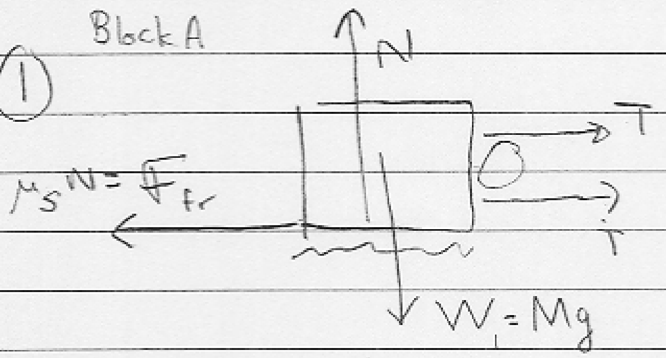
$\mu_s = 0.5$

$\mu_k = 0.4$

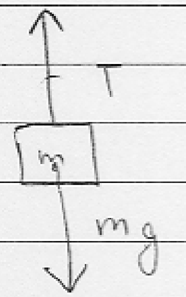
Determine for what mass m it will break free

Block A

(1)



Block b



$x|$ $\quad 2T - F_{fr}^{max} = M a_x^A$

$y|$ $\quad T - mg = m a_y^B$

$y|$

$$-Mg + N = M a_y^A$$

$$F_{fr}^{max} = \mu_s N$$

(2) Equations of Constraint

$x = Const \Rightarrow a_x^A = 0$

$y_A = const \Rightarrow a_y^A = 0$

$y_B = const \Rightarrow a_y^B = 0$

Now $\underbrace{\text{work} = \mu_s N}$

$$2T - F_{fr}^{max} = 0$$

$$T - mg = 0$$

$$T = mg$$

$$-Mg + N = 0$$

$$N = Mg$$

So

$$2mg - \mu_s Mg = 0$$

$$m = (0.5) \times \frac{20kg}{2} = 5kg$$

$$\boxed{m = \mu_s \frac{M}{2}}$$

→ See next page for additional analysis of the pin

Now suppose it Breaks free what is the accele

Co

(1) The free Body diagram is the same as before with μ_k instead of μ_s

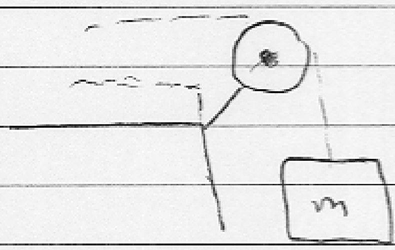
x] $2T - \mu_k N = Ma_x^A$

y] $T - mg = ma_y^B$

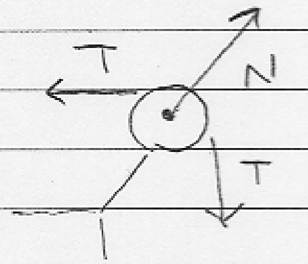
y] $-Mg + N = M\cancel{a_y^A}^{\,0}$

(2) Constraints $y^A = \text{const} \Rightarrow a_y^A = 0$

Determine the magnitude of the force on the pin



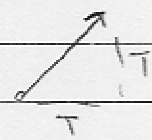
Solution since wheel is not moving



The acceleration is zero

$$\vec{F}_{net} = m\cancel{\vec{a}}^{\,0}$$

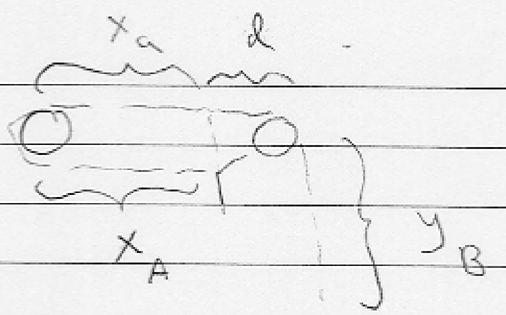
$$\vec{N} - T\hat{j} - T\hat{i} = 0$$



$$\vec{N} = T\hat{i} + T\hat{j}$$

$$|\vec{N}| = \sqrt{T^2 + T^2} = \sqrt{2}\,T = \sqrt{2}\,mg$$

$$= \sqrt{2}\,(5kg)(9.8) \simeq 70N$$



$$x_A + x_A + l + y_B = \text{Length of rope}$$

$$2x_A + l + y_B = \text{'' \quad --}$$

$$2\frac{d^2x_A}{dt^2} + 0 + \frac{d^2y_B}{dt^2} = 0 \Rightarrow dx^A = -\frac{dy^B}{2}$$

$$2a_x^A = -a_y^B$$

(3) Now Solve

$$2T - \mu_k Mg = Ma_x^A \qquad T - mg = ma_y^B$$

$$T - mg = -2ma_x^A$$

$$2(mg - 2ma_x^A) - \mu_k Mg = Ma_x^A$$

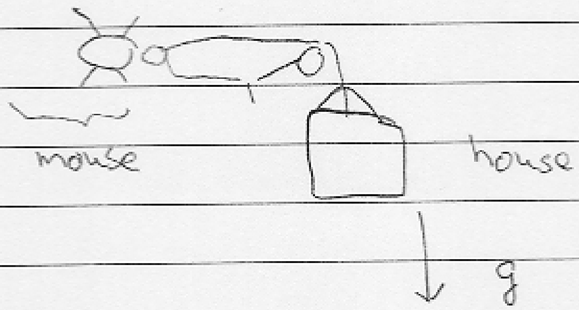
$$T = mg - 2ma_x^A$$

$$2mg - 4ma_x^A - \mu_k Mg = Ma_x^A$$

$$2mg - \mu_k Mg = (M + 4m)\, a_x^A$$

$$\frac{2mg - \mu_k Mg}{M + 4m} = a_x^A$$

Check: Suppose



Expect

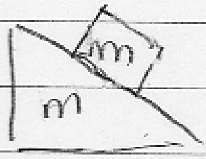
$a_x^A = g/2$ $\Leftarrow$ everytime the house moves abit the mouse half as much

Check $a_x^A = \dfrac{2mg - \mu_k Mg}{M + 4mg}$

$$a_x^A \simeq \frac{2(\infty)g - \mu_k Mg}{M + 4(\infty)g}$$

$$a_x^A \simeq \frac{2(\cancel{\infty})g}{4(\cancel{\infty})g} \simeq \frac{g}{2}$$

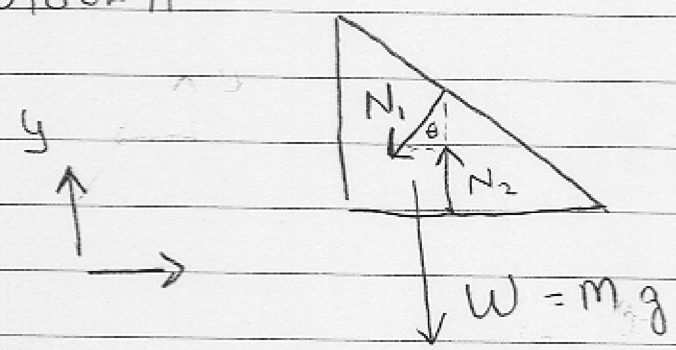
Two Blocks: equal mass



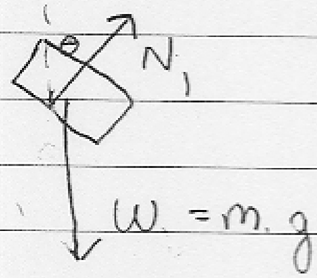
Determine the acceleration:

(1) Draw Free Body diagram

Block A

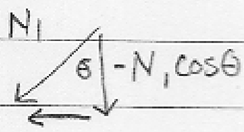


Block B



and write down Newton Laws in x and y direction

Block A:



x: direction

$$-N_1 \sin\theta = m a_x$$

(Eq #1)

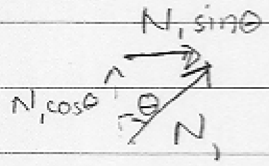
y: direction

$$-N_1 \cos\theta - m g + N_2 = m a_y$$

(Eq #2)

Block (B)

x) direction



y) direc

$$N_1 \sin\theta = m\, a^b_x \qquad \text{(Eq \#3)}$$

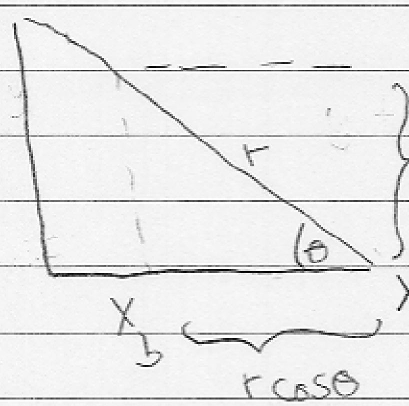
$$N_1 \cos\theta - m g = m a^b_y \qquad \text{(Eq \#4)}$$

(2) Constraints

$y = \text{constant}$

$a_y = \ddot{y} = 0 \leftarrow$ Block a doesn't fall into ground

Now one needs to recognize a relation between the coords



$$r\sin\theta = y_b \Rightarrow r = \frac{y_b}{\sin\theta}$$

$$x_b = x_a - r\cos\theta = x_a - y_b \frac{\cos\theta}{\sin\theta}$$

Thus

$$\frac{d^2x_b}{dt^2} = \frac{d^2x_a}{dt^2} - \frac{d^2y_b}{dt^2}\,\frac{\cos\theta}{\sin\theta} \qquad \text{(Eq \#5)}$$

$$a_x^b = a_x^a - a_y^b\,\frac{\cos\theta}{\sin\theta} \implies a_y^b = (a_x^a - a_x^b)\tan\theta$$

③ Count unknowns and equations

<u>Unknowns:</u> $N_1,\ a_x^a,\ N_2,\ a_x^b,\ a_y^b$ ✓

<u>Equations:</u> Eq #1 – Eq #5 ✓

Now work

$$N_1 = -\frac{m a_x^a}{\sin\theta} \qquad \text{(Used Eq \#1)}$$

So

$$+N_1 \sin\theta = m a_x^b \qquad \text{(Used Eq \#3)}$$

$$-\left(\frac{\cancel{m} a_x^a}{\cancel{\sin\theta}}\right)\cancel{\sin\theta} = \cancel{m} a_x^b$$

$$-a_x^a = a_x^b \equiv a \qquad \text{(Eq \#6)}$$

Then $\boxed{a_y^b = -2a_x^b \tan\theta}$ (Eq #7)

Now use Eq 4

$$N_1 \cos\theta - mg = m a_y^b$$

$$-\frac{m a_x^a}{\sin\theta}\cos\theta - mg = m a_y^b$$

$$-a_x^a \frac{\cos\theta}{\sin\theta} - g = a_y^b$$

$$-a_x^a \frac{1}{\tan\theta} - g = -2a_x^b \tan\theta$$

From Eq #5

$$a_y^b = (a_x^a - a_x^b)\tan\theta$$

$$a_y^b = -2a_x^b \tan\theta \quad \text{(Eq\#}$$

So Eq 6 Eq #7

$$+a_x^b \frac{1}{\tan\theta} - g = -2a_x^b \tan\theta$$

$$a_x^b \quad -g\tan\theta = -2a_x^b \tan^2\theta$$

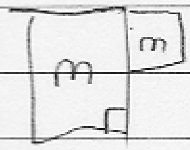
$$a_x^b (1 + 2\tan^2\theta) = g \tan\theta$$

Thus,

$$\boxed{a_x^b = g\frac{\tan\theta}{1+2\tan^2\theta}}$$

$$\boxed{a_y^b = -g\left(\frac{2\tan^2\theta}{1+2\tan^2\theta}\right)}$$

Check:



When $\theta = \pi/2$ expect $a_x = 0$ $a_y = -g$

$\tan \pi/2 \longrightarrow \infty$

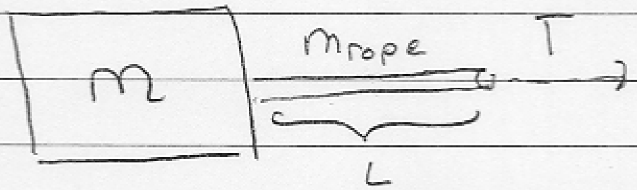
So

$$a_x^b = g \frac{\infty}{1 + 2\infty^2} \simeq \frac{g\,\infty}{2\infty^2} \simeq \frac{g}{2\infty} = 0$$

$$a_y^b = -g\left(\frac{2\infty^2}{1 + 2\infty^2}\right) \simeq -g\left(\frac{\cancel{2\infty^2}}{\cancel{2\infty^2}}\right)$$

$$a_y^b \simeq -g$$

Massive Rope



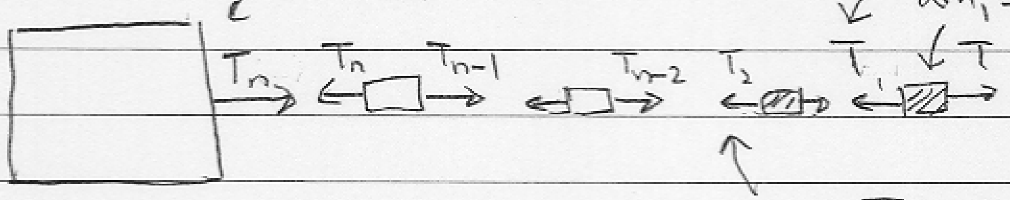
Whats the accel?

The mass has constant density:

$$\rho = \frac{m_{rope}}{L}$$

Break the rope up into $\quad T_1 \approx T + dT$

$T(L) = T_n \qquad dm_1 = \rho\, dx = \frac{m_{rope}}{L} dx$



$$T_2 = T_1 - dT$$

Draw a free body diagram

$$T - T_1 = dm \cdot a$$

$$T - (T + dT) = (\rho\, dx)\; a$$

$$-dT = (\rho\, dx)\, a$$

$$dT = -\frac{m_{rope}}{L}\, a\, dx$$

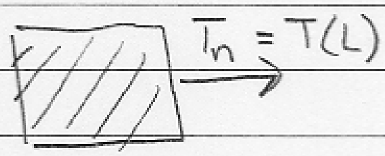
Integrating

$$\int_{T}^{T(L)} dT = -\frac{m_{rope}}{L} a \int_{0}^{L} dx$$

$$T(L) - T = -m_{rope} \frac{a}{\cancel{L}} \cdot \cancel{L}$$

$$T(L) = T - m_{rope} a$$

So



← Free body diagram of box

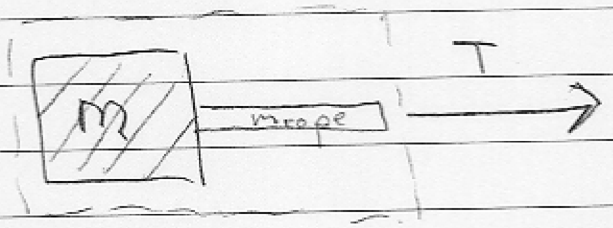
$$T(L) = Ma$$

$$T - m_{rope} a = Ma$$

$$T = (M + m_{rope}) a$$

$$\boxed{\frac{T}{M + m_{rope}} = a}$$

Discussion:



① Draw a fictitious box around the whole system.

② Then apply Newtons Law to the whole box

③ All the internal forces (forces in the box) cancel each other since they are equal and opposite

e.g. Force on box by rope = −F on rope by box

④ So

$$F_{net}^{external} = M_{TOT}\, a$$

$$T = (M + m_{rope})\, a$$

$$\boxed{\frac{T}{M + m_{rope}} = a}$$