Rocket Problem 10/15

$V_A = (4\hat{i} + 5\hat{j} - 2\hat{k})$ m/s

$\vec{V}_B = 0$ m/s

$\vec{V}_A' = (-2\hat{i} + 3\hat{k})$

Momentum conservation

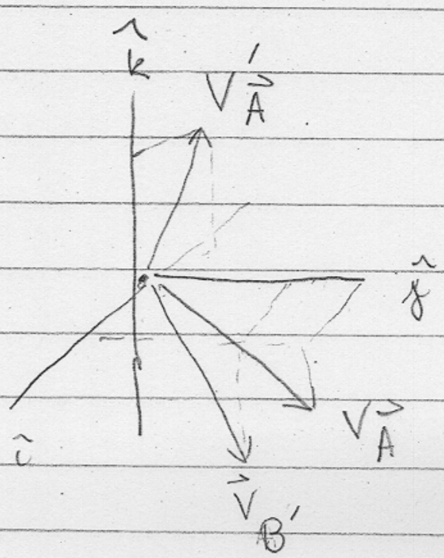
$$m_A \vec{V}_A + m_B \vec{V}_B^{\,0} = m_A \vec{V}_A' + m_B \vec{V}_B'$$

$$\text{kg}\frac{m}{s} \cdot 2 \begin{pmatrix} 4 \\ 5 \\ -2 \end{pmatrix} = \left[2 \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} + 3 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \right] \text{kg m/s}$$

$x = 4$

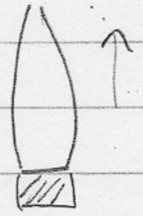
$y = 10/3 \quad \longrightarrow \quad \vec{V}_B' = (4\hat{i} + \frac{10}{3}\hat{j} - \frac{10}{3}\hat{k})$

$z = -10/3$



$$\text{speed} = \left(4^2 + \left(\frac{10}{3}\right)^2 + \left(\frac{10}{3}\right)^2\right)^{1/2} \simeq 6.2 \text{ m/s}$$

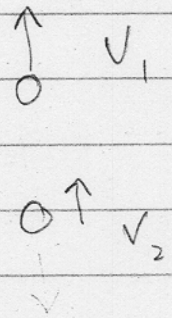
The initial rocket



$v_R = 6 \times 10^3 \, m/s$

$m_R = 925 \, kg$

Splits into two pieces of equal mass: the relative speed is



$v_{Rel} = 2.8 \times 10^3 \, m/s$

Determine the speeds of each fragment and the work done during the explosion

From momentum conservation

$$m_R V_R = m_1 v_1 + m_2 V_2 \qquad v_1 - V_2 = v_{rel}$$

$$m_R v_R = \frac{m_R}{2} v_1 + \frac{m_R}{2}(-v_{rel} + v_1) \qquad V_2 = -V_{rel} + v_1$$

$$v_R = v_1 - v_{rel}/2 \implies \boxed{v_1 = v_R + v_{rel}/2}$$

$$V_2 = -v_{rel} + v_1 = -v_{rel} + (v_R + v_{rel}/2)$$

$$\boxed{V_2 = v_R - v_{rel}/2}$$

According to work - theorem

$$W = \Delta K + \Delta \cancel{PE}^{\,0}$$

the rocket explodes at one hight, i.e the

$$W = + \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{1}{2} m_R v_R^2$$

$$W = \frac{1}{2} \frac{m_R}{2} \left(v_R + \frac{v_{rel}}{2}\right)^2 + \frac{1}{2} \frac{m_R}{2} \left(v_R - \frac{v_{rel}}{2}\right)^2 - \frac{1}{2} m_R v_R^2$$

$$W = \frac{1}{2} \frac{m_R}{2} \left(v_R^2 + 2 v_R v_{rel} + \frac{1}{4} v_{rel}^2\right) + \frac{1}{2} \frac{m_R}{2} \left(v_R^2 - 2 v_R v_{rel} + \frac{1}{4} v_{rel}^2\right) - \frac{1}{2} m_R v_R^2$$

$$W = \frac{1}{8} m_R v_{rel}^2$$