1 Problems

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$$15.27, 15.37, 15.41, 15.79 \tag{1}$$

1. (**Estimates**) Estimate the mass per length of a double bass string. The lowest note on the bass is an $E1$, ringing in at a cool 41 Hz just above human hearing. Based on the picture below, estimate the wavelength of this fundamental mode, and the velocity of a wave in the double bass string. Estimate the tension in the double bass low E string – give your answer in pounds and newtons.

2. (**Sinusoidal averages**) We have argued that $\overline{\sin^2(kx-\omega t)} = \frac{1}{2}$ using the fact that $\sin^2+\cos^2 = 1$. Here we would like to show result this more directly. The average of $\sin^2$ and $\cos^2$ over a period T is

$$\overline{\sin^2\left(\frac{2\pi t}{T}\right)} \equiv \frac{1}{T}\int_0^T \mathrm{d}t \, \sin^2\left(\frac{2\pi t}{T}\right) \tag{2}$$

$$\overline{\cos^2\left(\frac{2\pi t}{T}\right)} \equiv \frac{1}{T}\int_0^T \mathrm{d}t \, \cos^2\left(\frac{2\pi t}{T}\right) \tag{3}$$

These integrals are computed by using the identity

$$\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x) \tag{4}$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x) \tag{5}$$

(a) Draw a graph of $\sin^2(x)$ and $\cos^2(x)$ and give a graphical explanation of the indentities given in Eq. 4 and Eq. 5.

(b) Use these triginometic identities given to perform the integrals in Eq. 2 and Eq. 3 and show the result

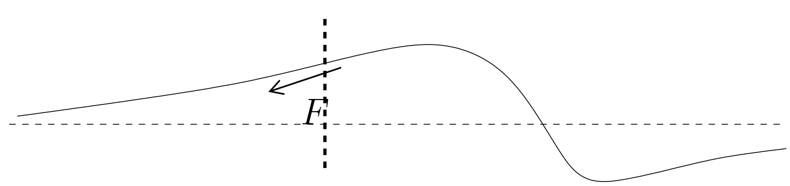
$$\overline{\sin^2} = \overline{\cos^2} = \frac{1}{2}$$

3. (**Power**)

(a) Derive the formula

$$P = -F_T \frac{\partial y}{\partial x}\frac{\partial y}{\partial t} \tag{6}$$

for the energy transported from left to right per unit time at position x and time t. Start with the following picture and the formula $dW/dt = \mathbf{F}\cdot\mathbf{v}$. You should use the small angle approximaion $\sin(\theta) \approx \tan\theta$ and relate $\tan\theta =$ slope of y vs. x. The picture shows the wave to the left of the dashed line doing work (per time) on the right hand side of the string.



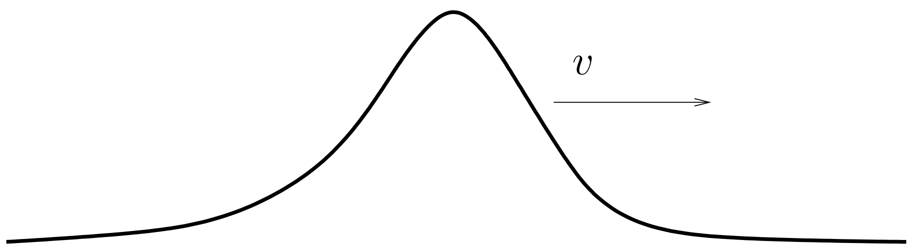
(b) For the sinusoidal wave $y = A\sin(kx \mp \omega t)$ show starting from Eq. 6 that the time averaged power is

$$\overline{P} = \pm\frac{1}{2}\mu A^2\omega^2 v\,. \tag{7}$$

You might find it instructive to compare this derivation to the one given in class. The minus sign for the power delivered from left to right in the left moving case, means that the left mover delivers positive power from right to left

$$\overline{P}_{R\to L} = -\overline{P} = +\frac{1}{2}\mu A^2\omega^2 v$$

4. (**Travelling waves**) This problem concerns a simple right moving $y(x,t) = f(x - vt)$ wave shown below:



(a) Show that the up and down "particle velocity" can be related to the slope of the function

$$\frac{\partial y}{\partial t} = -v\frac{\partial y}{\partial x} \tag{8}$$

Sketch this velocity for the wave given above.

(b) For the wave shown above sketch the kinetic, potential energies, and total energies.

(c) Show that the kinetic energy equals the potential energy:

$$\frac{dK}{dx} = \frac{dU}{dx}$$

(d) Show that the energy density is

$$u_E = F_T\left(\frac{\partial y}{\partial x}\right)^2 \tag{9}$$

and the power is

$$P = u_E v \tag{10}$$

5. (**Energetics of a standing wave**)

(a) Consider a travelling wave:

$$A\sin(kx - \omega t)\,. \tag{11}$$

At time $t = 0$, make a plot of the kinetic and potential and total energies in the string.

(b) Now consider a standing waves and focus on the second harmonic $n = 2$ for definiteness.

$$y(x) = 2A\sin(k_n x)\cos(\omega_n t)$$

Determine the kinetic and potential energies per length

(c) Determine the time averaged potential and kinetic energies per length and make a graph of the both of these.

(d) Show that the averaged energy per unit length is indendependent of x and is given by

$$u_E = F_T A^2 k_n^2 = \mu A^2\omega_n^2 \tag{12}$$

(e) Determine the power transmitted in the standing wave pattern. What is the power transmitted at the nodal and anti-nodal points of the standing wave pattern? Explain.

(f) What is the time averaged power transmitted in the standing wave pattern?

6. (**Transmission and Reflection**) The relfection coefficient is defined as:

$$R = \frac{\text{Time averaged power delivered by the reflected wave from right to left}}{\text{Time averaged power delivered by the incident wave from left to right}} \tag{13}$$

The transmission coefficient coefficient is defined as:

$$T = \frac{\text{Time averaged power delivered by the transmitted wave from left to right}}{\text{Time averaged power delivered by the incident wave from left to right}} \tag{14}$$

(a) Compute the reflection and transmission coefficients. To check your work you can do the next problem.

(b) Show that

$$R + T = 1\,. \tag{15}$$

Explain the physical significance of this result.

(c) Examine formula for the the reflected amplitude (i.e. B in the handout)

$$B = A\left[\frac{k_L - k_R}{k_L + k_R}\right] \tag{16}$$

Show that when the incident wave reaches a wall $\mu_R \to \infty$ the reflected wave is inverted. Show that when the incident end reaches a light string $\mu_R \to 0$ the reflected wave is not inverted but recoils with the same amplitude as the incident wave.

(d) Make a graph of the transmission and reflection coefficient as a function of μ_R/μ_L with μ_R/μ_L varying from zero to infinity.