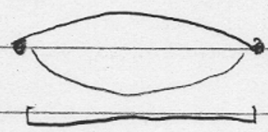
Estimates

Now $L \sim 1m$

$m \sim 10g$

$\mu = \frac{10g}{1m} = 0.01 \frac{kg}{m}$

Then



$1m = \frac{\lambda}{2}$

$\lambda = 2m$ $f = 41 Hz$

$f\lambda = v$

$\left(41 \frac{1}{s}\right)\left(2m\right) = 82 m/s = v$

Now

$v = \sqrt{\frac{F_T}{\mu}}$ so $v^2 \mu = F_T$

So

$(82 m/s)^2 \left(0.01 \frac{kg}{m}\right) = F_T$

$67 \text{ newtons} = F_T$

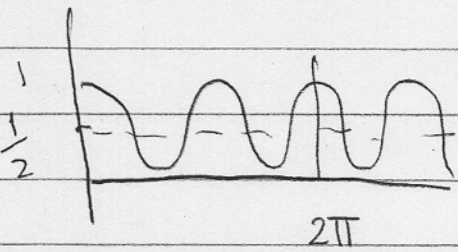
$15.0 lbs = F_T$

Or

$F_T = 15 \text{ lbs} \left(\frac{f}{41 Hz}\right)\left(\frac{1L}{1m}\right)^2 \left(\frac{m}{10g}\right)\left(\frac{1m}{L}\right)$

Sinusoidal Ave

$\cos^2(x)$



$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$$

theres is a $\frac{1}{2}\cos$ oscillating around $\frac{1}{2}$ @ twice the freq

Similarly

$$\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos 2x$$

So

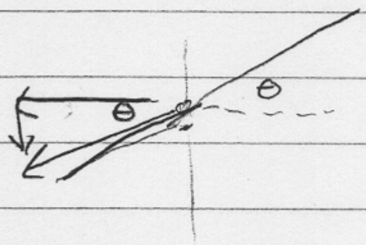
$$\overline{\sin^2\left(\frac{2\pi t}{T}\right)} = \frac{1}{T}\int_0^T \sin^2\left(\frac{2\pi t}{f}\right) = \frac{1}{T}\int_0^T \frac{1}{2} - \frac{1}{2}\cos\frac{2\pi t}{T}$$

$$= \frac{1}{T}\left(\frac{T}{2} + 0\right)$$

Sim for $\overline{\cos^2}$

Power

a)



$$P = \vec{F} \cdot \vec{v}$$

$$P = F^y v^y$$

$$P = (-F_T \sin\theta)\left(\frac{\partial y}{\partial t}\right)$$

Now $\sin\theta \simeq \tan\theta = \frac{\partial y}{\partial x}$

So

$$P = -F_T \frac{\partial y}{\partial x}\frac{\partial y}{\partial t}$$

b)

$$P = -F_T \frac{\partial y}{\partial x}\frac{\partial y}{\partial t} \qquad y = A\sin(kx - \omega t)$$

$$P = -F_T A k \cos(kx - \omega t)\left[-A\omega\cos(kx - \omega t)\right]$$

$$P = F_T A^2 k\omega \cos^2(kx - \omega t)$$

Now $\overline{\cos^2(kx-\omega t)} = \frac{1}{2}$, $k = \frac{\omega}{v}$, $v^2 = \frac{F_T}{\mu}$, so

$$\overline{P} = \frac{1}{2} F_T A^2 k\omega = \frac{1}{2} F_T \frac{\omega^2}{v} A^2 = \frac{1}{2}\mu v^2 \frac{\omega^2}{v} A^2 = \boxed{\frac{1}{2}\mu A^2 \omega^2 v = P}$$

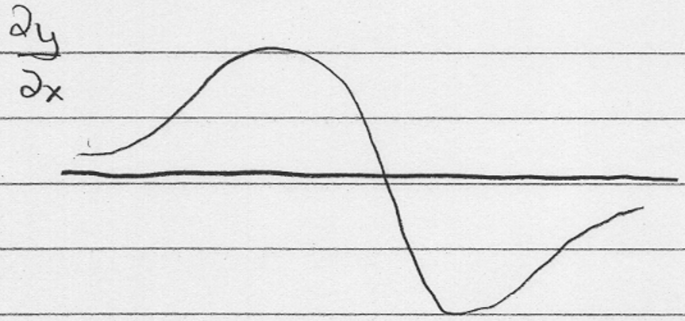
Travelling

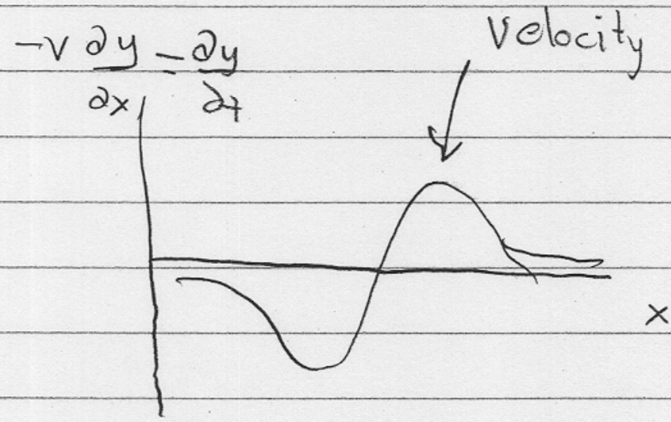
(a) $y = f(x - vt)$

$$\frac{\partial y}{\partial x} = f' \qquad \frac{\partial y}{\partial t} = (f') \cdot (-v)$$

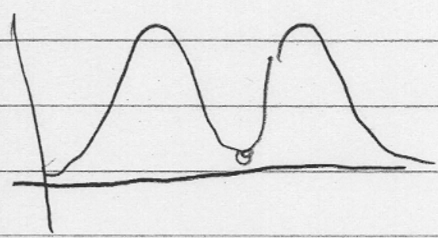
So

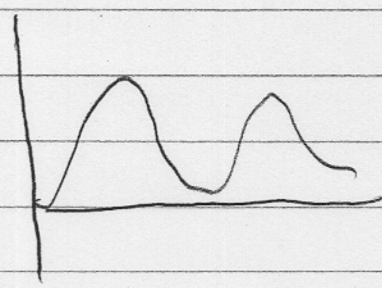
$$\frac{\partial y}{\partial t} = -v \frac{\partial y}{\partial x}$$





(b) $$\frac{dK}{dx} = \frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2 \qquad \frac{du}{dx} = \frac{1}{2}F_T\left(\frac{\partial y}{\partial x}\right)^2$$





Now (c)

$$\frac{dK}{dx} = \frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2 = \frac{1}{2}\mu\left(-v\frac{\partial y}{\partial x}\right)^2 =$$

$$= \frac{1}{2}\mu v^2\left(\frac{\partial y}{\partial x}\right)^2 = \frac{1}{2}F_T\left(\frac{\partial y}{\partial x}\right)^2 = \frac{dU}{dx}$$

Now (d)

$$u_E = \frac{dU}{dx} + \frac{dK}{dx} = 2\frac{dU}{dx} = F_T\left(\frac{\partial y}{\partial x}\right)^2$$

The power

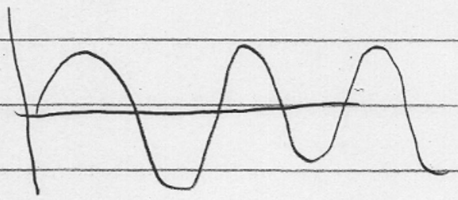
$$P = -F_T\frac{\partial y}{\partial x}\frac{\partial y}{\partial t} = -F_T\left(\frac{\partial y}{\partial x}\right)\left(-v\frac{\partial y}{\partial x}\right)^2$$

$$= F_T\left(\frac{\partial y}{\partial x}\right)^2 \cdot v = u_E v$$

(5) Standing Waves

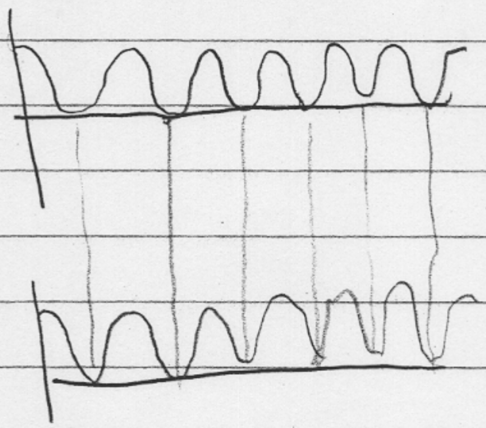
$y = A \sin(kx - \omega t)$

$t = 0$



KE

PE



(b))

$$y(x,t) = 2A \sin(kx) \cos(\omega t)$$

$$\frac{\partial y}{\partial t} = -2A\omega \sin(kx)\sin(\omega t) \qquad \frac{\partial y}{\partial x} = +2Ak\cos(kx)\cos(\omega t)$$

$$K = \frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2 = 2\mu A^2\omega^2 \sin^2(kx)\sin^2(\omega t)$$

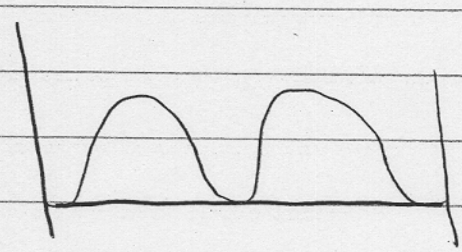
$$U = \frac{1}{2}F_F\left(\frac{\partial y}{\partial x}\right)^2 = 2F_T A^2 k^2 \cos^2(kx)\cos^2(\omega t)$$

$$\bar{K} = \mu A^2 \omega^2 \sin^2(kx)$$

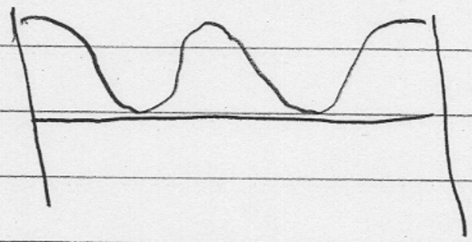
$$\bar{U} = F_T A^2 k^2 \cos^2(kx)$$

Then c)

K



U



d)

$$u_E = \mu A^2 \omega^2 \sin^2(kx) + F_T A^2 k^2 \cos^2(kx)$$

Note:

$$F_T A^2 k^2 = F_T A^2 \frac{\omega^2}{V^2} = \mu A^2 \omega^2$$

$$u_E = \mu A^2 \omega^2 \left(\sin^2(kx) + \cos^2(kx)\right) = \mu A^2 \omega^2 = F_T A^2 k^2$$

e

$$P = -F_T \frac{\partial y}{\partial x} \frac{\partial y}{\partial t}$$

$$= -F_T \left(-2A\omega \sin(kx)\sin(\omega t)\right)\left(+2Ak\cos(kx)\cos\omega t\right)$$

$$\boxed{P = F_T A^2 k^2 v \left[\sin(2kx) \sin(2\omega t) \right]}$$

$$\overline{P} = F_T A^2 k^2 v \left[\sin(2kx) \overline{\sin(2\omega t)} \right]$$

$$= 0$$

Power is zero at the nodal and anti-nodal points

Trans + Reflect

a) $$T = \frac{\frac{1}{2}\mu_R C^2 \omega^2 v_R}{\frac{1}{2}\mu_L A^2 \omega^2 v_L} = \frac{\text{transmitted pow}}{\text{incident pow}}$$

Note $$\frac{\mu_R v_R}{\mu_L v_L} = \sqrt{\frac{\mu_R}{\mu_L}}$$

$$T = \sqrt{\frac{\mu_R}{\mu_L}} \left(\frac{C}{A}\right)^2 = \sqrt{\frac{\mu_R}{\mu_L}} \left(\frac{4\mu_L}{(\sqrt{\mu_L} + \sqrt{\mu_R})^2} \right)$$

$$T = \frac{4\sqrt{\mu_L \mu_R}}{(\sqrt{\mu_L} + \sqrt{\mu_R})^2}$$

$$R = \frac{\frac{1}{2}\mu_L B^2 \omega^2 v_L}{\frac{1}{2}\mu_L A^2 \omega^2 v_L} = \frac{\text{Reflected pow}}{\text{incident pow}}$$

So

$$R = \frac{B^2}{A^2} = \left[\frac{(\sqrt{\mu_L} - \sqrt{\mu_R})^2}{(\sqrt{\mu_L} + \sqrt{\mu_R})^2} \right]$$

b) $$R + T = \frac{1}{(\sqrt{\mu_L} + \sqrt{\mu_R})^2} \left[(\sqrt{\mu_L} - \mu_R)^2 + 4\sqrt{\mu_L}\sqrt{\mu_R} \right]$$

$$R + T = \frac{}{(\sqrt{\mu_L} + \sqrt{\mu_R})^2} \left[(\sqrt{\mu_L} + \sqrt{\mu_R})^2 \right] = 1$$

This result expressed energy consv:

$$E_{in} = E_{reflected} + E_{transmitted}$$

c) $$B = A \left[\frac{k_L - k_R}{k_L + k_R} \right]$$

$$\omega = v k$$

$$\frac{\omega}{v} = k$$

So

$$\frac{B}{A} = \left[\frac{k_L/k_R - 1}{k_L/k_R + 1} \right]$$

$$\sqrt{\mu} \left(\frac{\omega}{\sqrt{F_T}} \right) = k$$

$$\frac{k_L}{k_R} = \frac{\sqrt{\mu_L}}{\sqrt{\mu_R}}$$

$$B = A\left[\frac{\sqrt{\mu_L} - \sqrt{\mu_R}}{\sqrt{\mu_L} + \sqrt{\mu_R}}\right]$$

So

$$\sqrt{\mu_R} \to 0$$

We have

$$B = A\left[\frac{\sqrt{\mu_L}}{\sqrt{\mu_L}}\right] = A \quad \leftarrow \text{not inverted } \mu_R \to 0$$

Then

$$\sqrt{\mu_R} \to \infty$$

$$B = A\left[\frac{-\sqrt{\mu_R}}{\sqrt{\mu_R}}\right] = -A \quad \leftarrow \text{Reflected wave inverted } \sqrt{\mu_R} \to \infty$$

Now

$$T = \frac{4\sqrt{\frac{\mu_R}{\mu_L}}}{\left(1 + \sqrt{\frac{\mu_R}{\mu_L}}\right)^2} \qquad R = \frac{\left(1 - \sqrt{\mu_R}/\sqrt{\mu_L}\right)^2}{\left(1 + \sqrt{\mu_R}/\sqrt{\mu_L}\right)^2}$$

$$T = \frac{4x}{(1+x)^2} \qquad R = \frac{(1-x)^2}{(1+x)^2} \qquad x = \sqrt{\frac{\mu_R}{\mu_L}}$$

So