Quantity	**Symbol**	**Value**
Coulombs Constant	$\frac{1}{4\pi\epsilon_o}$	$8.98 \times 10^9\,\mathrm{Nm^2/C^2}$
Electron Mass	m_e	$9.1 \times 10^{-31}\,\mathrm{kg}$
Proton Mass	m_p	$1.67 \times 10^{-27}\mathrm{kg}$
Electron Charge	e	$-1.6 \times 10^{-19}\,\mathrm{C}$
Electron Volt	eV	$1.6 \times 10^{-19}\,\mathrm{J}$
Permitivity	ϵ_o	$8.85 \times 10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{Nm^2}}$
Magnetic Permeability	μ_o	$4\pi \times 10^{-7}\,\mathrm{N \cdot A^2}$
Speed of Light	c	$3.0 \times 10^8\,\mathrm{m/s}$
Planck's Constant	h	$6.6 \times 10^{-34}\mathrm{m^2kg/s}$
Planck's Constant/2π	$\hbar$	$1.05 \times 10^{-34}\mathrm{m^2kg/s}$

Integrals	**Value**	
$\int_{-\infty}^{\infty} du\, e^{-\alpha u^2}$	$\sqrt{\frac{\pi}{\alpha}}$	
$\int_{-\infty}^{\infty} du\, u^2 e^{-\alpha u^2}$	$\frac{1}{2\alpha}\sqrt{\frac{\pi}{\alpha}}$	
$\int_0^{\infty} du\, u^n e^{-\alpha u}$	$\frac{n!}{\alpha^{n+1}}$	
$\int du\, \sin^2(\alpha u)$	$\frac{u}{2} - \frac{\sin(2\alpha u)}{4\alpha}$	
$\int du\, \cos^2(\alpha u)$	$\frac{u}{2} + \frac{\sin(2\alpha u)}{4\alpha}$	
$\int_{-\frac{1}{2}}^{+\frac{1}{2}} du u^2 \sin^2(n\pi u)$	$\frac{-6+n^2\pi^2}{24n^2\pi^2}$	$n = 2, 4, 6, 8$
$\int_{-\frac{1}{2}}^{+\frac{1}{2}} du u^2 \cos^2(n\pi u)$	$\frac{-6+n^2\pi^2}{24n^2\pi^2}$	$n = 1, 3, 5, 7$
$\int (\cos(\theta))^\alpha \sin(\theta) d\theta$	$\frac{-1}{\alpha+1}\,(\cos(\theta))^{\alpha+1}$	
$\int (\sin(\theta))^\alpha \cos(\theta) d\theta$	$\frac{+1}{\alpha+1}\,(\sin(\theta))^{\alpha+1}$	

n	ℓ	m	$\Phi_m(\varphi)$	$\Theta_{lm}(\theta)$	$R_{nl}(r)$	Ψ_{nlm}
1	0	0	1	1	$\frac{1}{\sqrt{\pi a_o^3}} e^{-r/a_o}$	$\frac{1}{\sqrt{\pi a_o^3}} e^{-r/a_o}$
2	0	0	1	1	$\frac{1}{\sqrt{32\pi a_o^3}} \left(2 - \frac{r}{a_o}\right) e^{-r/2a_o}$	$\frac{1}{\sqrt{32\pi a_o^3}} \left(2 - \frac{r}{a_o}\right) e^{-r/2a_o}$
2	1	0	1	$\sqrt{3}\cos(\theta)$	$\frac{1}{\sqrt{96\pi a_o^3}} \frac{r}{a_o} e^{-r/2a_o}$	$\frac{1}{\sqrt{32\pi a_o^3}} \frac{r}{a_o} e^{-r/2a_o} \cos(\theta)$
2	1	±1	$e^{\pm i\varphi}$	$\sqrt{\frac{3}{2}}\sin(\theta)$	$\frac{1}{\sqrt{96\pi a_o^3}} \frac{r}{a_o} e^{-r/2a_o}$	$\frac{1}{\sqrt{64\pi a_o^3}} \frac{r}{a_o} e^{-r/2a_o} \sin(\theta) e^{\pm i\varphi}$

For potential $V = \frac{1}{2}\, k\, x^2$ the lowest wave functions and energies

$$
\begin{aligned}
\Psi_0 &= \left(\frac{1}{\sqrt{\pi} L}\right)^{1/2} e^{-y^2/2} \\
\Psi_1 &= \left(\frac{1}{\sqrt{\pi} L}\right)^{1/2} \sqrt{2} y e^{-y^2/2} \\
\Psi_2 &= \left(\frac{1}{\sqrt{\pi} L}\right)^{1/2} \frac{1}{\sqrt{2}} (2y^2 - 1) e^{-y^2/2}
\end{aligned}
$$

where

$$
y \equiv \frac{x}{L} \qquad L = \sqrt{\frac{\hbar}{M\omega_o}} \tag{1}
$$

1. Write down the electronic configuration of atomic iron $Z = 26$ and the electronic structure of sodium $Z = 11$.

2. Using an energy level diagram, explain how you specified the electron structure of iron.

3. The wave functions $u_{n\ell}$ of the $3d$ electrons of hydrogen obey a radial schrodinger equation which has a different effective potential from the the effective potential of the $3s$ electrons. Sketch the effective potential potential for the $3s$ and $3d$ electrons.

4. Using the Bohr model estimate the average radius of the valence electrons of the Na. Compare this average radius to the average radius for the electrons of the Neon core. Give your result in $\mathring{A}$.

5. The additional electron outside the Ne core can be excited to its first excited state and then this electron decays back to the ground state yielding a photon which has energy which is typical for atomic spectra.

   (a) What is the energy of a typical photon from an atomic spectral line, and what is the typical wavelength of the electromagnetic radiation?

   (b) Draw a figure which is approximately to scale showing the atom and the wavelength of the emitted photon.

   (c) What is the size of the sodium nucleus?

Make a model of an atom as an electron in an infinite square well potential of length L

$$V(x) = \begin{cases} 0 & |x| < L/2 \\ \infty & |x| > L/2 \end{cases} \tag{2}$$

1. Sketch the lowest three wave functions and their associated probability densities for this potential.

2. For an electron in the first excited state, determine the probability to find the electron between $-L/4$ to $L/4$. (Hint: don't calculate, graph!)

3. (symbol + number) Estimate the radius of an atom. To model the atom with a 1D box potential, take the size of the box to be the circumference associated with this radius. Determine the energy of the photon that emitted as the electron decays from its second excited state down to the first state. Evaluate your result in eV.

4. (symbol) For the ground state, estimate the variance in position Δx and indicate this variance in your probability graphs of part 1.

5. (symbol) Determine exactly the variance Δx for the ground state wave function of the particle in the box.

6. (Symbol + number) For sufficiently small box sizes L relativistic corrections become important. Explain qualitatively why this is so? Estimate the box length where the typical relativistic corrections to the kinetic energy exceed 10%. Evaluate your answer in $\mathring{A}$. The kinetic energy relativistically is $K = \sqrt{(cp)^2 + (mc^2)^2} - mc^2 \simeq \frac{p^2}{2m} + \frac{3}{8}\frac{(cp)^4}{(mc^2)^3}$

Consider the wave functions of hydrogen:

1. Take wave function of the $3d$ state with $m = 1$. What is the angular momentum squared and z component of angular momentum for this state?

2. Draw a graph of the radial wavefunctions of the $2s$ and $2p$ wave functions and there associated probability.

3. Determine the average radius of the $1s$ state of hydrogen.

4. Show that the $1s$ state satisfies the radial Schroödinger equation and determine the energy. (Hint differentiate really, realy carefully, check, check, check)

5. Determine a series expansion for the radial probability distribution $P(r)\,dr$ for small r for an electron in the $1s$ state. Work to leading order in r/a_o.

6. (symbol) The size of a proton is $r_p = 1\,\mathrm{fm}$ ($1 =$ fm is known as a femptometer or a Fermi and is $1 \times 10^{-15}\,\mathrm{m}$). Determine the probability that the $1s$ electron will be inside a radius of $1\,\mathrm{fm}$. (The series expansion determined in the last question may be useful.) If pressed for time, you can skip the numerical evaluation of this answer.

Hydrogen binds to Bromide in an ionic bond. Since Bromium is heavy it may be considered fixed, while the hydrogen nucleus vibrates around its equilibrium position The potential $V(x)$ experienced by hydrogen nucleus is given by

$$V(x) = V_0 \left(1 - \exp(-x/a)\right)^2 , \tag{3}$$

which is illustrated below. For simplicity take $V_o = 0.56\,\mathrm{eV}$ and $a = a_o$ the Bohr radius.

1. The picture shown below is the n-th excited state. What is the value of n? Explain.
2. Estimate the classical turning points based on this figure.
3. Estimate the energy of the wave function based on these figures.
4. Using the Schrödinger equation explain why oscillations seen in the figure have longer wavelength at larger x, and shorter wavelength at smaller x.
5. For small amplitude oscillations, show that the potential can be approximated with a simple harmonic oscillator potential and determine the spring constant.
6. Estimate the energy of a photon that is emitted when the molecule decays from its first vibrational state down to its ground state using the simple harmonic oscillator approximation for the potential? How would you describe this radiation, radio, microwave, infrared, visible, uv, xray, gamma-ray,. . .

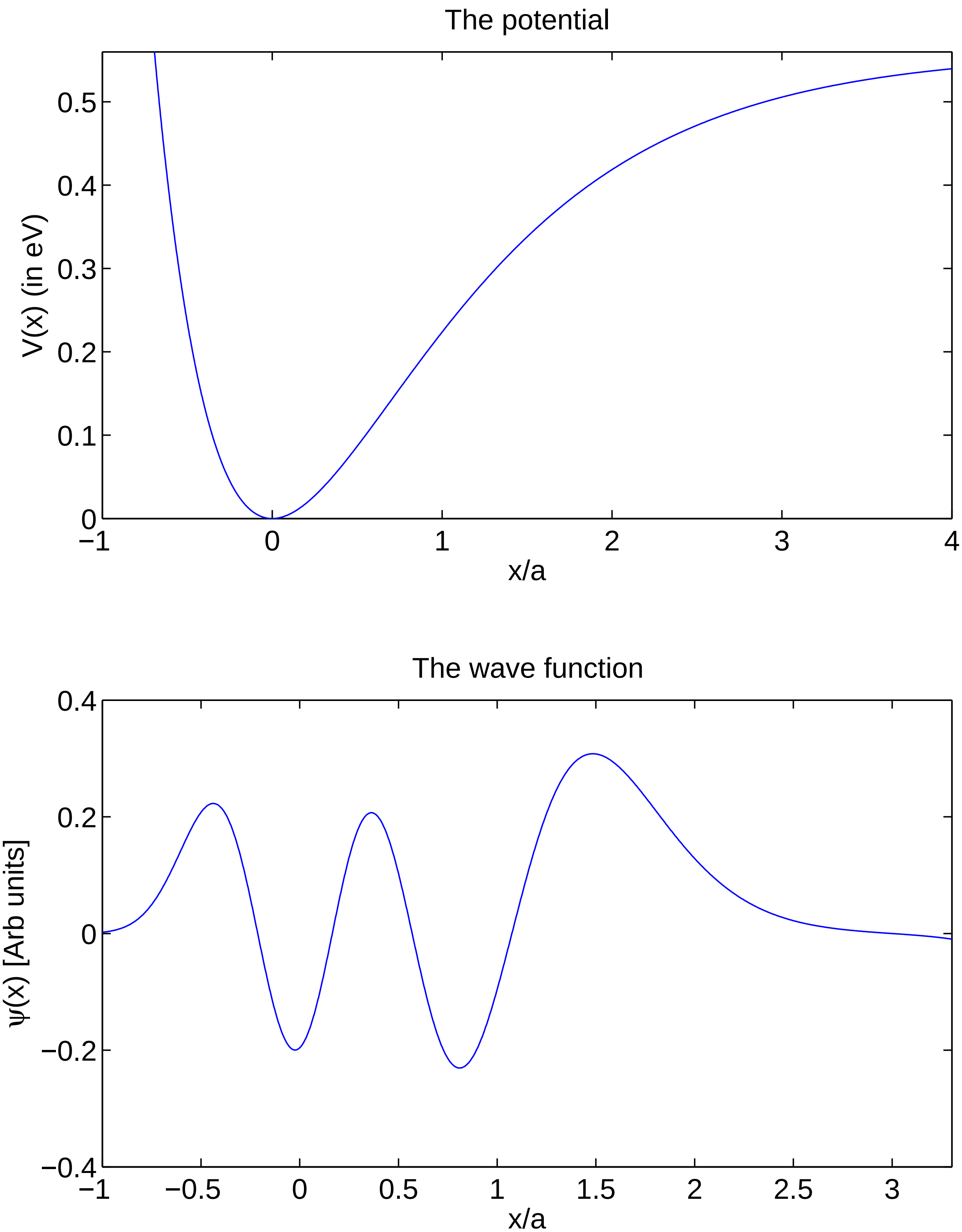

Figure 1: Figure for problem 4.

A mysterious sun-like star radiates a with total power $P = 3.8 \times 10^{26}\,\mathrm{W}$ and assume (for simplicity) that the wavelength of light is monochromatic, $\lambda = 550\,\mathrm{nm}$. However, the number of photons radiated from the star is not distributed uniformly over the solid angle. Rather the number per unit solid angle is proportional to $1 + \cos^2\theta$

$$\frac{dN}{d\Omega} = C\left(1 + \cos^2\theta\right) \tag{4}$$

where θ is the angle measured from the north pole of the star, and C is the proportionality constant. An equally mysterious earth-like planet orbits the star at distance of eight-light minutes from the sun. The radius of the earth planet equals the radius of the earth $R_e \simeq 6300\,\mathrm{km}$.

1. What is the total number of photons emitted per second, and what is the proportionality constant, C?

2. What is the solid angle of the earth-like planet as seen from the sun-like star?

3. When the earth-like planet is 30^o from the north pole, what is the total number of photons collected per second by the planet.