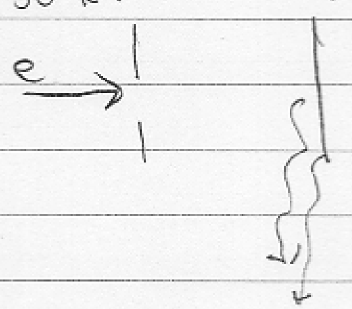
Last Time

① Photons

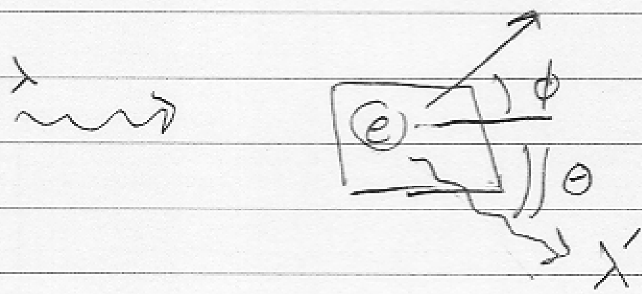
$E = h\nu$ ← sorry I used to use f but now I follow the book

② 50-kV metal



- See pictures

③



Used energy & mom consv

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\theta)$$

$\frac{h}{m_e c}$ = compton wavelength, only combo of h, m_e, c which has units length. This is the size of the electron

(3) There is a nucleus 1912-1913

(4) The Bohr Model of an Atom

- Describes many things -- the periodic table

What is Stuff?

Atom $\quad 1\,\mathring{A} = 10^{-10}\,m$

$\frac{h}{m_e c} \simeq 2\times10^{-2}\,\mathring{A} \qquad R_A \sim 5\times10^{-5}\,\mathring{A}$

$1 a_0 \simeq 0.5\,\mathring{A}$ $\qquad$ Nucleus 10^{-4} smaller

How this came to be

① Atoms

- Unity of Chemical Laws
- Study of Gasses - Maxwell Boltzman

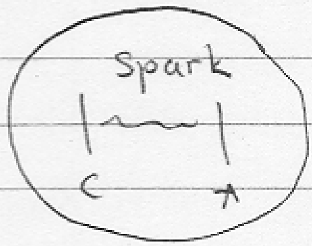
$$pV = nRT$$

② Atoms are divisible into positive and negative charge

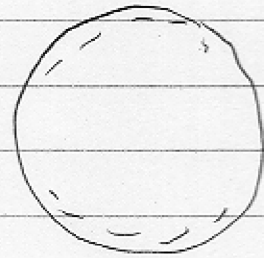
- Thompson measure e/m
- Millikan charges come in discrete units to better than 1%

Thompson Experiment

Green Light - Cathode rays



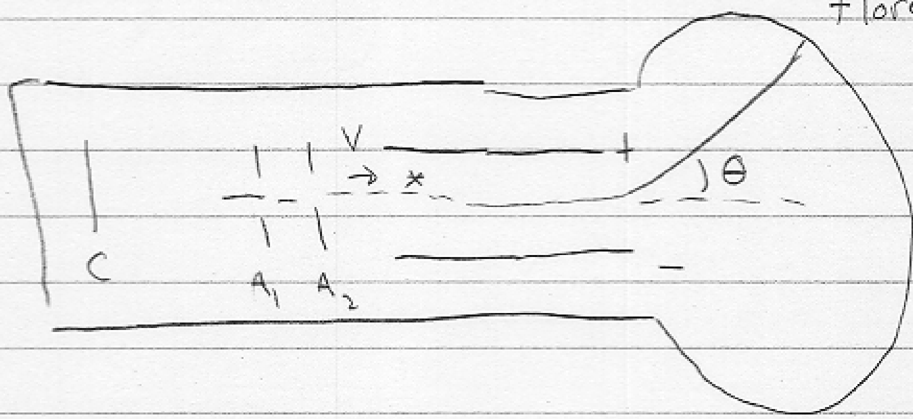
Then later



- electron striking phosphorous screen

The Experiment:

florescent screen

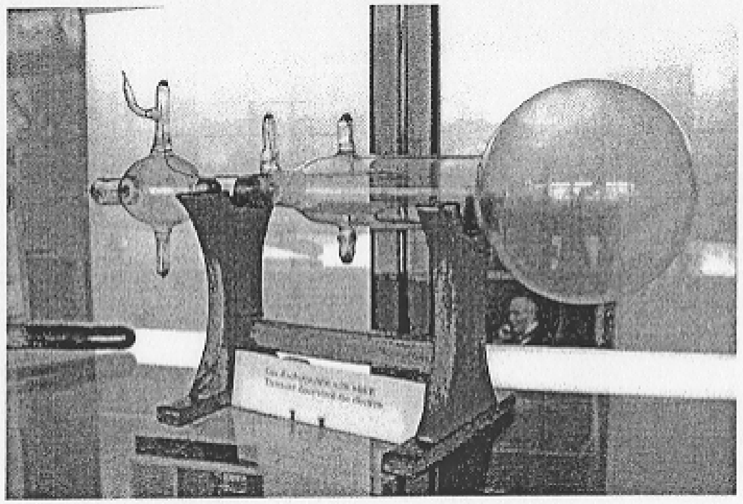


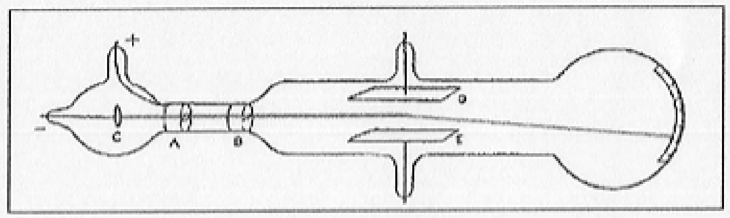
- By knowing how fast it was going, the electric field and bending θ, can determine the mass

- Generally don't know the speed But if add a magnetic field then the force $F = q\vec{v} \times \vec{B}$ and bending is also velocity dependent can determine, v and $\frac{q}{m}$

Electrons are independent of gas etc

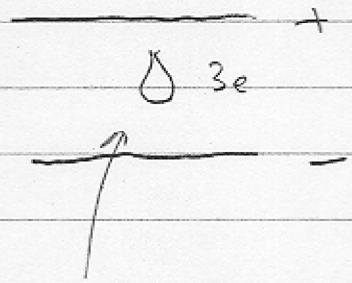
Thompson Experiment





Millikan Experiment - Charge Quantization

-- See Slides



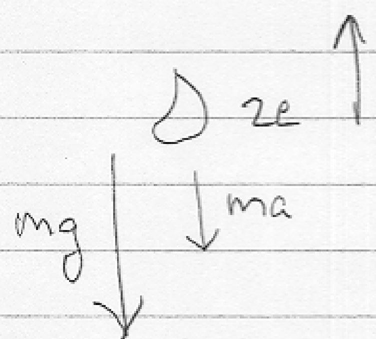
Oil Dropplet - Suspended in air

$$F_E = qE \simeq 3eE$$

mg

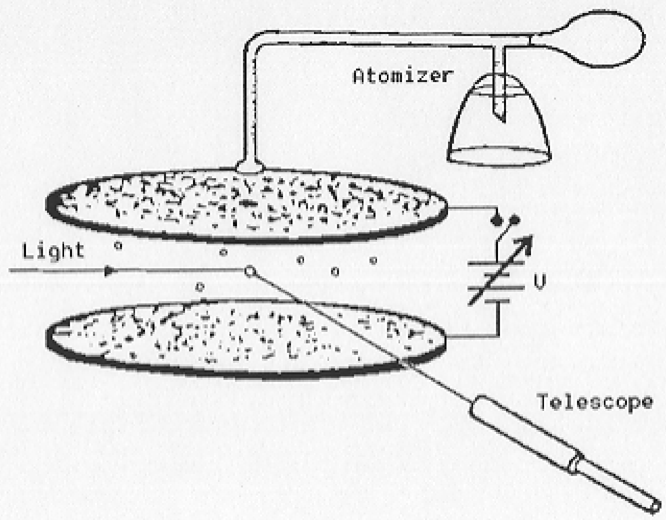
Then if oil dropplets bounces into air and looses one unit of charge

- It will souddenly start to move



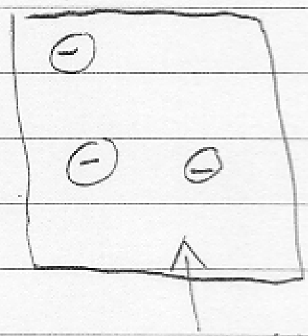
- Then it bounces again and changes direction again

Millikan Experiment



Rutherford Model of The atom

- Radio-activity is discovered and partially understood, α particles
  2 protons + 2 neutrons
- What is stuff?



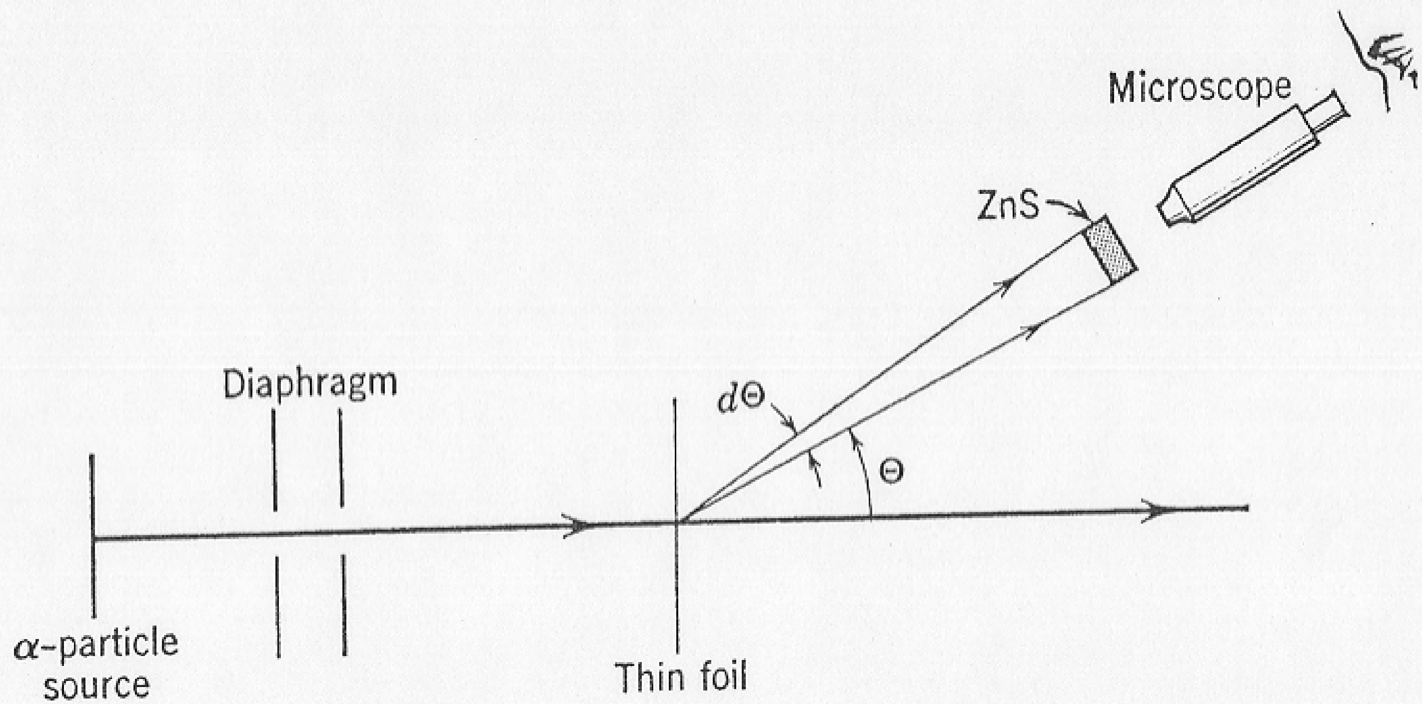
"Plum pudding"

positive charge uniformly

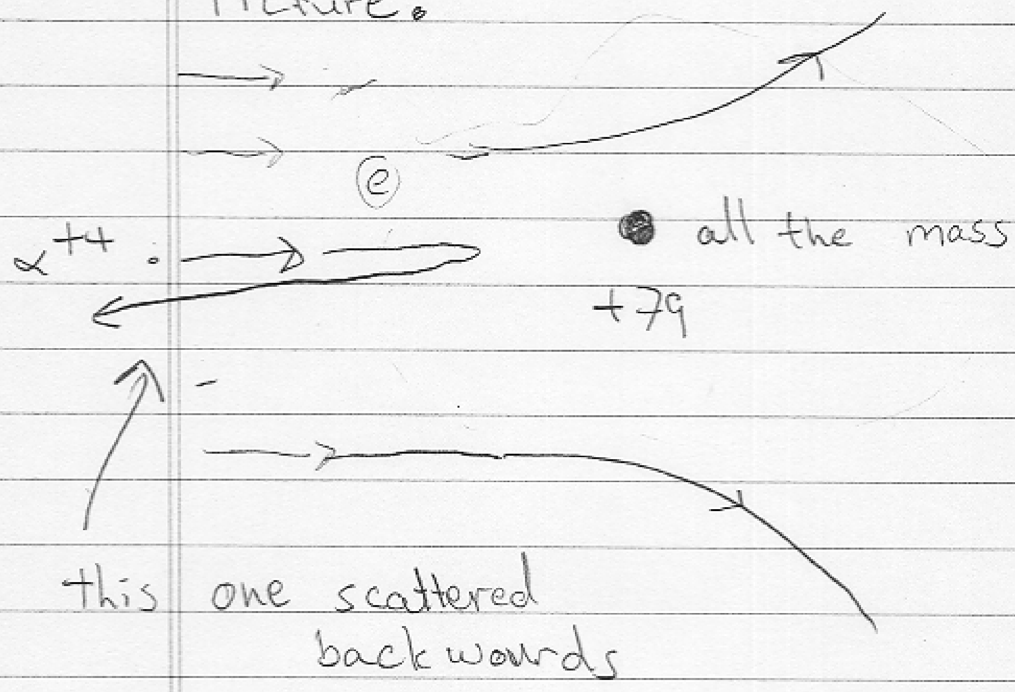
Rutherfords Setup:

- See Slides   KE-α particles ~8 MeV
- "It was if you fired a 15 inch shell at a piece of tissure paper and it came back and hit you"
- Rutherford concludes that all of the mass and positive charge is in a very small spot

Rutherford Experiment

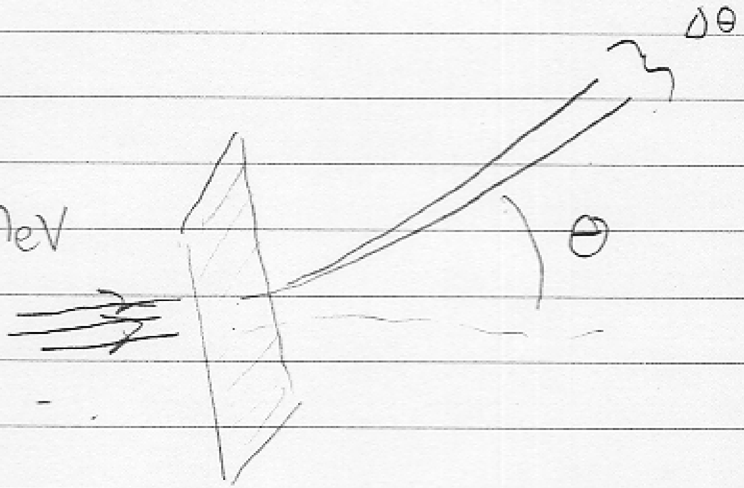


Picture:



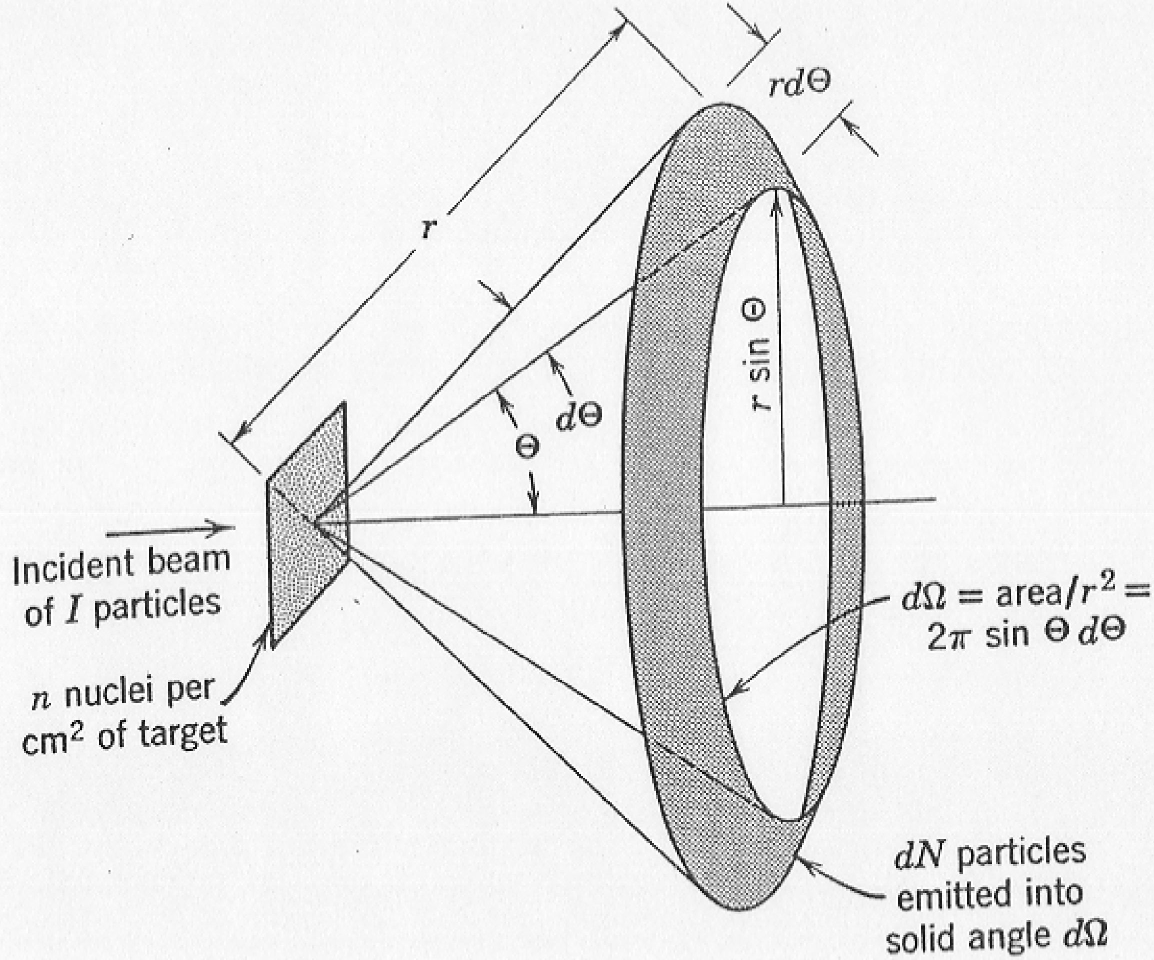
Goal:

$K \sim 8\,MeV$



Want to Calculate (Based on picture above
if N_α come in then a certain
number ΔN scatter into an angle $\Delta\theta$
at angle θ

Goal of the Rutherford Calculation



The distance of closest approach:

All KE

α^{++} $\xrightarrow{\quad}$

$\frac{1}{2}mv^2$

Au $79 = Z_{Au}$

All PE

α^{++}

$\underbrace{\bullet \qquad \bullet}_{D}$

$Z_{Au} = 79$

Lets evaluate:

+79 for Au

+2 for α

$$KE_i = PE_f \Rightarrow \frac{1}{2}mv^2 = \frac{Z_{Au}e \; ze}{D}$$

Then

$$\boxed{D = \frac{1}{4\pi\varepsilon_0} \frac{Z_{Au}e \; ze}{\frac{1}{2}mv^2}} \underset{\text{Substitute \#}}{=} 28 \text{ fm} = 28 \times 10^{-15} \text{ m}$$

How to substitute #'s

$$D = \left(\frac{e^2}{4\pi\varepsilon \hbar c}\right) Z_{Au} z \frac{\hbar c}{\frac{1}{2}mv^2}$$

$\hbar \equiv \frac{h}{2\pi}$ $\quad \hbar c = 197 \text{ eV nm}$ $\quad hc = 1240 \text{ eV nm}$

$= 197 \text{ MeV fm}$

h-bar = planck's constant $/2\pi$

Then

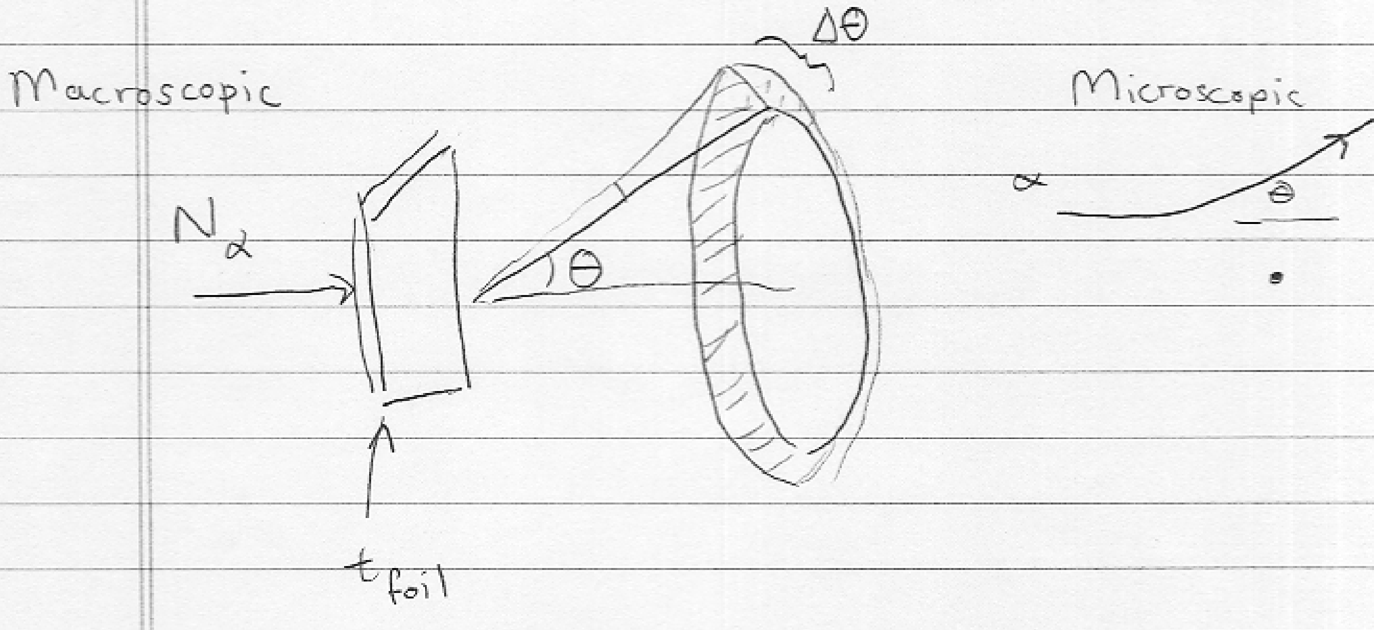
$$\alpha \equiv \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{1}{137} \equiv \text{fine struct}$$

So

$$D = \frac{1}{137\ \text{eV nm}} \cdot 79 \cdot 2 \cdot \frac{197\ \text{MeV fm}}{8\ \text{MeV}}$$

$$D = 28\ \text{fm}$$

Now Rutherford Calculated



Then Rutherford Showed that

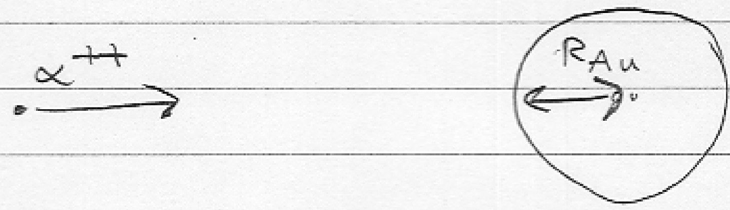
distance of Classest approa

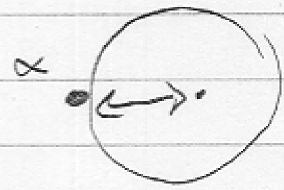
$$\Delta N = \left[N_\alpha \rho_{Au} t_{foil} \right] \frac{D^2}{16 \sin^4 \theta/2} \, 2\pi \sin\theta \, \Delta\theta$$

Number of α particles scattered into the detector

- The formula worked really well!
  Geiger + Marsden
- Ultimately determined the Z of many elements

Eventually for high enough α-particle energy (~1920's)





The α particle gets inside the nucleus and the formula stops working, find $\frac{1}{2}mv^2_{max} \simeq 20\,\text{Me}$

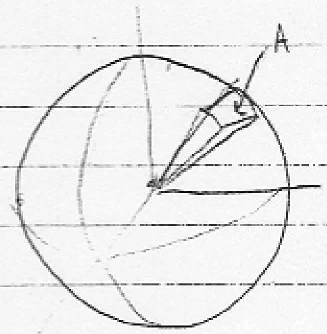
Then when

$$R_{Au} \sim D = \frac{e^2}{4\pi\epsilon_0} \frac{Z_{Au} z}{\frac{1}{2}mv^2_{max}}$$

Find for $\frac{1}{2}mv^2_{max} \simeq 20\,\text{MeV}$

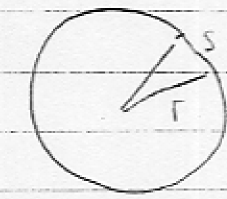
$$\boxed{R_{Au} \simeq 14\,\text{fm}}$$

Solid Angle:



patch on a sphere of radius r

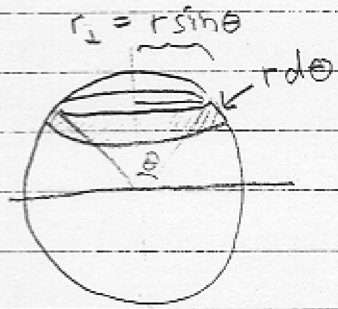
$$\Omega = \frac{A}{r^2}$$



analgous to

$$\theta = \frac{s}{r}$$

Example I



$$d\Omega = \frac{dA}{r^2} = \frac{(2\pi r\sin\theta)(r\,d\theta)}{r^2}$$

$$d\Omega = 2\pi\sin\theta\,d\theta$$

Example 2 (See Hand Out)

a small patch at an agle θ and azimuthal angle φ is shown on the handout, determine the solid angle

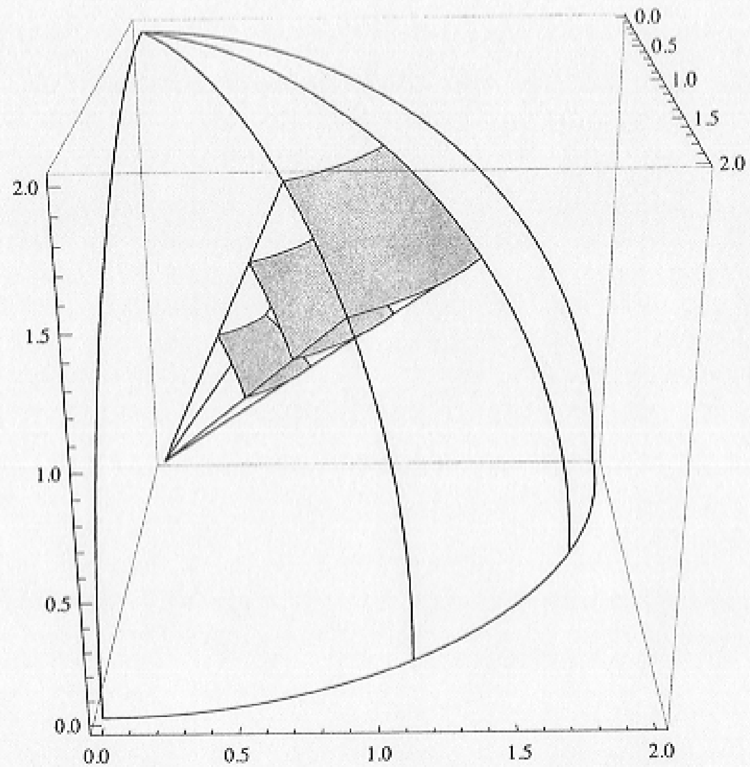
$$dA = (r\sin\theta\,d\varphi)(r\,d\theta)$$

So

$$d\Omega = \frac{dA}{r^2} = \boxed{\sin\theta\,d\theta\,d\varphi = d\Omega}$$

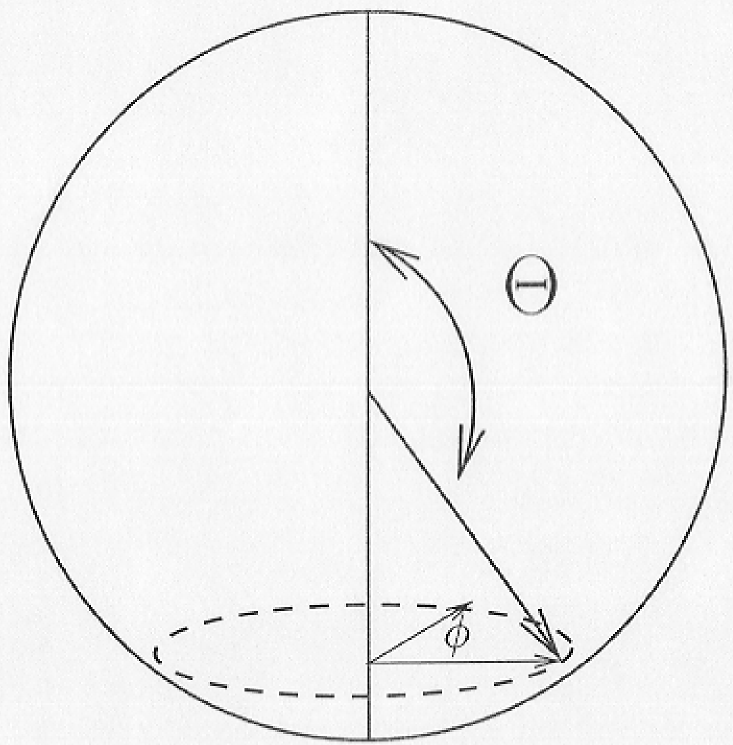
later $dV = \underbrace{r^2 d\Omega}_{dA}\,dr = r^2\sin\theta\,d\theta\,d\varphi\,dr$

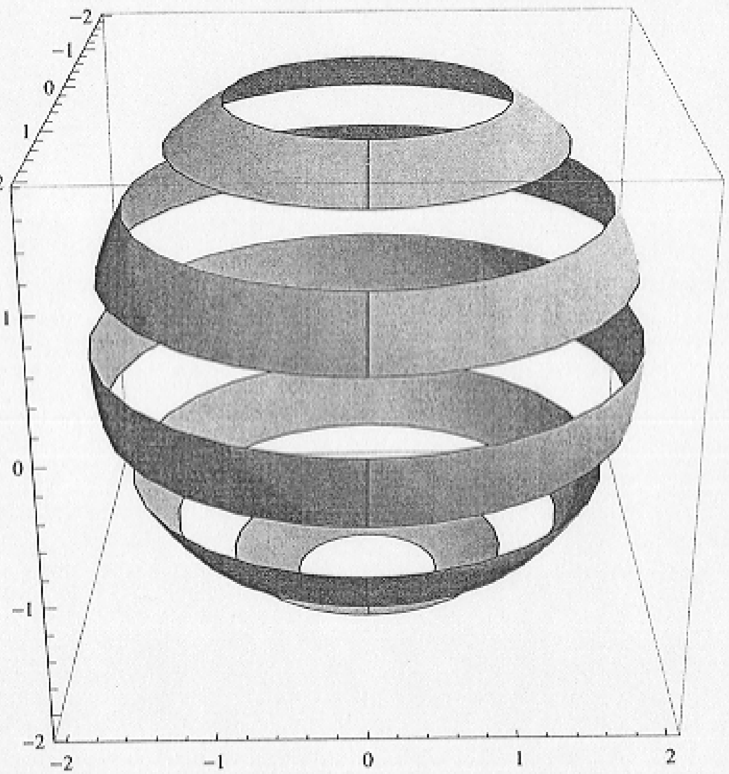
Solid Angle



$\Omega \equiv \frac{A}{r^2}$ is the same for these three surfaces

Solid Angle





Example 3

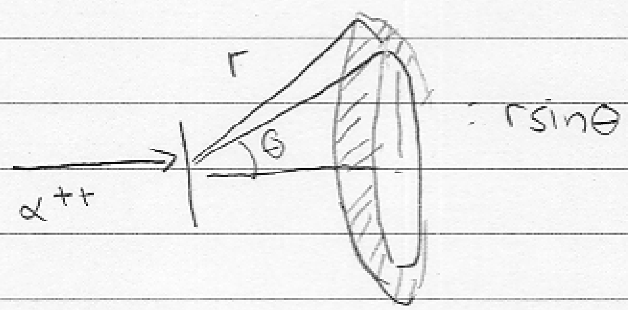
- Compute the area of a sphere using the notion of solid angle

$$A = \int dA = \int_{sphere} r^2 \, d\Omega = \int_{sphere} r^2 \sin\theta \, d\theta \, d\phi$$

$$A = r^2 \int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi$$

$$= r^2 \left[-\cos\theta\right]_0^{\pi} 2\pi = r^2 [1+1] \, 2\pi = 4\pi r^2$$

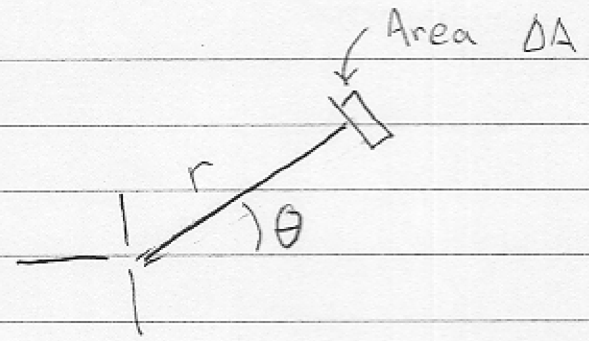
Rutherfords Result and Solid Angle



$$\Delta N = [N_\alpha \rho_{Au} t_{foil}] \frac{D^2}{16 \sin^4 \theta/2} \underbrace{2\pi \sin\theta \, \Delta\theta}_{= \Delta\Omega}$$

$$\Delta N = [N_\alpha \rho_{Au} t_{foil}] \frac{D^2}{16 \sin^4 \theta/2} \, \Delta\Omega$$

S



$$\Delta N = \left[N_\alpha \rho_{Au} t_{foil}\right] \frac{D^2}{16\sin^4\theta/2} \quad \frac{\Delta A}{r^2}$$

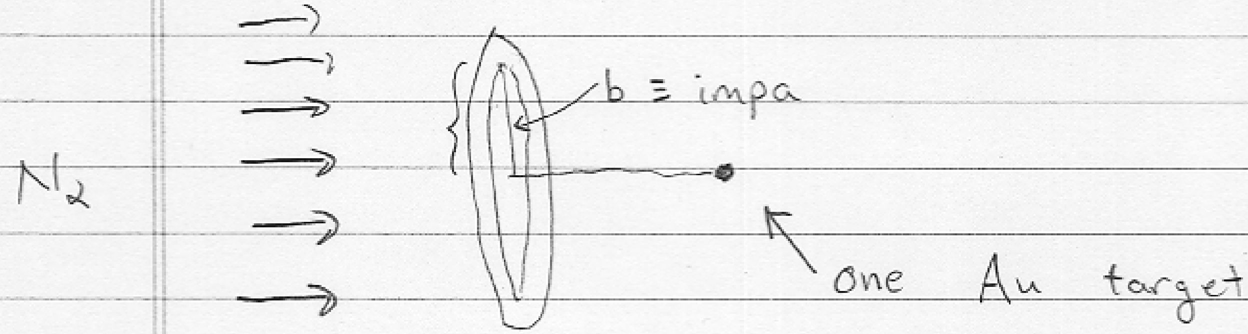
In general

$$\frac{dN}{d\Omega} = \left[N_\alpha \rho_{Au} t_{foil}\right] \frac{D^2}{16\sin^4\theta/2}$$

↑

number of α's per unit solid angle

Derivation of Rutherford's formula



$$\Delta N = \left(\frac{N_\alpha}{A}\right) 2\pi b\, |db|$$ ← number which scatter with impact parameter between b and $b+db$

Now if I have many gold targets

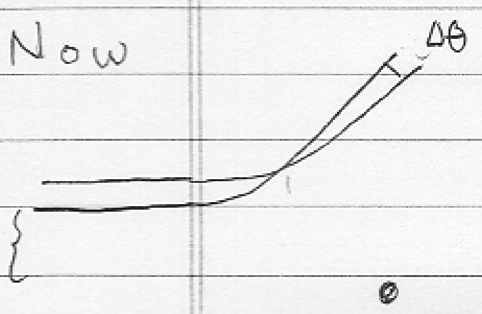
$$\Delta N = \left(\frac{N_\alpha N_{Au}}{A}\right) 2\pi b\, |db$$

density

$$\frac{N_{Au}}{A} = \left(\frac{N_{Au}}{A\, t_{foil}}\right) t_{foil}$$

$$\Delta N = (N_\alpha \rho_{Au} t_{foil})\, 2\pi b\, db$$

Now



We need to work out the scattering angle for a given impact parameter using $\vec{F} = m\vec{a}$

$$b = \frac{D}{2} \frac{\cos(\theta/2)}{\sin(\theta/2)}$$ ← appendix E

here

D = distance of clossest approach $= \frac{e^2}{4\pi\varepsilon_0} \frac{Z_{Au} z}{\frac{1}{2} m v^2}$

So

$$2\pi b \, |db| = \frac{D^2}{16} \frac{2\pi \sin\theta \, d\theta}{\sin^4(\theta/2)}$$

$$b = \frac{D}{2} \frac{\cos\theta/2}{\sin\theta/2}$$

$$db = -\frac{D}{4} \frac{d\theta}{\sin^2\theta/2}$$

Then

$$\Delta N = [N_\alpha \rho_{Au} \, t_{foil}] \underbrace{\frac{D^2}{16 \sin^4(\theta/2)} 2\pi \sin\theta \, d\theta}_{= 2\pi b \, |db|}$$

$$\Delta N = [N_\alpha \rho_{Au} \, t_{foil}] \frac{D^2}{16 \sin^4 \theta/2} \cdot 2\pi \sin\theta \, \Delta\theta$$