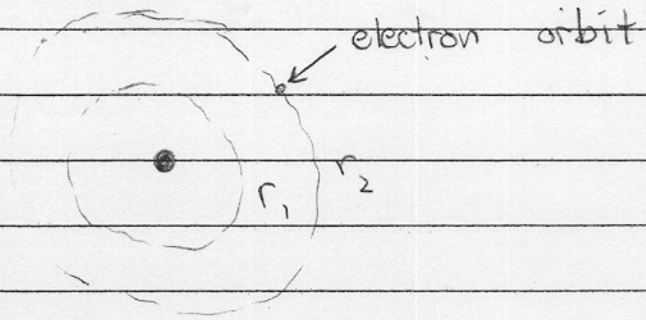
Last Time - Bohr Model



- The electrons are attracted to the proton

$$L = m v_n r_n = n\hbar \qquad n = 1, 2, 3, \ldots$$

- Then can determine the radius of the orbit

$$F_c = m_e \frac{v_n^2}{r}$$

$$\frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_n^2} = m_e \frac{v_n^2}{r_n}$$

$$\frac{e^2}{4\pi\varepsilon_0} \frac{1}{m v_n^2} = r_n$$

So $\quad L = m v_n r_n = n\hbar$

$$\cancel{m} \cancel{v_n} \cdot \left(\frac{e^2}{4\pi\varepsilon_0} \frac{1}{\cancel{m_e} v_n^{\cancel{2}}} \right) = n\hbar$$

Simplify

$$V_n = \frac{e^2}{4\pi\varepsilon_0 \hbar} \quad \frac{1}{n}$$

$$\frac{V_n}{c} = \left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right) \cdot \frac{1}{n} = \frac{\alpha}{n} \Rightarrow$$

$$\frac{V_1}{c} = \frac{1}{137}$$ ⇐ The electrons are non-relatistic to a good approximation

The

$$r_n = \frac{e^2}{4\pi\varepsilon_0} \frac{1}{m_e V_n^2} = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \cdot \frac{\hbar c}{m_e c^2 \left(\frac{V_n}{c}\right)^2}$$

$$r = \cancel{\alpha} \frac{\hbar \cancel{c}}{m_e c^{\cancel{2}} \alpha^{\cancel{2}} \frac{1}{n^2}}$$

$$r_n = n^2 \left[\frac{\hbar}{m_e c \alpha}\right] \equiv n^2 a_0$$

$$a_0 \equiv \frac{\hbar}{m_e c \alpha} = 137 \left(\frac{\hbar}{m_e c}\right)$$

up to 2π this is the electron compton wavelength

$$a_0 = 137 \cdot \left(\frac{\hbar c}{m_e c^2}\right) = 137 \left(\frac{197\ eV \cdot nm}{511000\ eV}\right) = 0.05\ n$$

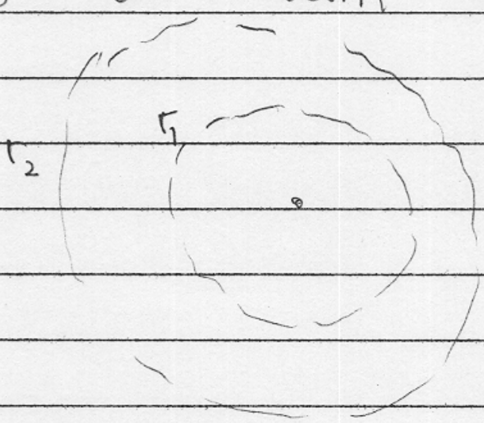
$$a_0 = 0.5\ \mathring{A}$$

Picture

$$a_0 = 0.5\,\text{Å} = \frac{\hbar}{m_e c\alpha}$$

$$\frac{\hbar}{m_e c} = 0.02\,\text{Å}$$

Now we want to determine the energies



$$PE_n = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} = -\left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right)\frac{\hbar c}{a_0 n^2}$$

$$= -\left(\frac{\alpha \hbar c}{a_0}\right)\frac{1}{n^2}$$

$$= -\left(\frac{1}{137}\cdot\frac{1970\,\text{eV Å}}{0.5\,\text{Å}}\right)\frac{1}{n^2} = \frac{-27.2\,\text{eV}}{n^2}$$

For the KE

$$KE = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{\alpha c}{n}\right)^2 = \frac{1}{2} m_e c^2 \frac{\alpha^2}{n^2} = \frac{13.6\,eV}{n^2}$$

Half of |PE|

Why?

$$\frac{e^2}{4\pi\varepsilon_0} \frac{1}{r^2} = \frac{m v^2}{r}$$

$$\underbrace{\frac{e^2}{4\pi\varepsilon_0} \frac{1}{r}}_{-PE} = \underbrace{m v^2}_{2\,KE}$$

$$-PE = 2\,KE$$

$$-\frac{1}{2} PE = KE$$

Energy summary

$$E_{Tot_n} = PE + KE$$

$$= -\frac{1}{4\pi\varepsilon_0} \frac{e^2}{a_0} \left(\frac{1}{n^2}\right) + \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0} \frac{e^2}{a_0}\right) \frac{1}{n}$$

$$= -\frac{1}{4\pi\varepsilon_0} \left(\frac{e^2}{2a_0}\right) \left[\frac{1}{n^2}\right] \equiv \boxed{-R_\infty \frac{1}{n^2} = E_n}$$

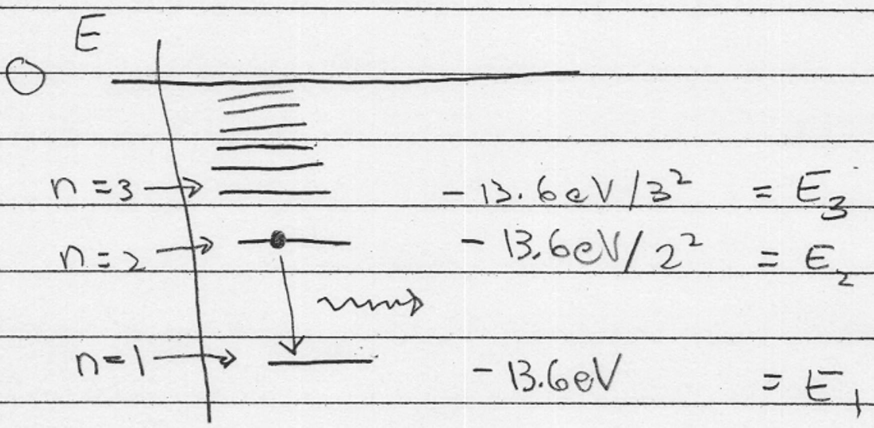
energy of n-th orbit

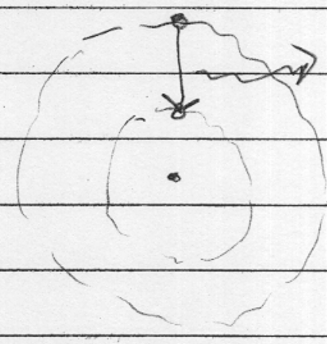
$$R_\infty = -\frac{e^2}{4\pi\varepsilon} \frac{e^2}{2a_0} = -13.6\,eV \leftarrow$$ Energy of Lowest orbit

$$E_1 = -13.6\,\text{eV}$$

$$E_1 = \underbrace{-27.2\,\text{eV}}_{PE_1} + \underbrace{13.6\,\text{eV}}_{KE_1}$$

Energy Diagrams





- Consider an atom which transitions from the $n=2$ orbit to the $n=1$ orbit emitting a photon

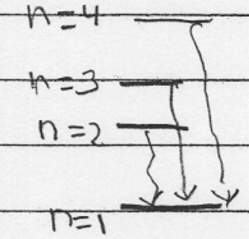
$$E_i = E_f + E_\gamma \quad \leftarrow \text{energy conservation}$$

$$E_\gamma = E_i - E_f$$

$$= \left(-\frac{R_\infty}{2^2}\right) - \left(-\frac{R_\infty}{1^2}\right) = R_\infty\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R_\infty \cdot \frac{3}{4}$$

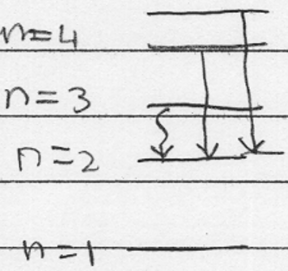
$$E_\gamma = 13.6\,\text{eV} \cdot \frac{3}{4} = 10.2\,\text{eV} \leftarrow \lambda = 124\,\text{nm} \quad \text{deap UV}$$

Others:



(Lyman Series)

Still Others



(Balmer Series)

Determite the wave lengths of the Balmer Lines

$$E_\gamma^{n\to 2} = E_i - E_f$$

$$E_\gamma^{n\to 2} = \left(-\frac{R_\infty}{n^2}\right) - \left(-\frac{R_\infty}{2^2}\right) = R_\infty\left[\frac{1}{2^2} - \frac{1}{n^2}\right] = R_\infty\left[\frac{n^2-2^2}{2n^2}\right]$$

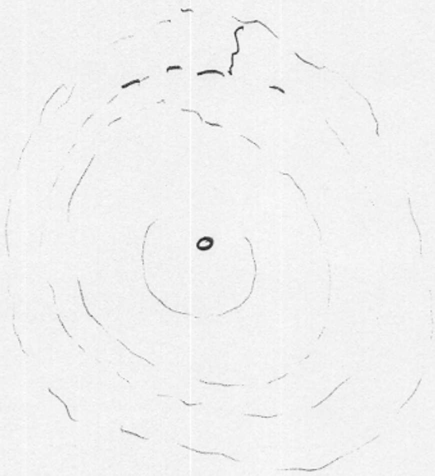
$$E_\gamma^{n\to 2} = \frac{hc}{\lambda_n} \Rightarrow \lambda_n = \frac{hc}{E_\gamma^{n\to 2}} = \frac{hc}{R_\infty\left(\frac{n^2-2^2}{2n^2}\right)}$$

$$\lambda_n = \frac{2hc}{R_\infty}\cdot\left(\frac{n^2}{n^2-2^2}\right)$$

Explains empirical rules

$$E = \left(\frac{2\cdot 1240\ \mathrm{eV\,nm}}{13.6\ \mathrm{eV}}\right)\left(\frac{n^2}{n^2-2^2}\right) = (363\ \mathrm{nm})\left(\frac{n^2}{n^2-2^2}\right)$$

The Correspondence Principle



$$r_n = n^2 a_0$$

$$\frac{r_{n+1} - r_n}{r_n} \approx \frac{2n a_0}{n^2 a_0} = \frac{2}{n}$$

- When n becomes large $r \rightarrow \infty$ and the spacing gets smaller (in a relative sense)
- So we should have

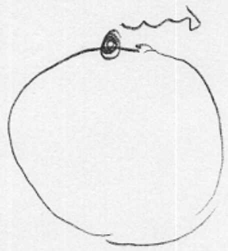
$$\lim_{n \to \infty} \text{quantum physics} = \text{classical physics}$$

- This enough to say that if photons come in discrete units

$$E = hf = \hbar\omega$$

Then the angular momentum changes in discrete steps

When



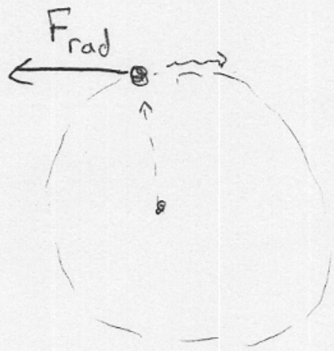
Facts

① $V = R\omega_e$

② For an electron going around in a circle (classically)

Maxwell equations say that:

$$\omega_{photon} = \omega_e \equiv \omega$$



$$\frac{\Delta E_{Lost}}{\Delta t} = F_{rad} \cdot V$$

$$\frac{\Delta E_{Lost}}{\Delta t} = F_{rad} \, R \, \omega$$

$F_{Rad} R$ = Torqe from radiation $= \frac{\Delta L}{\Delta t}$ so $\frac{\Delta E_{lost}}{\Delta t} = \frac{\Delta L_{lost}}{\Delta t} \omega$

$$\Delta E = \Delta L \cdot \omega$$

Now planck say Energy Is emitted in discrete units

$n\hbar\omega_{photon} = n\hbar\omega$

$$n\hbar\cancel{\omega} = \Delta L \cancel{\omega}$$

$$n\hbar = \Delta L$$

Change in L during emission of one light packet

Says L Comes in discrete + units

DeBroglie ~ 1920

- Bohr theory clearly inadequate
  - Want something like: given electron at time one where is electron at time #2
  - Only limitted success at describing multi-electron atoms. Just specifies L_{TOT}. not enough to describe each individual electron

Think about light:

$$E = h\frac{c}{\lambda} \qquad E = cp$$

$$\cancel{c}p = h\frac{\cancel{c}}{\lambda} \qquad p = \frac{h}{\lambda} \Rightarrow \boxed{\lambda = \frac{h}{p}}$$

Also remember the Bohr model: $\frac{v}{c} = \alpha \qquad v = \alpha c$

$$p = m v = m\alpha c = m\left(\frac{ke^2}{\hbar \cancel{c}}\right)\cdot \cancel{c} = \hbar \underbrace{\left(\frac{mke^2}{\hbar^2}\right)}_{\frac{1}{a_0}}$$

$$p = \frac{\hbar}{a_0}$$

call it wave length

Thin

$$a_0 = \frac{\hbar}{p} \qquad \text{so} \qquad \overbrace{2\pi a_0}^{\lambda} = \frac{h}{p}$$

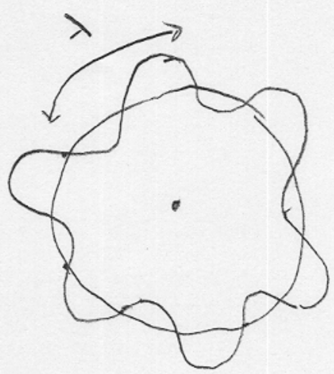
Then Bohr quantization condition

$$\overbrace{mvr} = n\frac{h}{2\pi}$$

$$2\pi \cdot \underbrace{p}\, r = nh$$

$$2\pi \frac{\cancel{h}}{\lambda} r = n\cancel{h} \qquad \boxed{2\pi r = n\lambda}$$

Picture



have to have complete wave lengths going around the circle

$$\lambda = \frac{h}{p} \qquad p = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \boxed{\hbar k = p}$$