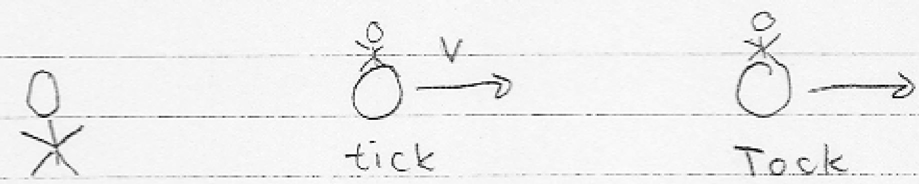
Last Time: The speed of light is constant in all frames

Time dilation:

$\Delta\tau \leftarrow$ A person sitting on clock measures a proper time interval $\Delta\tau$

A person watching the clock move sees

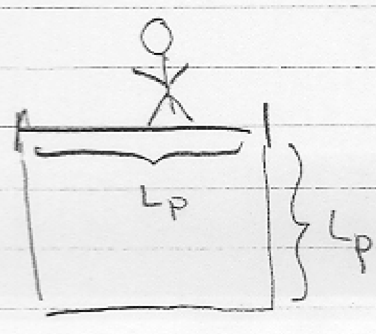


And measures the time between tick and tock is Δt

$$\Delta t = \gamma\, \Delta\tau \qquad \gamma = \frac{1}{\sqrt{1-(V/c)^2}} \qquad \gamma > 1$$

Length Contraction

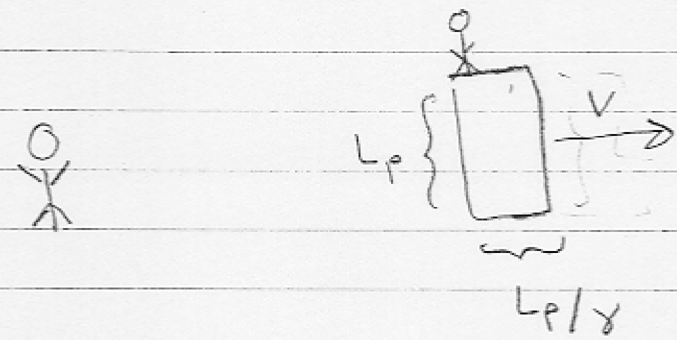
A person sitting on a square stick measures length and width



L_p by L_p

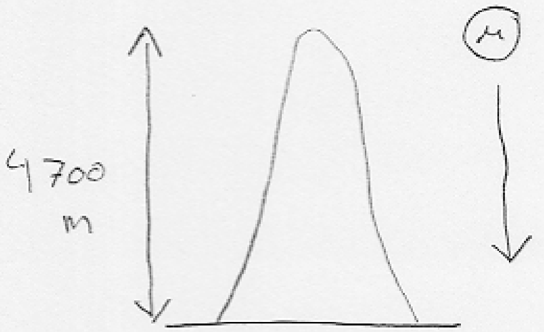
These are propper lengths

Wile a person sitting on the ground measures one of the sides length contracted



See the last lecture for a discussion of the muon and the mountain

Muon and mountain : Earth Observer



$V_{muon} = 0.99c$

- The μ decays in 2.2 μs in its own frame (a proper time)

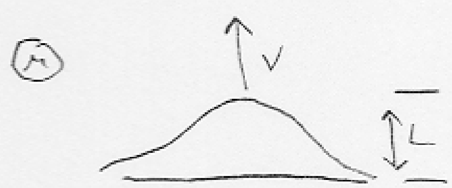
- To an observer on earth the muon decays in

$$\Delta t = \gamma \, \Delta\tau \qquad \gamma = \frac{1}{\sqrt{1-(V/c)^2}} = 7.1$$

$$\Delta t = (7.1)(2.2\,\mu s) \simeq 16\,\mu s$$

- The distance travelled is $d = V\,\Delta t \approx c \cdot 16\,\mu s = 4700$ the muon reaches the bottom !

Muon and mountain: Muon Observer



$$L = L_0/\gamma = \frac{4700\,m}{7.1} \simeq 650$$

The muon says $x = \Delta\tau\, V$ amount of mount passes him

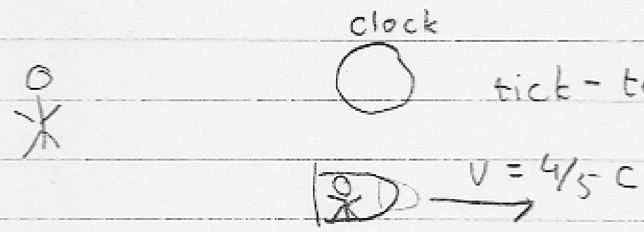
$$x = 2.2\,\mu s\,(0.99c) \simeq 650\,m$$

The muon agrees he reaches the bottom

Lorentz Transformations:

Classically: (Not Correct)

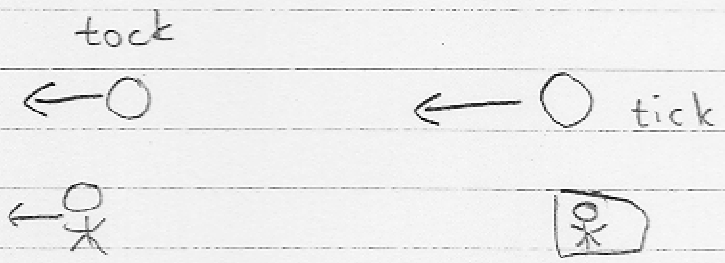
A person on the ground sees a clock at rest and a spaceship moving at 4/5c. He measures the time between tick and tock to be 1s



$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{5}{3}$$

$$\beta = 4/5$$

The person on the space ship sees



Let the two observers agree that "tick" happens at the origin

$(t, x) = (0, 0)$ ← coords as meas. by earth

$(t', x') = (0, 0)$ ← coords as meas. by spaces

"Tock" happens at according to person on earth

$(t, x) = (1s, 0)$

The coordinates according to someone on space ship

$$t' = t \quad = \quad 1s$$
$$x' = x - vt \quad = (0) - \frac{4}{5}c\,(1s) = -\frac{4}{5}cs$$

1 cs = The distance light travels in a second $\simeq 3 \times 10^8$ m

<u>But we know this isn't right:</u>

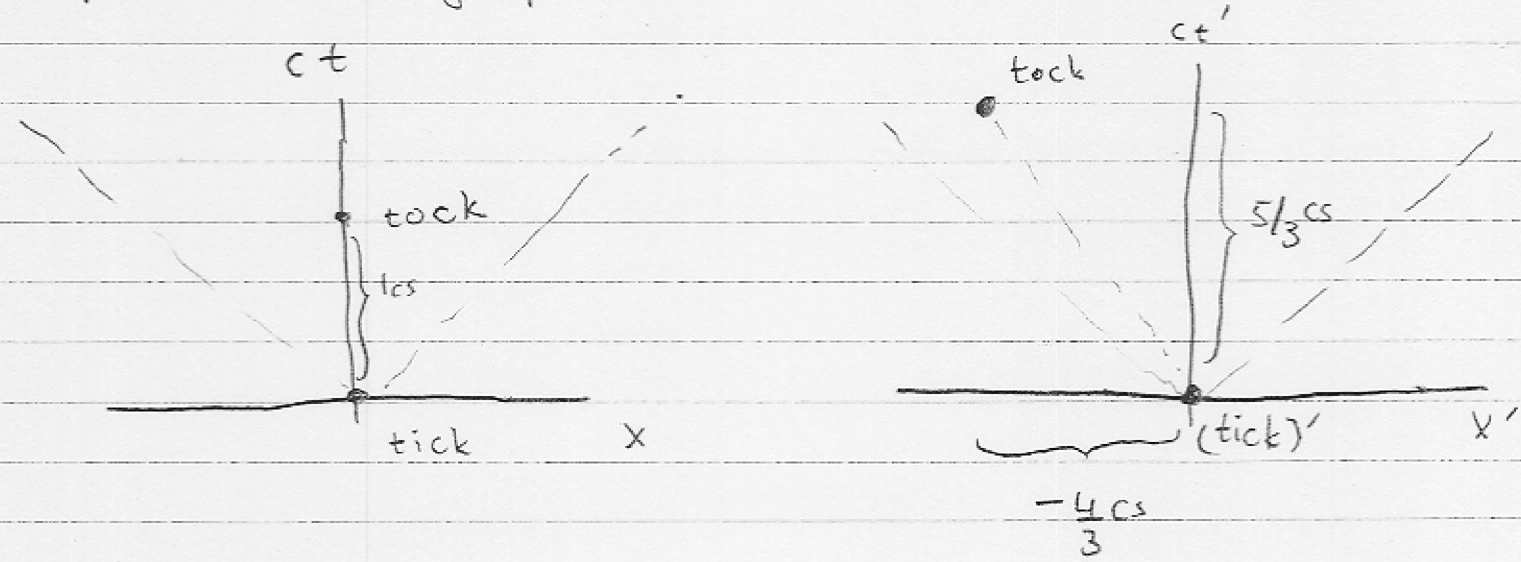
$$t' = \gamma t \quad = \frac{5}{3} \cdot 1s$$ ← time of tock after tick

<u>In</u> this time interval the

$$x' = -vt'$$

$$x' = -v(\gamma t) = -\frac{4c}{5} \cdot \frac{5}{3} \cdot (1s) = -\frac{4}{3}\,cs$$

Space - Time graph:



Classically we have the coordinates relations

$$t' = t$$

$$x' = x - vt$$

- Relation between coordinates of someone on ground (t, x) and some one moving to right (t', x')

- But this does not keep the speed of light constant

Lorentz Transformations:

$$\beta = V/c$$

$$ct' = \gamma\,(ct) - \gamma\beta\,x$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$x' = -\gamma\beta\,(ct) + \gamma\,x$$

(If you know what a matrix is:

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

Don't worry if you don't know what matrix is

- Keeps the speed of light constant as we will show shortly

Review

○ Tick - Tock

$v = 4/5\,c$

Observer moving to right sees

$\leftarrow$ O $\qquad$ $\leftarrow$ O

tock $\qquad$ tick

Tick and tock

- tick = $(ct, x) = (0,0)$
- tick' $= (ct', x') = (0, 0)$

$$ct' = \gamma\ ct - \gamma\beta\ x = \gamma\ (0) - \gamma\beta\,(0) = 0$$
$$x' = -\gamma\beta\ ct + \gamma\ x = -\gamma\beta\ (0) - \gamma\ (0) = 0$$

- tock $= (ct, x) = (1s, 0)$
- tock' $= (ct', x')$

$$ct' = \gamma \cdot (ct) - \gamma\beta\, x = \gamma\ (c \cdot 1s) - \cancel{\gamma\beta(0)} = \frac{5}{3}\ cs$$

$$x' = -\gamma\beta \cdot (ct) - \gamma\, x = (-\gamma_\beta)(c 1s) = -\left(\frac{5}{3}\right)\left(\frac{4}{5}\right) 1cs$$
$$= -\frac{4}{3}\ cs$$

$$(ct', x' =) = \left(\frac{5}{3}\ cs\ ,\ -\frac{4}{3}\ c \cdot s\right)$$ $\leftarrow$ agrees ⓦ before

Proof that Speed of Light Const under Lorentz transformation:

Light travelling to right:

$$\rightsquigarrow \quad x = ct$$ ← parametrizes a sequence of events

Want to see that according to an observer moving to right:

$$x' = ct'$$

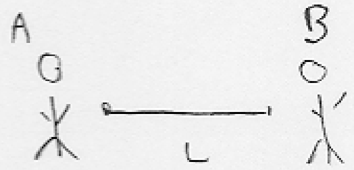
$$ct' = \gamma\, ct - \gamma\beta \overbrace{(ct)}^{x} = (\gamma - \gamma\beta)\, ct$$

$$x' = -\gamma\beta\, ct + \gamma \underbrace{(ct)}_{x} = (-\gamma\beta + \gamma)\, ct$$

So $x' = ct'$

Speed of Light is constant

- Now Consider two guys who measure distances at the same time separated by a distance of 1m, What does the observer moving with speed $v = \frac{4}{5}c$ see.



Solution: $\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(4/5)^2}} = \frac{5}{3}$ $\quad \beta = \frac{v}{c} = \frac{4}{5}$

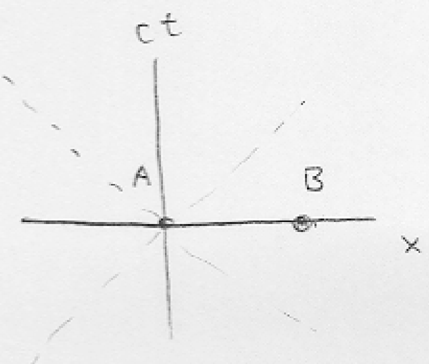
- The event A occurs at the origin $(ct, x) = (0,0)$
- The event B " " $(ct, x) = (0, 1m)$

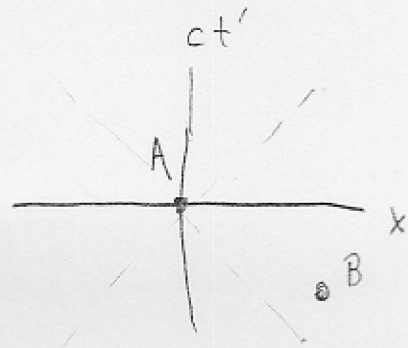
Lets See where event B occurs to the right mover

$$ct' = \gamma\,(\cancel{ct}^{\,0}) + -\gamma\beta\,(L) = -\gamma\beta L = -\frac{4}{3}\,m$$

$$x' = -\gamma\beta\,(\cancel{ct}^{\,0}) + \gamma\,(L) = \gamma L = \frac{5}{3}\,m$$

- An observer moving to right sees event B happen before A

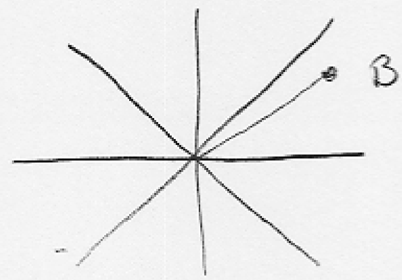




- B happens before A
- Order and simultaneity of events depends on the observer

Next consider an observer moving to the left then we have

$$\underbrace{\beta}_{\frac{v}{c}} \longrightarrow -\beta$$

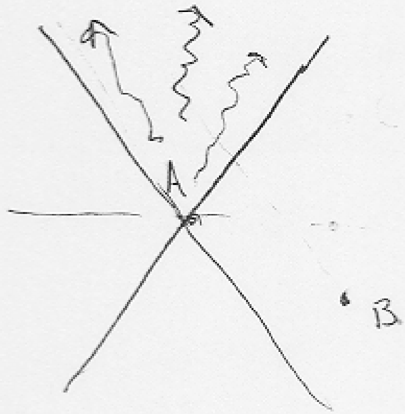


So

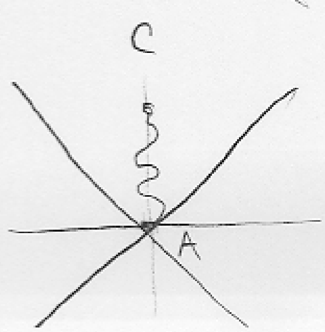
$$ct' = +\gamma\beta L = +\frac{4}{3}\,m$$

$$x' = \gamma L = \frac{5}{3}\,m$$

Why Does this not matter?



- There is no way for point A to be causally connected to points B. Different observers can disagree about order.



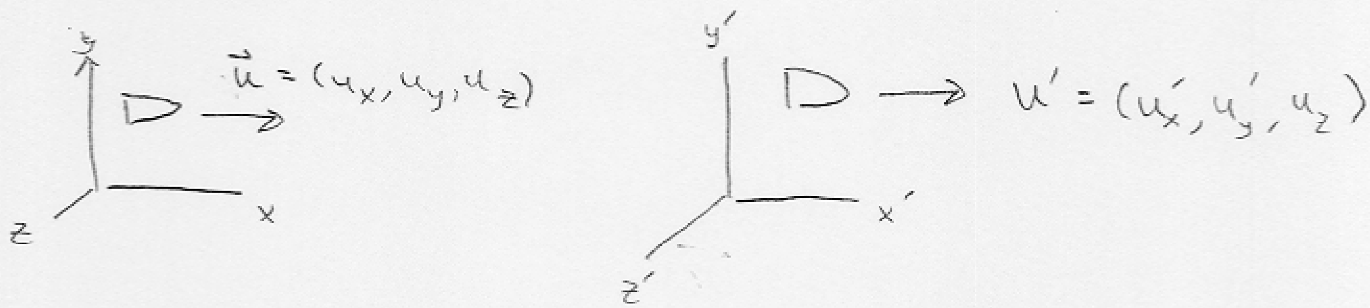
- Point A can influence the events of C. All observers agree C is after A

Addition of Velocities:

- So far we have left out transverse directions if (ct, x, y, z) are coords in one frame

$$
\begin{aligned}
ct' &= \gamma\, ct - \gamma\beta\, x \\
x' &= -\gamma\beta\, ct + \gamma\, x \\
y' &= y \\
z' &= z
\end{aligned}
$$

Question:



A space-ship moves (w) velocity $\vec{u} = (u_x, u_x, u_z)$ in one frame then according to an observer moving to right with speed v the space-ship moves with velocity $\vec{u}' = (u_x', u_y', u_z')$

$$
u_x' = \frac{u_x - v}{1 - u_x v/c^2}
$$

$$
u_y' = \frac{u_y}{\gamma(1 - u_x v/c^2)}
$$

$$
u_z' = \frac{u_z}{\gamma(1 - u_x v/c^2)}
$$

Proof: (x coords only)

Suppose a ship moves with speed u_x to the right

$$u_x = \frac{\Delta x}{\Delta t}$$

Then:

$$u_x' = \frac{\Delta x'}{\Delta t'} \qquad \begin{aligned} \cancel{c}\Delta t' &= \gamma \cancel{c}\Delta t - \gamma\beta\, \Delta x / c \\ \Delta x' &= -\gamma\beta\, c\Delta t + \gamma\, \Delta x \end{aligned}$$

So

$$u_x' = \frac{\Delta x'}{\Delta t'} = \frac{-\gamma \overbrace{\beta c}^{V} \Delta t + \gamma\, \Delta x}{\gamma\, \Delta t - \gamma \frac{\beta}{c}\, \Delta x} \qquad \beta = V/c$$

cancel γ from each term use $\beta = V/c$

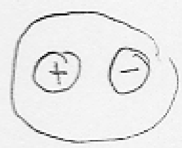
$$= \frac{-V \Delta t + \Delta x}{\Delta t - \frac{V}{c^2}\Delta x}$$

$$u_x' = \frac{-V + \overbrace{\Delta x / \Delta t}^{u_x}}{1 - \frac{V}{c^2}\underbrace{\Delta x/\Delta t}_{u_x}} \equiv \frac{u_x - V}{1 - u_x V/c^2}$$

Energy and Momentum:

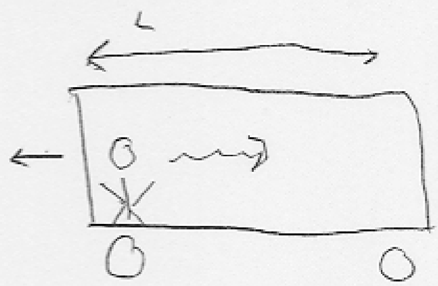
Rest Energy:

Consider positron + electron which annihilate



annihilate

Einsteins Box



$$P_{Box} = P_{Light}$$

$$Mv \simeq \frac{E}{c} \leftarrow \text{energy of phot}$$

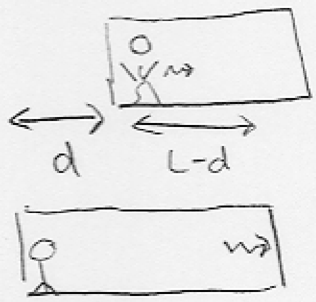
$$v = E/Mc$$

Now the system moves a distance

$$d = v\,t = v\,\frac{L}{c} = \frac{E}{Mc}\,\frac{L}{c} = \frac{E}{Mc^2}\,L \quad \text{(very sma}$$

time it takes light to move from one end to the other

But there are no external Forces so the CM can not change.



$M_{TOT} X_{cm} = M_1 x_1 + M_2 x_2$

Einsteins answer:

- photon carries away some of carts mass as energy

$$(M - \Delta m)(-d) + \Delta m (L - d) = 0$$

$$M d = \Delta m L$$

$$M \frac{E_\gamma}{Mc^2} L = \Delta m L$$

$$\boxed{E_\gamma = \Delta m c^2}$$

Energy of matter in its own rest frame