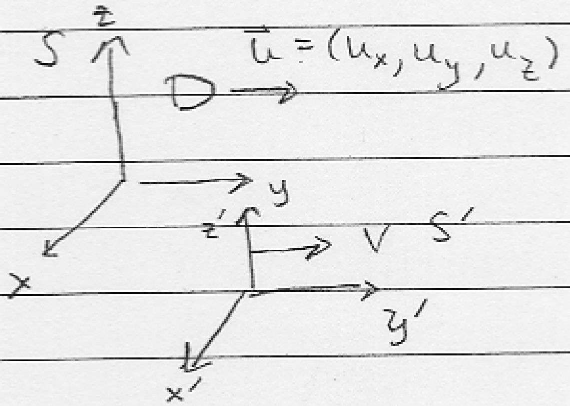
Last Time



- We have a bullet which moves ⓦ velocity $\vec{u}$ in frame S, an observer (S') (i.e. a guy in a spaceship), moving with speed V with respect to S measures a different velocity

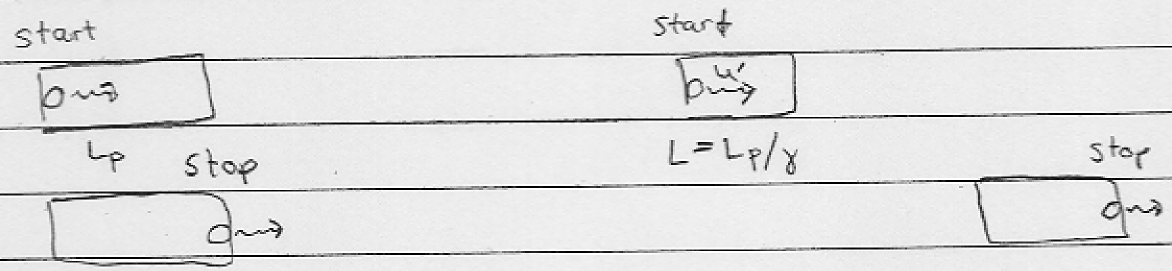
Relativistic	Classical
$u'_x = \dfrac{u_x - V}{1 - u_x V/c^2}$	$u'_x = u_x - V$
$u'_y = \dfrac{u_y}{\gamma(1 - u_x V/c^2)}$	$u'_y = u_y$
$u'_z = \dfrac{u_z}{\gamma(1 - u_x V/c^2)}$	$u'_z = u_z$

Example:

A train which moves at $4/5c$ and has propper length of 100m. A person on the train launches a μ meson @ a speed of $3/5c$ relative to him

Q1: What is the time it takes according to someone on earth for the μ to reach the other side

Train sees: Earth Sees:

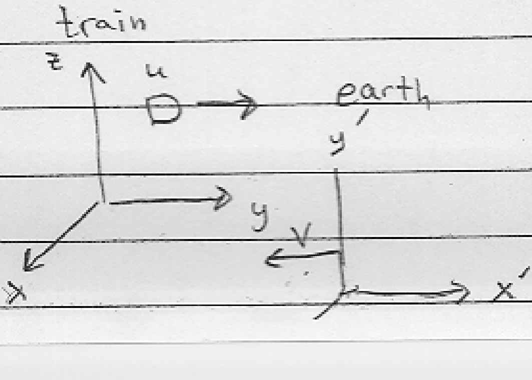


Earth measures the Length

$$L = L_p/\gamma = \frac{100\,m}{5/3} \qquad \gamma = \frac{1}{\sqrt{1-(4/5)^2}} = 5/3$$

$$L = 60m$$

To determine the velocity of μ meson in the earth frame



So

velocity μ rel to train

velocity of earth rel to train

$$u' = \frac{u - v}{1 - uv/c^2}$$

velocity of μ relative to earth

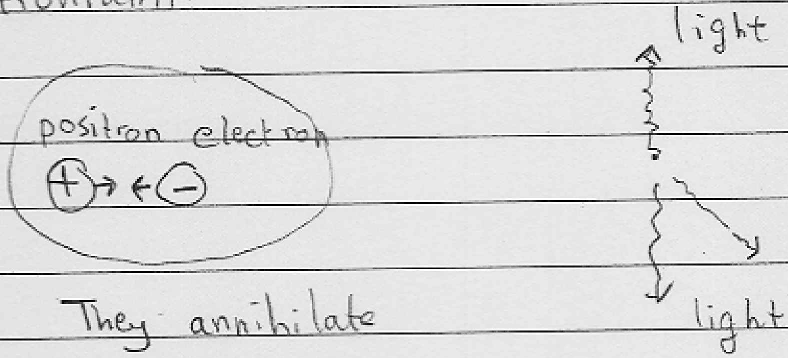
$$u' = \frac{\frac{3}{5}c - (-4/5\,c)}{1 - (3/5\,c)(-4/5\,c)/c^2} = \frac{7/5\,c}{1 + \frac{12}{25}} = \frac{35}{37}\,c$$

So the time

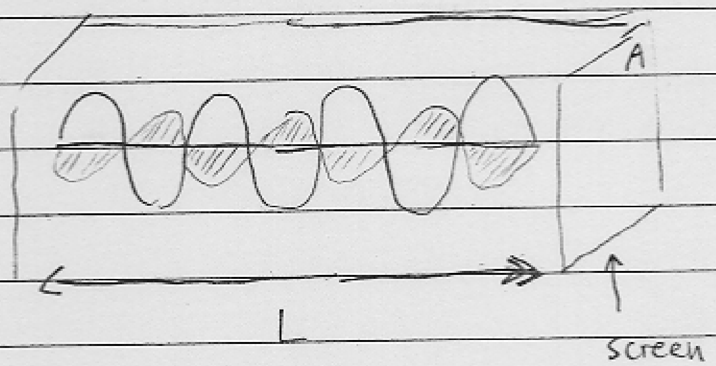
$$t_{earth} = \frac{L}{u'} = \frac{60\,m}{35/37\,c} = 0.211\ \mu s$$

Energy is Mass:

- Today its obvious, for instance consider positronium:



Electromagnetic Waves:



$\mathcal{E} = \mathcal{E}_0 \cos(kx - \omega t)$

$\mathcal{B} = \mathcal{B}_0 \cos(kx - \omega t)$

$\vec{S} = (\vec{\mathcal{E}} \times \vec{B})/\mu_0$ ← poynting vector

The momentum in the wave

$$\vec{P} = \frac{\vec{S}}{c^2}(\text{Volume}) = \frac{\vec{S}}{c^2}(LA)$$

The energy passing through the screen per unit time

$$\frac{\Delta E}{\Delta t} = \vec{S} \cdot \vec{A}$$

After a time $\Delta t = L/c$ the end of the wave passes through the screen

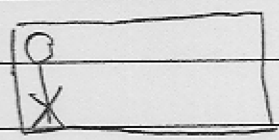
So

$$E = \vec{S} \cdot \vec{A} \, \Delta t = |\vec{S}| \, A \cdot \frac{L}{c}$$

$$E = c|\vec{P}|$$ (see next page)

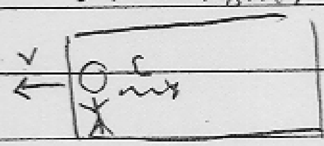
Einstein's Box:

Start:



M_{TOT} = initial mass of the guy. Assume the cart has neglible mass

Shoots light:



m = mass of guy after he shoots the light $< M_{TOT}$

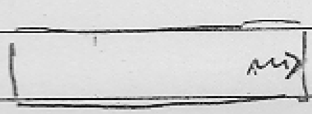
$$P_{BOX} = P_{light}$$

$$mv = \frac{E_{light}}{c}$$

$$v = \frac{E_0}{mc}$$

$E_0 = E_{light}$

Then some time later the light is absorbed and the car stops. The energy of the light gets converted into mass Δm at the back of the cart



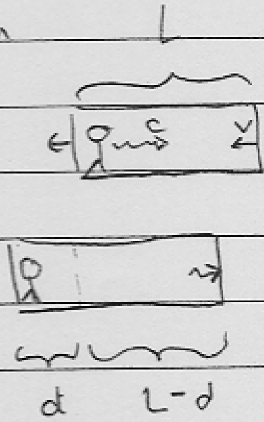
Photons:

- Later we will see that light is made of particle like things called photons

- For now in certain circumstances assume that light is made of particles, with

$$m = 0 \quad \text{and} \quad E = cp$$

Then



The time it takes to travel from one end to the other is

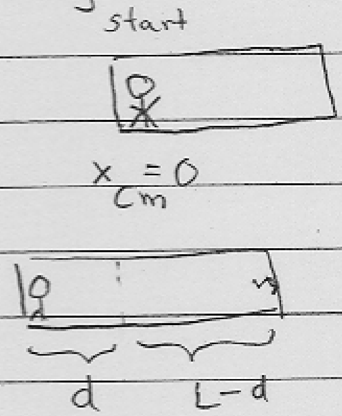
$$t = \frac{L}{c+v}$$

its "c+v" because light goes forward and back of cart goes backward

The Distance the cart moves is

$$d = vt = v\left(\frac{L}{c+v}\right) = \frac{E_0}{mc}\,\frac{L}{c+\frac{E}{mc}} = \frac{E_0 \cdot L}{mc^2 + E_0}$$

Initially



$$x_{cm}^{start} = x_{cm}^{stop}$$

$$0 = m(-d) + \Delta m\,(L-d)$$

Δm is the increase in mass at the back of the cart

So:
$$m\left(\frac{-E_0 L}{mc^2+E_0}\right) + \Delta m\left(L - \frac{E_0 \cdot L}{(mc^2+E_0)}\right) = 0$$

Solving for Δm: (simple algebra)

$$m\left(\frac{+E\cdot \cancel{V}}{mc^2+E}\right) = +\Delta m\,\cancel{V}\left(\frac{mc^2}{mc^2+E}\right)$$

$$\frac{\cancel{m}E}{\cancel{mc^2+E}} = \left(\frac{\cancel{m}c^2}{\cancel{mc^2+E}}\right)\Delta m$$

$$E_0 = \Delta m c^2$$

E = the energy absorbed by the back of the cart. The rest energy

Δm = the increase in mass of the back of the cart

Examples

Units: 1J or $eV = 1.6 \times 10^{-19}$ J

energy of an electron passing through one volt

$$\text{mass} = \frac{\text{Energy}}{c^2} \qquad \frac{eV}{c^2} = 1.783 \times 10^{-36}\ \text{kg}$$

1u = 1 atomic mass unit

Important Numbers:

$m_e = .511 \frac{meV}{c^2}$ = electron mass "half an meV"

$m_p = 938 \frac{meV}{c^2}$ = proton mass "1 GeV"

$1 MeV = 10^6 eV$ = mega eV
$1 GeV = 10^9 eV$ = giga eV

$m_p \simeq 2000\, m_e$

$m_n = 939.5 \frac{meV}{c^2}$ = neutron mass

$m_p \simeq m_n \simeq 1u$ $\qquad 1u \equiv 931 \frac{meV}{c^2}$ = 1 atomic mass unit

Q: Approximately how much does an $N_A = 6 \times 10^{23}$ protons weigh? 1g = Answer

Q: Oxygen 16 has eight protons and eight neutrons. How much does one mole weigh? 16g = Answer

The u = 1 atomic mass unit is defined so that

$N_A u \equiv 1g$ $\qquad$ Exact not approximate

Then we have the reaction

$$^{236}_{92}U \rightarrow {}^{90}_{37}Rb + {}^{143}_{55}Cs + 3n + \text{Energy}$$

(neutron)

- Determine the energy released when one $^{236}_{92}U$ is converted in this reaction:

$$Q_{\substack{\text{energy}\\\text{released}}} = m_u c^2 - (m_{Rb}c^2 + m_{Cs}c^2 + 3m_n c^2)$$

$$= 236.03\,uc^2 - (89.91 + 142.92 + 3(1.008))\,c^2$$

$$= 0.17\,uc^2 \simeq 160\ MeV \qquad (1uc^2 = 931\ MeV)$$

- Determine the energy released when 1kg of $^{236}_{92}U$ is converted in this reaction

$$\#\text{ of part} = 1000\,g \cdot \frac{\#\text{ part}}{\text{gram}}$$

$$\#\text{ of part} = 1000\,g \cdot \frac{6 \times 10^{23}}{236\,g} \simeq 25.423 \times 10^{23}$$

$$E\text{ released} = (E\text{ per pat}) \cdot (\#\text{ of parts})$$

$$= (160 \times 10^6\,eV)(25. \times 10^{23}) \simeq 4000 \times 10^{29}\,eV$$

$$\simeq 0.64 \times 10^{14}\,J$$

Then compare $1J = 2.7 \times 10^{-7}$ kWh

$0.64 \times 10^{14} J = 1.72 \times 10^{7}$ kWh

The electricity used By New York State in 1991 was $\simeq 36 \times 10^{9}$ kWh

Binding Energy:

An electron and a positron are very far away

very Large distance

$(e^-) \longleftrightarrow (e^+)$

$E_0^{e^-} = 0.511$ MeV/c^2

$E_0^{+} = 511 \frac{MeV}{c^2}$

$U_{12} = \frac{1}{4\pi\varepsilon_0} \frac{e_1 e_2}{r}$

$= \frac{1}{4\pi\varepsilon_0} \frac{(-e)(+e)}{(\text{very Large})}$

$U_{12} \simeq 0$

Now Bring the electron and positron close to each other

$(e^-) \xleftrightarrow[1\mathring{A}]{} (e^+)$

$\boxed{1\mathring{A} = 10^{-10} m}$

Important!

• Determine the mass shift when the electron and positron are close together

$$(m_{e^+e^-} \text{ far apart})c^2 = 2 \times (510998.9 \ \tfrac{eV}{c^2})c^2 = 2 \cdot m_e c^2$$

$$(m_{e^+e^-} \text{ close together})c^2 = 2m_e c^2 + U_{12}$$

$$(m_{e^+e^-} \text{ close together})c^2 = 2m_e c^2 - |U_{12}|$$

The potential energy is negati[ve] when they attrac[t]

$$U_{12} = \frac{1}{4\pi\varepsilon_0} \frac{(+e)(-e)}{r}$$

$$U_{12} = \left(8.99 \times 10^9 \ \frac{J \cdot m}{C^2}\right)\left[\frac{+(1.6 \times 10^{-19} C)(-1.6 \times 10^{-19} C)}{(1 \times 10^{-10} m)}\right]$$

$$U_{12} = -14.2 \text{ eV}$$

Thus the Energy difference is:

$$(m_{e^+e^-} \text{ close together})c^2 - (m_{e^+e^-} \text{ far apart})c^2 = \Delta m c^2 \equiv \text{Binding energy}$$

$$\left[2m_e c^2 - |U_{12}|\right] - \left[(2m_e c^2)\right] = \Delta m c^2 \equiv BE$$

$$-U_{12} = \Delta m c^2 \equiv BE$$

$$-14.2 \text{ eV} = BE$$

Usually one quotes $|BE|$ since BE is always negative!

Summary

Binding energy

$$(\text{mass of combo})c^2 = \underset{\text{far apart}}{(\text{mass of constituents})}c^2 + \overbrace{BE}$$

$$(\text{mass of combo})c^2 < \underset{\text{far apart}}{(\text{mass of constituents})}c^2 \quad \text{for bound states}$$

Energy and Momentum:

$E_0 = mc^2$ **Rest** • $E_0 = mc^2$ **Moving:** • $\longrightarrow$ $E = ?\ p = ?$

E_0 = Energy of a particle at rest

- What about kinetic energy & momentum when moving
- What is the velocity of a slow particle:

$$p = mu \implies u = \frac{p}{m} = \frac{c^2 p}{mc^2} = \boxed{c^2 \frac{p}{E} = u}$$

This also works for light:

$$u = c^2 \frac{p}{E} = c\left(\frac{cp}{E}\right) = c \quad \checkmark$$

So $\boxed{u = c^2 \frac{p}{E}}$