Last Time

1. Light is made of discrete packets - Photons
   \[ E = hf \]
   \[ \hbar c = 1240 \text{eV/nm} \quad 2 \text{eV} \sim 600 \text{nm} \sim \text{red} \]
   Visible light = a couple of eV's

2. When lots of photons are measured
   see the wavelike nature of light
   a) -- see slides
   b) A relation between the wavelike and Particle like properties:
      \[ \text{wave energy absorbed area time} \]
      \[ \Delta E = I A \Delta t \]

Intensity of light
\[ I = \langle \delta \rangle \propto \vec{E} \cdot \vec{B} \propto E^2 \]
Small Detector

Medium Detector

Big Detector

Counting Photons
Particle Picture

\[ \Delta E = \Delta N \cdot h \nu \]

\[ \text{energy absorbed} \quad \text{number of photons absorbed} \]

So,

\[ \Delta N = \frac{I \cdot A \cdot t}{h \nu} \]

(3) Photo-Electric Effect - Atoms are small enough to see the discrete nature of light.

\[ K = h \nu - \phi \]

\[ K = \text{energy required to strip electron of light} \quad \text{KE} \]

\[ \text{KE} = \text{energy of ejected electron} \quad \text{eV} \]

\[ K = \text{energy of ejected electron} \quad \text{eV} \]

One can measure the electron's energy by turning a stopping voltage.

\[ KE = PE \quad \text{eV} \]

\[ eV_{\text{stop}} = K \]
Practical Uses of the Photo-electric effect?

Photomultiplier Tube?

one photon in
one burst accelerating potential
of current out
Photomultiplier Tubes

(Janet Conrad - MIT professor, co-spokesperson for the Muons Experiments)
X-rays

50 - 100 kV

\[ \theta \rightarrow \text{metal} \]

\[ \sqrt{\sqrt{}} \]

X-rays

- Very high frequency \( \lambda \sim 10^{-10} \text{ m} \)
  - Compare to visible \( \sim 6000 \times 10^{-10} \text{ m} \)

- \( E = hf \) is a big step! This light is not continuous on human scales

- Problem: how to measure the wavelength?

Normal settings:

\[ \text{Light} \rightarrow \text{Diffraction Grating} \]

\[ \theta_{\text{blue}} - \theta_{\text{green}} - \theta_{\text{red}} \]

\[ \text{red} \rightarrow \text{green} \rightarrow \text{blue} \]
X-ray Tube - Used for security. 160 Kilovolts.
How to make a diffraction grating where the spacing is \( \Delta d \).

Answer: Make a diffraction grating out of a crystal (Bragg, von Laue).

\[ 2d \sin \theta = n \lambda \quad n = 1, 2, 3, 4, \ldots \]

Result
Sharp lines which reflect atomic structure known as $K\alpha$ lines

Continuous radiation "bremsstrahlung"

$\lambda_{\text{min}}$ is determined when all of the energy of the electron is converted to a single photon.

\[ eV = h\nu = h\frac{c}{\lambda_{\text{min}}} \]

The Compton Process (1922)

Before:
\[ x\text{-rays}, E, p_x \]

After:
\[ E_e, p_e \]

Use relativistic energy and momentum conservation and

\[ E = h\nu \]
The Compton Process

Before:

\[ \rightarrow \]
\[ X-rays \]
\[ \rightarrow \]
\[ p/\gamma \]

Particle Description

\( p \)-conservation:

\[ (2) \quad p = p' \cos \theta + p_e \cos \phi \]

\[ (3) \quad 0 = -p' \sin \theta + p_e \sin \phi \]

\( E \)-conservation:

\[ (1) \quad \frac{E + mc^2}{cp} = \frac{E'}{cp'} \quad \sqrt{(cp_e)^2 + (mc^2)^2} \]

Knowns:

\( \theta, \gamma \)

Unknowns:

\( \phi, p_e, p' \Rightarrow \) Use equations (1), (2), (3) solve for \( \phi, p_e, p' \) in terms of \( \gamma, \theta \)
Algebra: (See handout)

\[ \frac{1}{m_e c} \cdot (1 - \cos \theta) = \frac{1}{p'} - \frac{1}{p} \]

Now assume

\[ E = cp = \frac{hc}{\lambda} \quad \text{and} \quad E' = \frac{hc}{\lambda'} \]

\[ p = \frac{h}{\lambda} \quad \text{and} \quad p' = \frac{h}{\lambda'} \]

Then multiplying (*) by \( h \)

\[ \frac{h}{m_e c} \cdot (1 - \cos \theta) = h - \frac{h}{p'} \]

Or

\[ \frac{h}{m_e c} \cdot (1 - \cos \theta) = \lambda' - \lambda \]
Energy (Algebra: Skipped if pressed for time)

(1) \[ E + m_e c^2 = E' + E_c \]

Momentum

(2) \[ p = p' \cos \theta + p_e \cos \phi \]

(3) \[ \mathbf{p} = -p' \sin \theta + p_e \sin \phi \]

Also have:

\[ E = cp, \quad E' = cp', \quad E_e = \sqrt{(cp_e)^2 + m_e c^2} \]

Algebra: use the 1. equation to eliminate \( p_e, \phi \)

\[ p' \sin \Theta = p_e \sin \phi \quad (2) \]

\[ (p - p' \cos \Theta) = p_e \cos \phi \quad (3) \]

Squaring both equations and add:

\[ p^2 - 2pp' \cos \Theta + p'^2 \cos^2 \Theta = p_e^2 \cos^2 \phi \]

\[ p^2 \sin^2 \Theta = p_e^2 \sin^2 \phi \]

\[ p^2 - 2pp' \cos \Theta + p'^2 = p_e^2 \quad \text{so far used one eqn to elim } \phi \]
Now use $E = mc^2$

$$(E_p - E_{p'}) + mc^2 = E_e$$

$$\left[(E_p - E_{p'}) + mc^2\right]^2 = E_e^2$$

$$(E_p - E_{p'})^2 + 2(E_p - E_{p'})(mc^2) + (mc^2)^2 = E_e^2$$

$$(E_p - E_{p'})^2 + 2(E_p - E_{p'})mc^2 = E_e^2 - mc^2$$

$$(E_p - E_{p'})^2 + 2(E_p - E_{p'})mc^2 = (cp)^2$$

$$(p - p')^2 + 2(p - p')mc = p_e^2$$

Putting in:

$$p^2 - 2pp' \cos \theta + p'^2 = (p - p')^2 + 2(p - p')mc$$

**Working:**

$$\frac{1}{1 - \cos \theta} = \frac{1}{mc} \frac{1}{p - p'}$$

Now assume: $p = h\nu$, $p' = h\nu'$

$$\frac{1}{1 - \cos \theta} = \frac{1}{mc} \frac{1}{h\nu} \frac{1}{h\nu'}$$
Important Points

1. Used Relativity and Used
   \[ E = cp = \hbar f \]
   and it worked

2. Compton wavelength

\[ \lambda' \text{ is longer than } \lambda \text{ for } 90^\circ \]

\[ \frac{\hbar}{mc} (1 - \cos\Theta) = \lambda' - \lambda \]

\[ \frac{\hbar}{mc} = \lambda' - \lambda \]

The wavelength is longer by the Compton wavelength

\[ \lambda' = \frac{\hbar}{mc} = \text{the intrinsic size of the electron} \]
\[ \lambda_c = \frac{h}{mc} = \frac{hc}{mc^2} = \frac{1240 \text{ eV nm}}{0.511 \times 10^6 \text{ eV}} = 0.025 \text{ Å} = 0.0025 \text{ nm} \]

\[ \lambda \approx \frac{1}{40} \text{ the size of an atom} \]

3. Why use X-rays

For \( \theta = 90^\circ \)

\[ \lambda_c (1 - \cos \theta) = \lambda' - \lambda \]

\[ \lambda_c = \lambda' - \lambda \Rightarrow \lambda' = \lambda + \lambda_c \]

For \( \lambda \approx 500 \text{ nm} \) (visible light)

\[ \lambda' \approx 500 \text{ nm} + 0.0025 \text{ nm} \approx 500 \text{ nm} \]

Small shift

Thus it was essential to use X-rays

4. What is the energy \( E' \) of the recoiling electron when the incoming X-ray has energy \( h \lambda_c \) at \( \theta = 90^\circ \)

\[ E_{\text{cons}} + mc^2 = E' + E_e \]
Then for $\Theta = 90^\circ$

\[
\lambda' \cos^2 \Theta = \lambda' - \lambda
\]

$= \lambda$ for this problem

$2\lambda' = \lambda'$

\[
E = hf = \frac{hc}{\lambda} = \frac{hc}{\lambda'} = \frac{m_e c^2}{2}
\]

\[
E' = hf' = \frac{hc}{\lambda'} = \frac{m_e c^2}{2}
\]

So \[E + m_e c^2 = E' + E_e\]

\[
m_e c^2 + m_e c^2 = \frac{m_e c^2}{2} + E_e\]

\[
\frac{3}{2} m_e c^2 = E_e \implies \gamma = \frac{3}{2} \implies \beta = 0.74
\]

Thus electrons are pretty relativistic

Q: How important is the binding energy?