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Lab Section: 01
Date: 01/01/01

Projectile Motion

1 Introduction

The purpose here is to convince the TA that you understood how the lab worked. Needlessly philosoph-
ical or lengthy remarks will cost you points.

The purpose of this lab was to measure the properties of projectile motion. A schematic
of the apparatus is shown below (you could/should simply draw this by hand. I used X-fig which is
free)
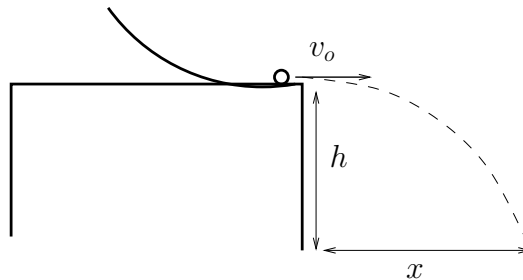
A small metal ball was released from a ramp at the edge of the table of height h The initial
velocity v_o of the ball was measured by measuring the time it took for the ball to cross the
photogate detector and knowing effective diameter of the ball. The final distance x that the
ball landed was recorded as a function of the initial velocity v_o. In the Newtonian theory of
projectile motion these quantities are related by (If you are using some program like word where
entering formulas is time consuming, simply leave a bit of space and write the formula by hand)

$$x = v_o \sqrt{\frac{2h}{g}} \, .$$

This formula is compared to the measured data in what follows.

2 Recorded and Derived Data

The purpose here is to record all relevant numbers and how they were obtained. The raw data should
be in the lab notebook. Often when making plots we need derived quantities, e.g. if you know the
length and width you could determine the area $A = LW$. you should explain how you propagated the
errors in L and W to determine the error in A. This is described in the error analysis writeup. The data
in this "Lab" is totally made up.

Before data taking started, the height of the table was $h = (1.01 \pm 0.005)$ m measured
using ruler stick. The error was estimated to the nearest half centimeter.

Ramp Setting	x (m)	Time t_{stop} (s)	$v_o(m/s)$
1	0.042 ± 0.003	0.1373	0.087 ± 0.003
2	0.059 ± 0.003	0.0924	0.130 ± 0.005
3	0.087 ± 0.003	0.0758	0.158 ± 0.007
4	0.089 ± 0.003	0.0660	0.181 ± 0.008
5	0.096 ± 0.003	0.0584	0.205 ± 0.009

Table 1: Summary of data taken.

The diameter of the ball $D_{\text{eff}} = (1.20 \pm 0.05)\,\text{cm}$ was determined by using the SALT translation stage and the photogate detector. Specifically, the every turn of the nob of the translation stage advanced the stage by $1/28$ of an inch. After the photogate detector registered off, an additional 13.5 turns were registered before the light crossed the photogate detector again. The uncertainty was estimated based on repeating the process.

To vary the velocity of the projectile, the ball was released from five different positions along the ramp. For a given ramp position the time the photogate detector was off t_{stop} was recorded by the photogate electronics. This together with the effective diameter of the ball was sufficient to determine the projectile velocity for each ramp position

$$v_o = \frac{D_{\text{eff}}}{t_{\text{stop}}} \,.$$

The uncertainty in t_{stop} was small and is neglected. The uncertainty in the initial velocity is then entirely due to D_{eff}, *i.e.* $\Delta v_o = \Delta D_{\text{eff}}/t_{\text{stop}}$. These initial velocities and uncertainties are recorded in Table 1.

The distance x from the edge of the table to the landing point recorded for every release using a the ruler stick and a plumb line. To estimate the uncertainty in this number, the process was repeated several times and the full data set is recorded in the notebook.

A tabular summary of the launch stopping times and ranges x for each launch position is given in Table 1.

3   Analysis and Conclusions

According to the Newtonian theory the projectile location is given by

$$x = v_o\sqrt{\frac{2h}{g}} \,. \tag{1}$$

To derive this, we first note the time the ball is in the air is found by the formula

$$h = \frac{1}{2}gt^2, \qquad \text{so,} \qquad t = \sqrt{\frac{2h}{g}} \,.$$

Then since the x and y directions are independent we can simply multiply by the x velocity v_o to obtain the formula given above.
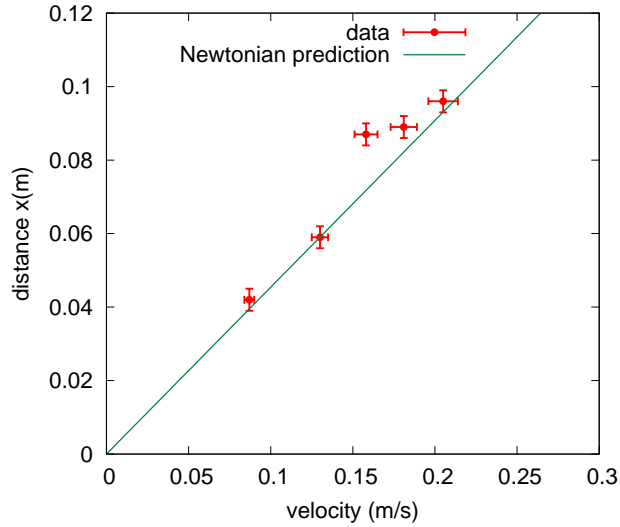
Figure 1: A graph of the projectile range versus its initial velocity.

Fig. 1 shows a graph of the measured landing distance x as a function of the initial velocity v_o. Also shown is the Newtonian prediction of Eq. 1. (In drawing this theoretical curve we have neglected the uncertainty in h which is small compared to the spread of data points.) The theoretical curve is slightly, but systematically, under the data points. This could arise due a number of reasons. First, it is difficult to avoid disturbing the photogate detector between calibration step (where D_{eff} is measured) and the measurement step (where v_o is determined). Such disturbances can systematically underestimate the initial velocity. Additionally, the track was not exactly level. If the launch angle is somewhat larger than 90°, this launch angle could systematically bias the comparison to the theoretical curve. Overall, the data agree with the Newtonian theory within these systematic uncertainties.

A The report

To physically make your lab report, you have several options

1. Neatly write your lab report (I think this is the slowest way!).

2. Use microsoft word or some other word processor to type up your lab report. If you need an equation, you can simply leave a space and write the equation in by hand. This option seems to be what most people are following. You can simply attatch graphs at the end, or in the middle

3. Finally, you could try to use LaTeXwhich is free and is the way professional people write mathematical documents. This document was made with LaTeX. I personally feel that LaTeXis the best choice so I have included a sample lab latex file on the web page. People who used it last year ultimately liked it. If you are having trouble getting the sample file to "compile" with latex, ask me, and Ii wll help you. If you don't feel comfortable googling and hacking with computers until LaTeXworks for you, the first two options are probably better a choice, but are ultimately very limitting.

B Error Analysis

Generally quantities have error. Consider computing the volume from $V = xyz$. Each of these quanties have error $x \pm \Delta x$, $y \pm \Delta y$ and $z \pm \Delta z$. We wish to determine the error in the volume

$$V \pm \Delta V \,.$$

To keep this general consider the volume a generalized function of x, y, z, i.e. $V = f(x, y, z)$. In the day of computers, the simplelest way to do error analysis on a column of numbers is to *use excel* or write a simple program (in *e.g. python*) to compute

$$\Delta V = \sqrt{(\Delta_x f)^2 + (\Delta_y f)^2 + (\Delta_z f)^2}$$

Here

$$
\begin{aligned}
\Delta f_x &= (f(x + \Delta x, y, z) - f(x - \Delta x, y, z))/2 & (2)\\
\Delta f_y &= (f(x, y + \Delta y, z) - f(x, y + \Delta y, z))/2 & (3)\\
\Delta f_z &= (f(x, y, z + \Delta z) - f(x, y, z - \Delta z))/2 & (4)
\end{aligned}
$$

This procedure works for any function $f(x, y, z)$ and not just for a simple product.

As an example $x = 2.0 \pm 0.1 \,\mathrm{cm}$, $y = 4.0 \pm 0.4 \,\mathrm{cm}$, $z = 8.0 \pm 1.6 \,\mathrm{cm}$, and $f(x, y, z) = xyz$. Following this procedure

$$V = 2 \cdot 4 \cdot 8 = 64 \,\mathrm{cm}^3$$

Yielding

$$\Delta f_x = 3.2 \,\mathrm{cm}^3 \qquad \Delta f_y = 6.4 \,\mathrm{cm}^3 \qquad \Delta f_z = 12.8 \,\mathrm{cm}^3$$

and the total error

$$V = 64. \pm 14.6 \,\mathrm{cm}^3$$

The above procedure always works. Usually formulas for error analysis are quoted a bit differently which we will discuss now. One can make the approximation

$$\Delta_x f \simeq \frac{\partial f}{\partial x}\Delta x \qquad \Delta_y f \simeq \frac{\partial f}{\partial y}\Delta y \qquad \Delta_z f \simeq \frac{\partial f}{\partial z}\Delta z$$

and the error on $f(x, y, z)$ is

$$\Delta f = \sqrt{(\frac{\partial f}{\partial x}\Delta x)^2 + (\frac{\partial f}{\partial y}\Delta y)^2 + (\frac{\partial f}{\partial z}\Delta z)^2}$$

This leads to the following rules:

- If $z = x + y$ is a sum of two numbers then the error in z is the errors of x and y added in quadrature

$$\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2} \qquad (5)$$

- If $z = x \cdot y$ is the product of two numbers then the percent error in z is the percent errors of x and y added in quadrature

$$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} \qquad (6)$$

C Plotting

There are a number of things to keep in mind when making a scientific graph.

- All axes of graphs should be clearly labeled and units given.

- Usually a graph should have a title though this is not always necessary.

To physically make graphs, here are some acceptable options:

1. Use your lab notebook that you purchased in the book store and carefully plot by hand the data-points using a ruler. Attach this hand made graph to your lab report. Usually we will want to find a trend line to these data. You can simply draw the best line by eye and the max and min line to estimate the error. This whole max min slope business is somewhat mickey-mouse, but does provide estimate of the uncertainty which will most likely be qualitatively correct. This procedure is described on the course web page in the error analysis writeup. An example of a graph made in this way is below. This procedure is perhaps initially fastest, but is ultimately slower.

2. Use Excel to make the graph. The procedure to make an XY graph using excel and to have it print out on a separate page is to first create the chart in the worksheet. After this you should copy the chart to a "Chart Sheet" so that when the Chart Sheet is printed it prints the whole graph. Unfortunately excel does not make error bars. (I guess financial people don't care about error) So If you make a graph with excel, you
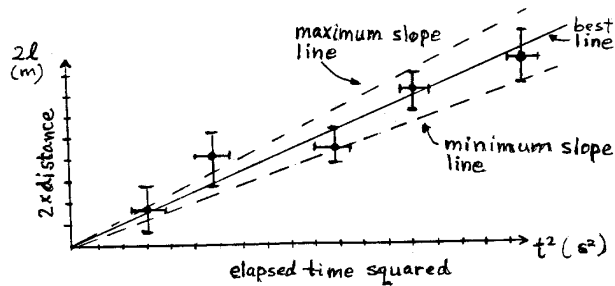
Figure 2: A scan of graph made by hand. You could make the graph and in the lab book and then tear it out and attach it to your report.

will have to draw in the x and y error bars by hand. You can use excel to draw the best fit line. However, it will not estimate the uncertainty in the slope. So to estimate the uncertainty in the slope you can draw by hand on your graph the line with the largest and smallest slope and estimate the uncertainty in the line from the difference. This procedure is described on the course web page in the error analysis writeup. A scan of a graph made in this manner is here.

3. A third and more professional option is to use a free program such as gnuplot to make the graph. Gnuplot will draw the errors in the x and y directions. If you choose this option you can use gnuplot to fit the best fit line through a set of data points. It will also report an uncertainty in this procedure (unlike excel). You can use this number as an estimate for the uncertainty in the fitted parameters. Gnuplot will not consider the uncertainty in the x-direction when making this fit. So should check that the resulting slope and error is reasonable by plotting the min slope line and the max slope line. The gnuplot option has the advantage that it will continue to grow with you. I (and many others!) use it for professional work. If you don't feel comfortable googling and hacking with computers (or do not want to bother), the first two options are probably better a choice.

A minimal procedure to make a gnuplot plot is first to put the data into a file. In this case it was called `fakeg.dat` and contained the data organized into columns

```
1.0 10.2980290186 0.5 0.98
4.0 36.6202589812 0.5 3.92
9.0 85.329410075 0.5 8.82
16.0 172.796164547 0.5 15.68
25.0 192.615373522 0.5 24.5
```

Then use a text editor (such as Notepad on Windows) to type in a script which I called `fakeg.gpi` which yields
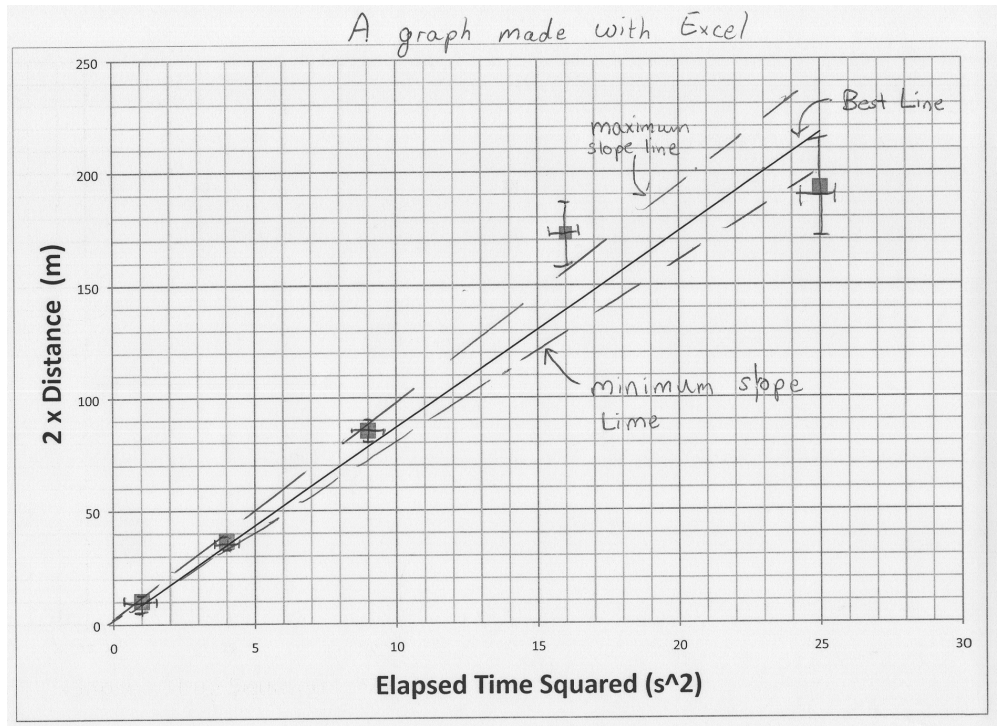
```
# turn of the legend
set nokey
```

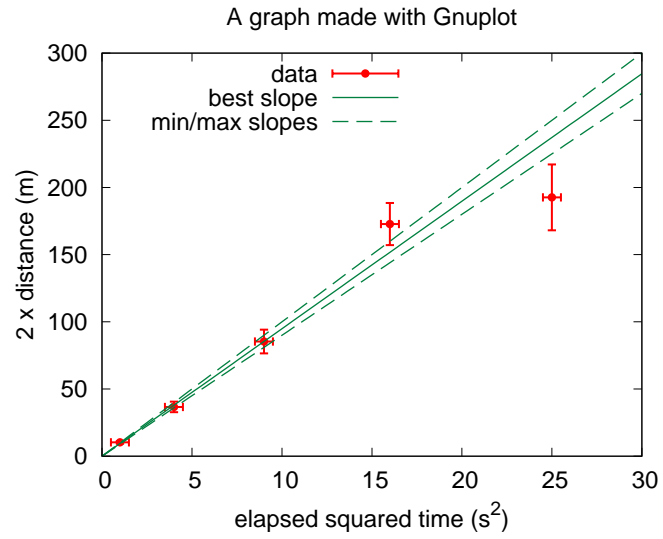Figure 3: A scan of graph made with excel. You would (of course) simply attach the real graph to the report.

```
# plot between 0 and 30 using columns 1 and 2 for x, y and 3, 4 for
# y and x errorbars
plot [0:30] "fakeg.dat" using 1:2:3:4 with xyerrorbars
# Uncomment these lines to make the output for the printer
# often a png terminal is good for windows
#set terminal postscript enhanced color
#set output "test.ps"
replot
```

Then at the gnuplot command line typing

```
load "fakeg.gpi"
```

will produce the figure. With enough practice and reading online and in the documentation one makes publishable plots such as this one

4. A final option is to use `matlab` to make your graphs and to perform a fit. As with `matlab` you should If you choose this option you can use `matlab` to fit the best fit line through a set of data points. It will also report an uncertainty in this procedure (unlike excel). You can use this number as an estimate for the uncertainty in the fitted parameters. `matlab` will not consider the uncertainty in the x-direction when making

A graph made with Gnuplot

this fit. So should check that the resulting slope and error is reasonable by plotting the min slope line and the max slope line.