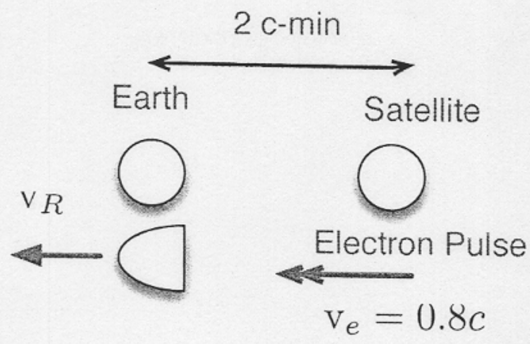
Quantity	Symbol	Value
Coulombs Constant	$k_C = 1/4\pi\epsilon_o$	$8.98 \times 10^{9}\,\mathrm{Nm^2/C^2}$
Electron Mass	m_e	$9.1 \times 10^{-31}\,\mathrm{kg}$
Electron Charge	e	$-1.6 \times 10^{-19}\,\mathrm{C}$
Electron Volt	eV	$1.6 \times 10^{-19}\,\mathrm{J}$
Permitivity	ϵ_o	$8.85 \times 10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{Nm^2}}$
Magnetic Permeability	μ_o	$4\pi \times 10^{-7}\,\mathrm{N \cdot A^2}$
Speed of Light	c	$3.0 \times 10^{8}\,\mathrm{m/s}$
Planck's Constant	h	$6.6 \times 10^{-34}\mathrm{m^2kg/s}$



According to observers stationary with respect to earth at time $t = 0$ an electron pulse is emitted with speed $v_e = 0.8c$ from an unknown satellite a distance $2c\,\text{min}$ from the earth as shown below (one c-minute is a unit of length). According these observers a rocketship traveling to the left flies past the earth at the exact same time as the electron pulse is emitted (see picture). The observers on the rocket ship measure the electron pulse moving to the left with speed $0.4c$. The earth and rocketship observers agree to place the rocket's passing of the earth at the the origin of spacetime $(ct, x) = (ct', x') = (0, 0)$.

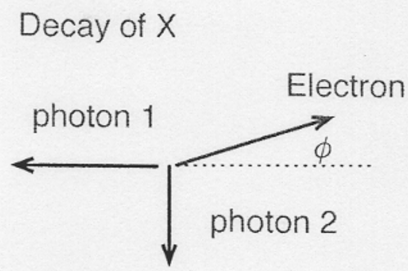
1. Classially (i.e. non-relativistically) what is the speed of the rocketship v_R in units of c relative to earth?

2. Relativistically what is the speed of the rocketship v_R in units of c relative to earth. (If you cant do this part take the speed $v_R = 0.5c$ in what follows.)

3. Draw a spacetime diagram of according to earth observers. Show the worldlines of the electron pulse, the earth, the rocketship, and the satellite.

4. Determine when and where the electron pulse was emitted according to the rocket ship. Give your answer in $c\,\text{min}$ and minutes.

5. Draw the space time diagram according to the rocketship; show the worldlines of the electron pulse, the earth, the rocketship, and the satellite. Breifly explain your picture in words.



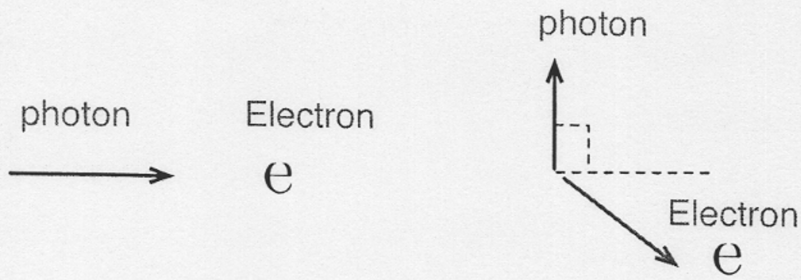
A mysterious particle X at rest spontaneously decays into two photons and an electron as shown below. Photon 1 has a wavelength which happens to be exactly equal to the electron Compton wavelength and is traveling in the negative X direction. Photon 2 has a wavelength which is half of the electrons Compton wavelength and is traveling in the negative y direction.

1. What is the electrons Compton wavelength in Angstroms and what is the electron mass ($\times c^2$) in MeV? If you dont know this you can ask so you can do the other parts.

2. Determine the energies of the photons in MeV.

3. Determine the momentum of the electron in MeV/c. Determine the angle ϕ.

4. Determine mass ($\times c^2$) of particle X in MeV.



An X ray photon with frequency ν collides into the electrons of a carbon target and the electrons are ejected. Neglect the binding of the electrons to the carbon. Give your final answers in terms of ν and the fundamental constants h, m_e and c

1. If the photon is scattered by an angle of 90^o as shown below, calculate the kinetic energy of the electron.

2. Under what conditions (on ν) will the electron be non-relativistic, *i.e.* the frequency should be large or small compared to something in order for the electron to be non-relativistic. (Hint: how does the kinetic energy compare the rest energy in the non-relativistic limit.)

3. Determine a Taylor series in ν for the kinetic energy of the electron divided by its rest energy K/m_ec^2 which is valid at small frequency. Your final expressions should be valid through and including ν^3.

Problem 1

① $-0.4c$

② We have

$$v_e' = -0.4c$$

$$v_e' = \frac{v_e - v_R}{1 - v_e v_R/c^2} \qquad v_e = -0.8c$$

Solving: $v_e'(1 - v_e v_R/c^2) = v_e - v_R$

$$v_e' - v_e' v_e v_R/c^2 = v_e - v_R$$

$$v_R(1 - v_e v_e'/c^2) = v_e - v_e'$$

$$v_R = \frac{v_e - v_e'}{1 - v_e v_e'/c^2}$$

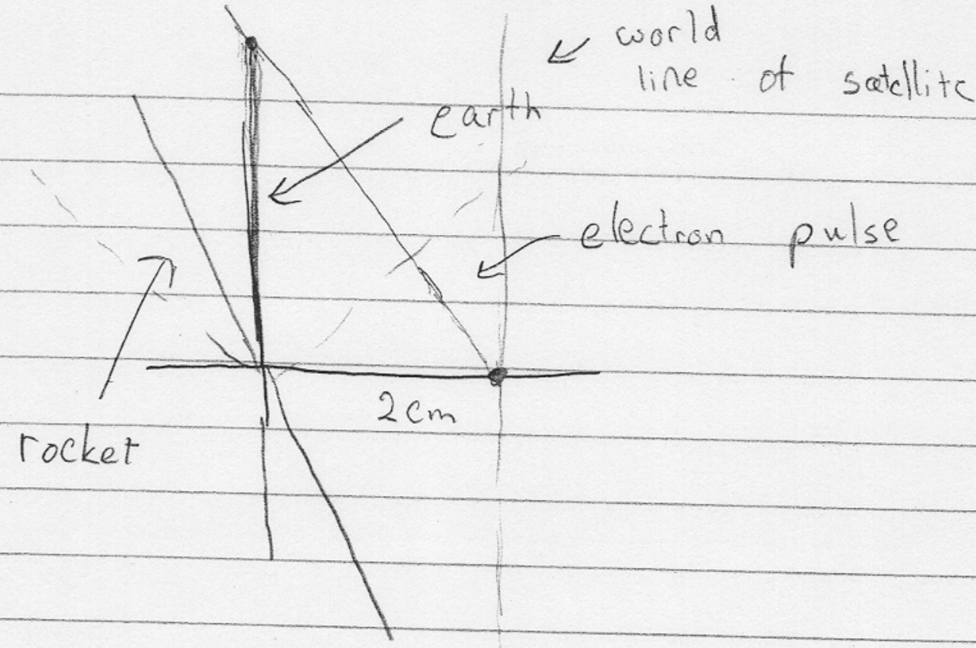
We have:

$$v_R = \frac{-0.8c - (-0.4c)}{1 - (-0.8)(-0.4)}$$

$$v_R = \frac{-0.4c}{1 - (0.8)(0.4)} = -0.59c$$

So the rocket must go slightly faster to get the same factor of two reduction of ~~ele~~ electron speed.

(3)



(4) According to Rocket

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

$$ct' = \gamma \cancelto{0}{ct} - \gamma\beta x$$

$$x' = -\gamma\beta \cancelto{0}{ct} + \gamma x$$

So

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = -0.59$$

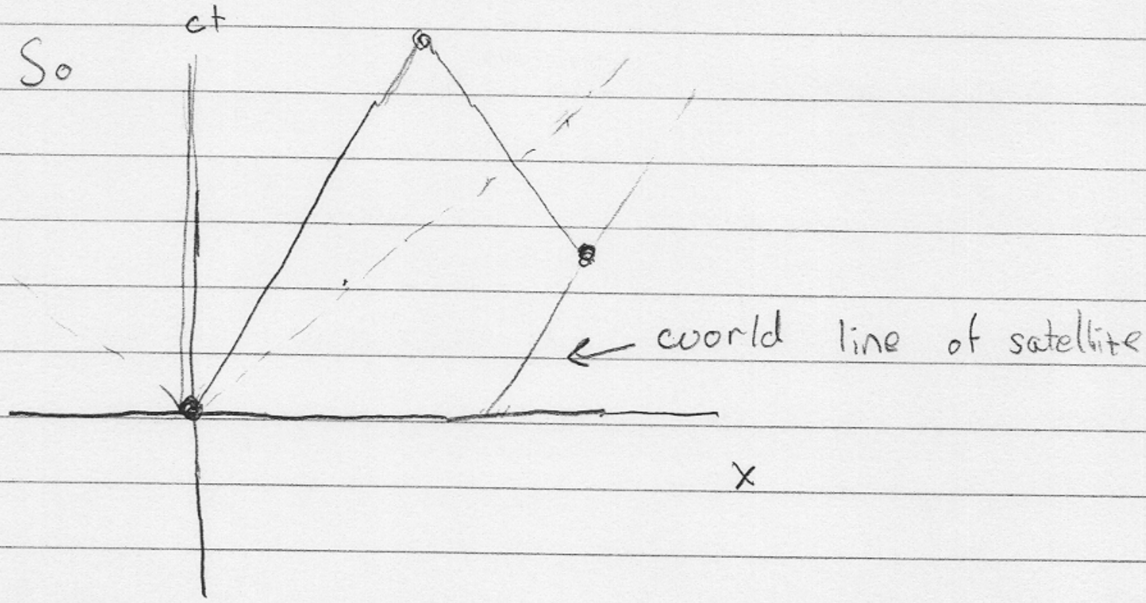
$$\gamma = 1.24$$

So

$$ct' = -(1.24)(-0.59)\, 2\,\text{cmin} = 1.46 \text{ cmin}$$

$$x' = 1.24 \cdot 2\,\text{cmin} = 2.48\,\text{cmin}$$

(4) So



Problem 2

① $\lambda_c = \frac{h}{m_e c} = \frac{hc}{m_e c^2} = \frac{1240 eV nm}{0.5 \times 10^6 eV} = 0.0024 nm$

$= 0.024 Å$

$m_e c^2 \simeq 0.5 \text{ MeV}$

② $E_1 = \frac{hc}{\lambda_1}$ $E_2 = \frac{hc}{\lambda_2}$

$\lambda_1 = \frac{h}{m_e c}$ $\lambda_2 = \frac{1}{2} \frac{h}{m_e c}$

So

$E_1 = \frac{hc}{\frac{h}{m_e c}} = m_e c^2$ $E_2 = \frac{hc}{\frac{1}{2}\frac{h}{m_e c}} = 2 m_e c^2$

$E = 0.5 MeV$ $= 1 MeV$

③ Using momentum conservation:

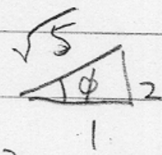
$0 = -p_1 + 0 + p_e \cos\phi$ $p_1 = p_e \cos\phi$

$0 = 0 + - p_2 + p_e \sin\theta$ $p_2 = p_e \sin\phi$

So

$$\tan\phi = \frac{p_2}{p_1} = 2$$

$$\phi = 63^\circ$$



Then

$$cp_e = \frac{cp_2}{\sin\phi} = \frac{2m_ec^2}{\sin\phi} = \frac{2m_ec^2}{2/\sqrt{5}} = \sqrt{5}\, m_ec^2$$

$$cp_e = 1.14 \text{ meV}$$

$$p_e = 1.14 \text{ meV}/c$$

(4) Using energ conservation

$$M_xc^2 = E_1 + E_2 + E_e$$

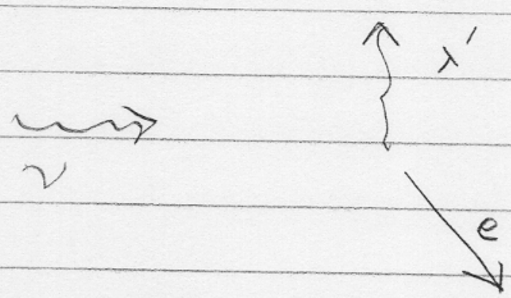
$$= E_1 + E_2 + \sqrt{(cp_e)^2 + (m_ec^2)^2}$$

$$= m_ec^2 + 2m_ec^2 + \sqrt{(\sqrt{5}m_ec^2)^2 + (m_ec^2)^2}$$

$$= m_ec^2\left(1 + 2 + \sqrt{5+1}\right)$$

$$M_xc^2 = 2.78 \text{ meV}$$

Problem 3



Writing down E and P-consv

(1) $E + m_e c^2 = E' + E_e$

$p = 0 + p_e \cos\varphi \implies p = p_e \cos\varphi$

$0 = p' - p_e \sin\varphi \implies p' = p_e \sin\varphi$

Then from (1)

$$E - E' = \underbrace{E_e - m_e c^2}$$

$$\frac{E - E'}{m_e c^2} = \frac{K}{m_e c^2}$$

Then

0 for 90°

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

So

$$\frac{h}{p'} - \frac{h}{p} = \frac{h}{m_e c} \implies \frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}$$

So

$$\frac{E}{E'} - 1 = \frac{E}{m_e c^2}$$

$$\frac{E}{E'} = 1 + \frac{E}{m_e c^2}$$

$$\frac{E}{1 + E/m_e c^2} = E'$$

So

$$\frac{K}{m_e c^2} = \frac{(E - E')}{m_e c^2} = \left(E - \frac{E}{1 + E/m_e c^2}\right) \frac{1}{m_e c^2} = \frac{K}{m_e c^2}$$

So

$$\frac{E^2/(m_e c^2)^2}{1 + E/m_e c^2} = \frac{K}{m_e c^2}$$

(2) For $K/m_e c^2 \ll 1$ we have

a non-relativistic electron:

Need, $\frac{h\nu}{m_e c^2} \ll 1$

(3) Using the taylor series

$$\frac{K}{m_e c^2} = \frac{(h\nu/m_e c^2)^2}{1 + (h\nu/m_e c^2)}$$

$$\frac{K}{m_e c^2} \simeq (h\nu/m_e c^2)^2 \left[1 - (h\nu/m_e c^2)^2 + \ldots\right]$$