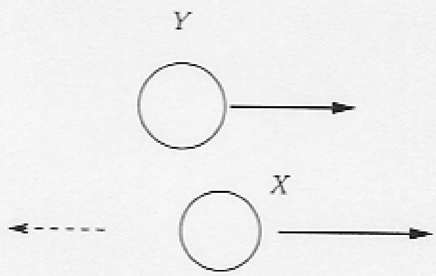
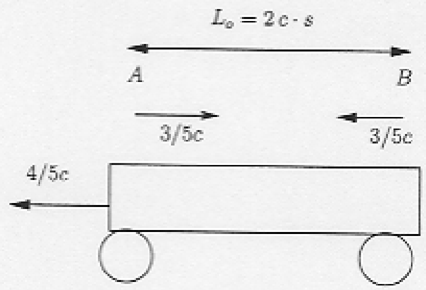
Quantity	Symbol	Value
Coulombs Constant	k_C	$8.98 \times 10^9\,\mathrm{Nm^2/C^2}$
Electron Mass	m_e	$9.1 \times 10^{-31}\,\mathrm{kg}$
Electron Charge	e	$-1.6 \times 10^{-19}\,\mathrm{C}$
Electron Volt	eV	$1.6 \times 10^{-19}\,\mathrm{J}$
Permitivity	ϵ_o	$8.85 \times 10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{Nm^2}}$
Magnetic Permeability	μ_o	$4\pi \times 10^{-7}\,\mathrm{N \cdot A^2}$
Speed of Light	c	$3.0 \times 10^8\,\mathrm{m/s}$
Planck's Constant	h	$6.6 \times 10^{-34}\mathrm{m^2kg/s}$

A mysterious new particle Y has a mass which is 8 times the electron mass and has kinetic energy, KE $= 2m_ec^2$. It decays into another mysterious particle X and a photon with a wavelength of one compton wavelength.

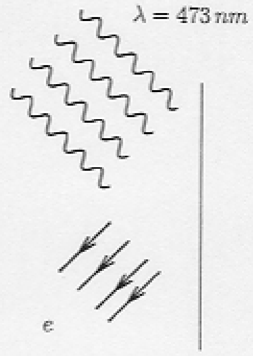


1. What is the electron mass in MeV/c^2 and what is the compton wavelength in meters. (Ask me if you don't know this – some points will be deducted.)
2. Determine the energy and momentum of the photon.
3. Determine the energy and momentum of particle Y
4. Determine the mass of the final particle X.

Two particle accelerators A and B are located a distance of $2\,cs$ apart according to an earth observer (never mind that $2\,cs$ is larger than the earth.) According to the earth observer, at time $t = 0$ to muons are launched from the accelerators each with a speed of $3/5\,c$ in opposite directions as shown below. Meanwhile a train moving to the left with speed $4/5c$ observes the same process. Both the train and earth observers agree to place the emission of muon A at the space time-origin, i.e. the emission of a muon A from particle accelerator A happens at $(ct, x) = (ct', x') = (0, 0)$



1. Determine when and where the muons cross according to an earth observer.
2. Draw a space time diagram showing the trajectories of the two muons according to an earth observer.
3. What is the length of between the accelerators according to a train observer
4. Determine when and where muon B is emitted. Explain your results with a short description (one or two sentences)
5. Determine the velocities of muon A and muon B according to the train observer
6. Write down the equation of motion of the two muons according to the train observer.
7. Determine when and where the two muons cross according to the train observer.
8. Draw a space time diagram showing the trajectories of the two accelerators and the trajectories of the two muons according to a train observers.



Consider the photoelectric effect produced by shining a blue $\lambda = 473$ nano-meters laser on the surface of a clean metal foil of sodium with work function 2.1 eV. A typical atomic radius is $r_o = 1\,A^o$ (ask me if you dont know what an Angstrom is) . Typical response times for an atom to receive a photon and eject an electron are say than $\Delta t = 100\text{ns} = 10^{-7}s$ (or less). The laser has a beam waist radius of $r = 1\,\text{mm}$. (For example a laser pointer make a small circle on the page with approximately this radius.)

1. Determine the energy of a photon with the blue wavelength.

2. For a 40 mW laser determine the number of photons which strike the metal per unit area per unit time.

3. Estimate the power that would be necessary for the atom in the laser beam to receive on average two photons during the course of the emission of an electron. Use the radius of the atom given above and the time for electron emission given above. Compare this power to a typical milliwatt laser.

4. Now lets assume that the power is sufficiently low that the atom sees only one photon at a time. Determine the v/c (velocity over the speed of light) of the ejected photo electron.

5. Normally when analyzing the photoelectric effect we use a non-relativistic expression for the kinetic energy $K_{nr} = \frac{1}{2}m_e v^2$. We define the error as

$$\%\texttt{error} \equiv \frac{K_{\text{rel}} - K_{\text{nr}}}{K_{\text{nr}}}$$

where K_{rel} is the relativistic expression for kinetic energy. By using the Taylor series expansion $(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \ldots$ determine the `% error` as a function of v/c.

6. Using this result, to estimate the error of using the non-relativistic expression in part 4.

Problem 1

① $m_e = 0.511\ \mathrm{MeV}/c^2$

$$\lambda_c = \frac{h}{m_e c} = \frac{hc}{m_e c^2} = \frac{1240\,\mathrm{eV\ nm}}{511000\,\mathrm{eV}} = 0.0024\,\mathrm{nm}$$

② $$E = \frac{hc}{\lambda} = \frac{hc}{\frac{h}{m_e c}} = m_e c^2 = 0.511\ \mathrm{MeV}$$

$$p = \frac{E}{c} = 0.511\ \mathrm{MeV}/c$$

③ $m_Y c^2 = 8 m_e c^2$

$K_Y = 2 m_e c^2$

$E_Y = 10 m_e c^2$

$$cp_y = \sqrt{E^2 - (m_Y c^2)^2} = \sqrt{(10 m_e c^2)^2 - (8 m_e c^2)^2}$$

$$= m_e c^2 (100 - 64)^{1/2} = 6 m_e c^2$$

④ Using E and p consv

$E_y = E + E_x$

$cp_y = -cp + cp_x$

So

$$cp_x = cp_y + cp = 6m_ec^2 + m_ec^2 = 7m_ec^2$$

$$E_x = E_y - E = 10m_ec^2 - m_ec^2 = 9m_ec^2$$

$$m_xc^2 = \sqrt{E^2 - (cp_x)^2} = \sqrt{(9m_ec^2)^2 - (7m_ec^2)^2}$$

$$= m_ec^2\sqrt{9^2 - 7^2}$$

$$= m_ec^2\, 4\sqrt{2} \simeq m_ec^2(5.65)$$

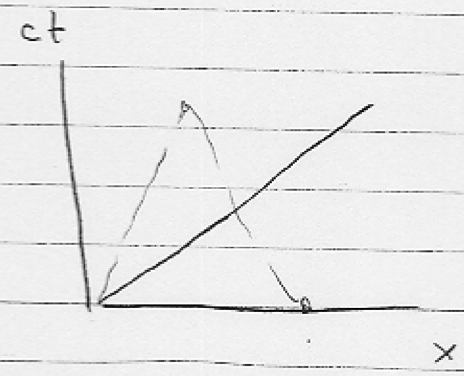
Problem 2

(1)

$x = 1cs$

$t = \frac{1cs}{\frac{3}{5}c} = \frac{5}{3}s$

(2)



(3)

$L = \frac{L_0}{\gamma}$

$= \frac{2cs}{5/3}$

$L = \frac{6}{5}cs$

$\gamma = \frac{1}{\sqrt{1-(4/5)^2}} = 5/3$

(4)

$ct' = \gamma ct - \gamma\beta x$

$x' = -\gamma\beta ct + \gamma x$

train moves to left

$(ct, x) = (0, 2cs)$ $\gamma = \frac{5}{3}$ $\beta = -4/5$

So

$$ct' = \gamma\,(0) - \frac{5}{3}\left(\frac{-4}{5}\right) 2cs = +\frac{8}{3}cs = 2.66\,cs$$

$$x' = -\beta\gamma\,(0) + \frac{5}{3}\cdot 2cs = \frac{10}{3}cs = 3.33\,cs$$

- The B muon is emitted after the A muon

(5)

$$u_A' = \frac{u_A - V}{1 - u_A V/c^2} = \frac{3/5\,c - (-4/5\,c)}{1 - \left(\frac{3}{5}c\right)\left(\frac{-4}{5}c\right)/c^2} = \frac{7/5\,c}{1 + \frac{12}{25}}$$

$$u_A' = \frac{35}{37}c = 0.95c$$

$$u_B' = \frac{u_B - V}{1 - u_B V/c^2}$$

$$u_B' = \frac{-3/5\,c - (-4/5\,c)}{1 - \left(\frac{-3}{5}c\right)\left(\frac{-4}{5}c\right)/c^2} = \frac{+1/5\,c}{1 - 12/25} = \frac{5}{13}c = 0.38\,c$$

(6) $x_B' = x_0' + u_B'(t' - t_0')$

$$x_B' = \frac{10}{3}cs + \frac{5}{13}c\left(t' - \frac{8}{3}s\right)$$

$$= 3.33\,cs + 0.38(ct' - 2.66\,cs$$

$$x'_A = \frac{35}{37} ct' = 0.946\ ct'$$

(7) $x_{A'} = x_{B'}$

$$\frac{35}{37} ct' = \frac{10}{3} cs + \frac{5}{13} ct' - \frac{40}{39} cs \quad \text{or} \quad .945\, ct' = 3.33 + 0.38 ct' - 1.02$$

$$\left(\frac{35}{37} - \frac{5}{13}\right) ct' = \left(\frac{10}{3} - \frac{40}{39}\right) cs$$

$$ct' = \frac{\left(\frac{10}{3} - \frac{40}{39}\right)}{\left(\frac{35}{37} - \frac{5}{13}\right)} cs$$

$$ct' = \frac{37}{9} cs \qquad x' = \frac{35}{37} ct' = \frac{35}{37} c \cdot \frac{37}{9} s = \frac{35}{9} cs = 3.88 cs$$

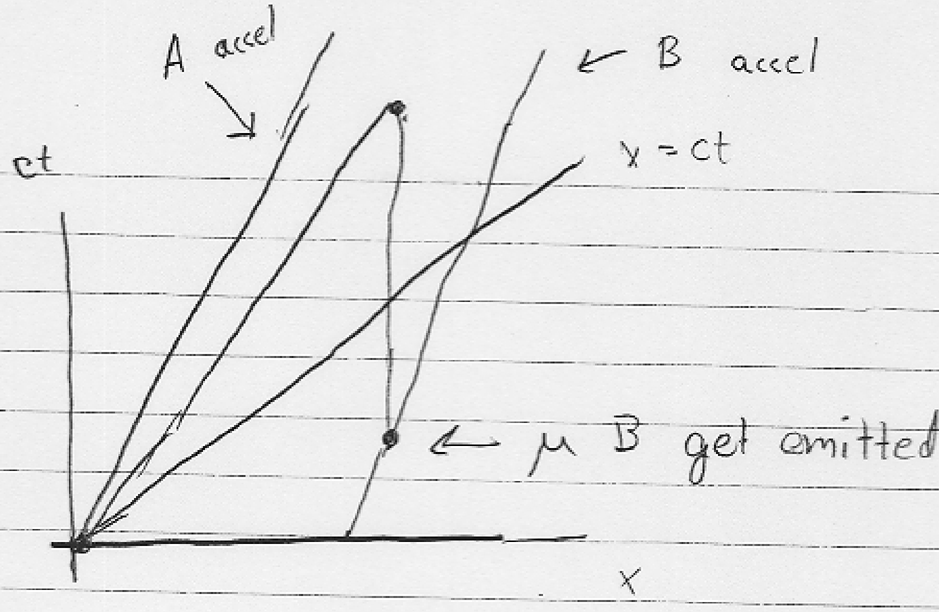
Note the position of the center-point between the two accelerators is

$$x_c = \frac{1}{2}\left(\frac{6 cs}{5}\right) + \frac{4}{5} ct$$

$$x_c = \frac{3}{5} cs + \frac{4}{5} \frac{37}{9} cs = \frac{35}{9} cs$$

I.e. the two muons meet in the center

8



Problem 3

(1)

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{473} = 2.62 \text{ eV}$$

(2)

$$\frac{\Delta N}{\Delta A\, \Delta t} = \frac{\mathcal{P}}{A\, h\nu}$$

$$= \frac{40 \times 10^{-3} \text{ W}}{\pi\, (1\times10^{-3} \text{ m})^2\; 2.62 \text{ eV}}$$

$$= 4.8\times10^{3}\ \frac{\text{W}}{\text{eV m}^2}$$

$$= 0.30 \times 10^{12}\ \frac{\text{photons}}{\text{m}^2\cdot\text{s}}$$

(3)

$$\Delta N = \frac{\mathcal{P}}{A}\,\frac{\Delta A\, \Delta t}{h\nu}$$

$$2 = \left(\frac{\mathcal{P}}{A\cdot h\nu}\right) \overbrace{\Delta A\, \Delta t}^{\text{area and time scales of atom}}$$

# of photons per area per time

$$\mathcal{P} = \frac{2\,h\nu}{\Delta t} \cdot \left(\frac{A}{\Delta A}\right)$$

$$\mathcal{P} = \frac{2 \cdot 2.6\,eV}{(10^{-7} s)} \; \frac{\cancel{\pi} r^2}{\pi r_0^2}$$

$$= \frac{2 \cdot 2.6 \cdot (1.6 \times 10^{-19}\, W)}{10^{-7}\, s} \cdot \frac{(1 \times 10^{-3}\, m)^2}{(1 \times 10^{-10} m)^2}$$

$$\mathcal{P} = 832{,}000 \text{ Watts}$$ ← its much greater than a milliwatt

(4) $K = h\nu - W_0$

$K = 2.62 eV - 2.1 eV = 0.5 eV$

$\frac{1}{2} m v^2 = K$

$$v = \sqrt{\frac{2k}{m}}$$

$$\frac{v}{c} = \sqrt{\frac{2k}{m_e c^2}} = \left(\frac{2 \cdot (0.5 eV)}{511000\, eV}\right)^{1/2}$$

$$\frac{v}{c} = 0.0014$$

$$K = \gamma m_e c^2 - m_e c^2$$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{1}{2!}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left[\left(\frac{-v^2}{c^2}\right)\right]^2$$

$$\gamma = 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{3}{8}\left(\frac{v}{c}\right)^4$$

$$K = \left[1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{3}{8}\left(\frac{v}{c}\right)^4\right] m_e c^2 - m_e c^2$$

$$K_{rel} = \frac{1}{2} m_e v^2 + \frac{3}{8} m_e c^2 \left(\frac{v}{c}\right)^4$$

$$\% err = \frac{K_{rel} - K_{nr}}{K_{nr}} = \frac{\frac{3}{8} m_e c^2 \left(\frac{v}{c}\right)^4}{\frac{1}{2} m_e v^2}$$

$$\% err = \frac{3}{4}\left(\frac{v}{c}\right)^2$$

(c) $\% err = \frac{3}{4}\left(0.0014\right)^2 = 0.147 \times 10^{-5}$

$= 0.147 \times 10^{-3}\ \%$