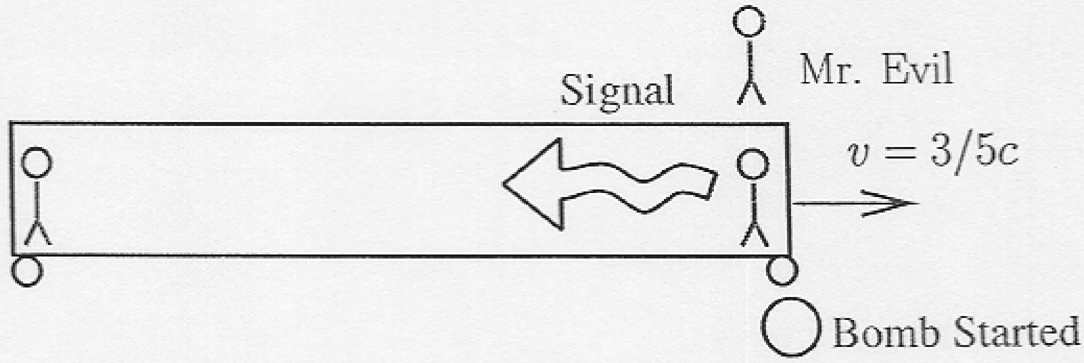
25/25

Consider the following situation. (a) A train of proper length $2cs$ traveling at speed $3/5c$ to the right (relative to earth). (b) When the front of the train reaches the location of Mr. Evil (see below), Mr. Evil starts the internal clock of a stationary time bomb next to the track. (c) The time bomb's internal clock is set to explode after a time $2.5s$ once the clock is started. (d) The driver at the head of the train sends out a warning signal to the passenger riding in the back of the train so he can escape. (e) The warning signal travels at speed $0.75c$ (relative to the driver of the train) and is sent towards the back of the train. The signal is sent the instant the front of the train passes Mr. Evil. Mr. Evil and the train driver synchronize there clocks to $t = t' = 0$ when they cross paths.



4/4 1. When and where relative to the train driver does the signal reach the passenger?

5/5 2. Does the bomb explode before the signal arrive?

6/6 3. What is the speed of the signal according to the Mr. Evil?

5/5 4. When and where relative to Mr. Evil does the signal reach the passenger?

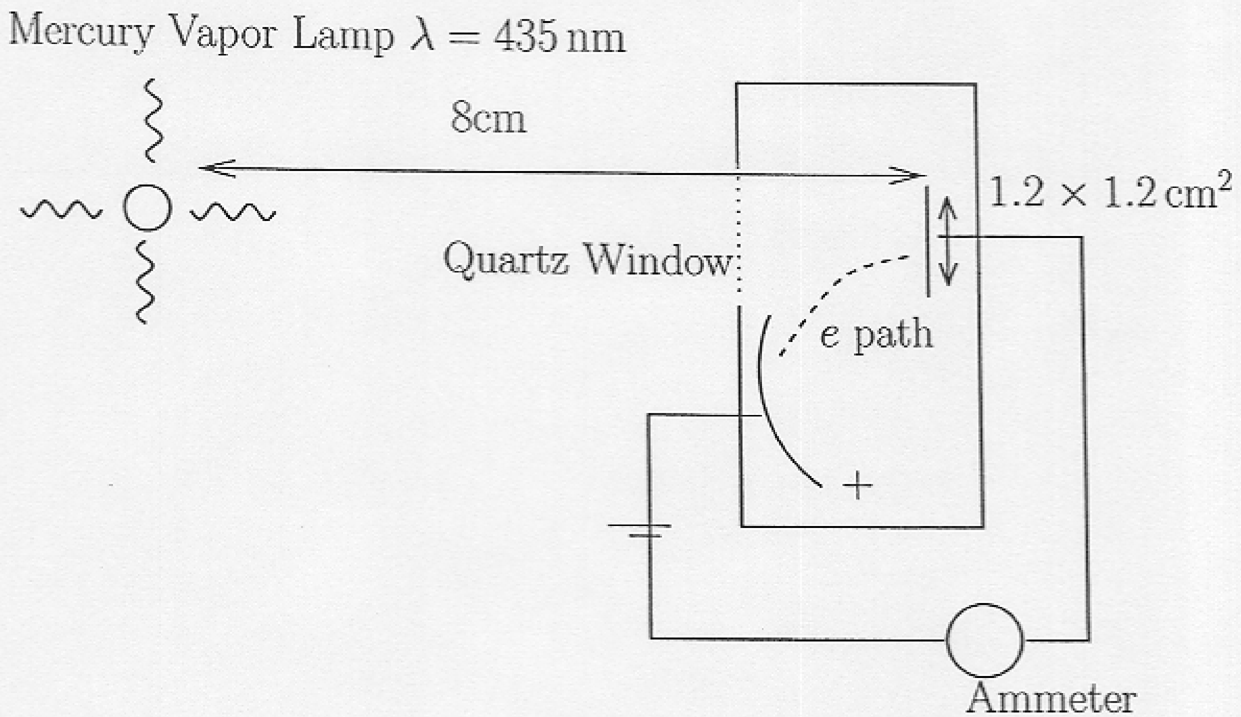
5. Draw a space time diagram according to the earth. Indicate the following on your graph:

5/5 (a) The signal being sent from the front of the train, the signal arriving at the back of the train, the bomb exploding.

(b) The world lines of the front of the train, the back of the train, and the bomb.

20/20

A mercury vapor lamp is positioned a distance of 8 cm from a $1.2 \times 1.2\,\text{cm}^2$ metal foil as shown below. For simplicity, assume that the vapor lamp emits only the characteristic blue light of mercury vapor 435 nm. The lamp emits light with a total power of 1.8 mW uniformly in all directions. A metal foil is placed in the photo electric effect apparatus shown below. The work function of the metal foil is 2.1 eV. The electrons are collected by a plate biased to attract the emitted photo-electrons as shown below.



6/6 1. What is the number of photons which hit the metal foil per second? (The foil only covers a small fraction of the total area of an $R = 8\,\text{cm}$ sphere surrounding the light source.)

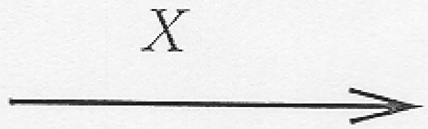
4/4 2. Assuming that every photon which reaches the metal foil produces a photo-electron, what is the measured current in Amps? This is an upper bound of the photo current. (The first page contains a table of constants)

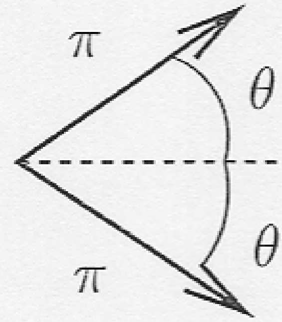
6/6 3. What is $\beta = v/c$ of the emitted electron?

4/4 4. Make an order of magnitude estimate for the size of relativistic corrections to the answer in part (3). Explain your reasoning.

15/15

A mysterious particle X is traveling to the right with energy of 825 MeV and momentum of 300 MeV/c. It decays symmetrically into two pions which each have a mass of $m_\pi c^2 = 140\,\text{MeV}$.





4/4 1. Determine the opening angle (*i.e.* θ) of the two pions.

6/6 2. According to an observer (Observer A) moving to the right with respect to earth, particle X was stationary before it decayed. What is the velocity of this observer?

3. According to Observer A:

3/3 (a) What is the opening angle of the two pions?

2/2 (b) What is the energy of these pions?

Solution - P1

① According to train observer the recieve of signal happens at

$x = -L_p = -2cs$ ← receive happens to the Left of driver

$t = +\frac{L_p}{u_s} = \frac{2cs}{\frac{3}{4}c} = \frac{8}{3}s$ $x_{driver} = 0$

signal speed

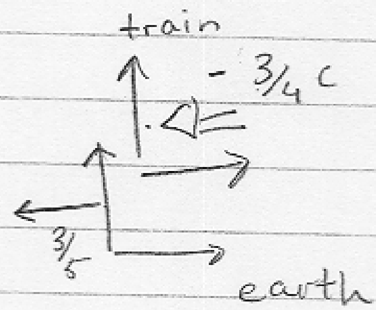
So the receive coordinates are $(ct, x) = \left(\frac{8}{3}cs, -2cs\right)$

② $T_{train} = \gamma T_B = \frac{5}{4} \cdot (2.5s) = 3.125s > \frac{8}{3}cs$

So there is enough time to escape

③ So

$u' = \frac{u - v}{1 - uv/c^2}$



$u' = \frac{-\frac{3}{4}c - \left(-\frac{3}{5}c\right)}{1 - \left(-\frac{3}{5}c\right)\left(-\frac{3}{4}c\right)/c^2} = \frac{-\frac{3}{20}c}{\left(1 - \frac{9}{20}\right)} =$

$u' = -\frac{3}{11}c = -0.272$

(4) According to ms evil:

a) The back of train has the trajectory

$$x_B = x_B^0 + v\,(t - \cancel{t_B^0})$$

where x_B^0 = is the coordinates of Back $= -\frac{L_p}{\gamma}$

at time $t_B^0 = 0$, and $v = \frac{3}{5}c$ is the train velocity

$$x_B(t) = -\frac{L_p}{\gamma} + vt$$

b) the signal has trajectory

$$x_S(t) = -u_S t \qquad \text{where} \quad u_S = \frac{3}{11}c$$

Setting them equal, we find when they meet; at time t_* and place x_*

$$x_B(t) = x_S$$

$$-\frac{L_p}{\gamma} + vt_* = -u_S t_*$$

$$\frac{L_p}{\gamma(v+u_S)} = t_* \qquad x_* = \frac{-u_S L_p}{\gamma(v+u_S)}$$

So we have

$$t_* = \frac{2\,cs}{\frac{5}{4}\left(\frac{3}{5}c + \frac{3}{11}c\right)} \qquad x_* = -0.5\,cs \qquad \text{(EQ A)}$$

$$t_* = 1.83\,s \qquad \text{(EQ B)}$$

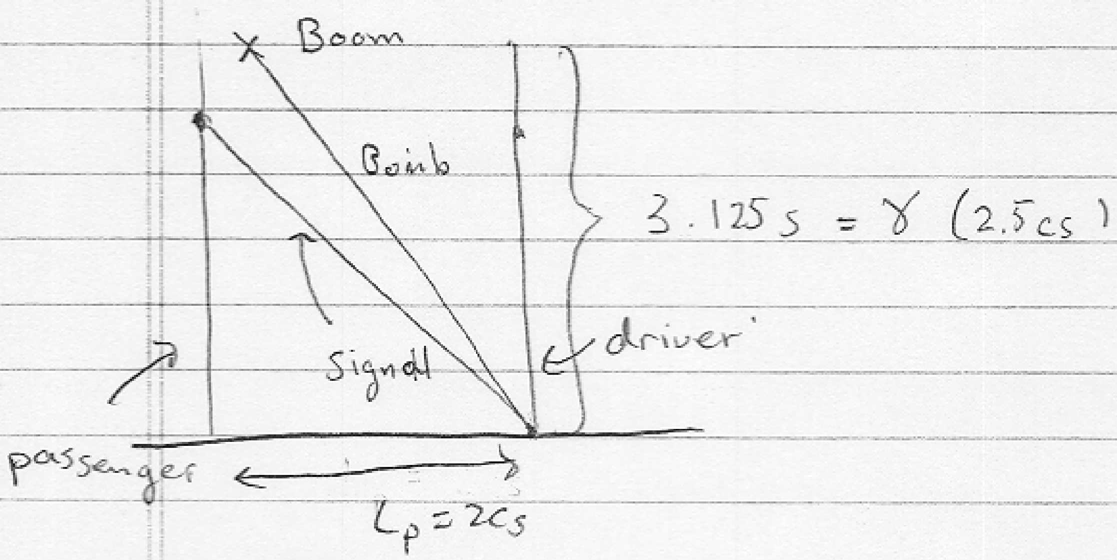
Lets check: we can Lorentz transform the coordinates found in ①:

$$ct' = \gamma\, ct - \gamma\beta\, x = \frac{5}{4}\left(\frac{8}{3}\right) - \frac{5}{4}\left(\frac{-3}{5}\right)(-2) = 1.833c$$

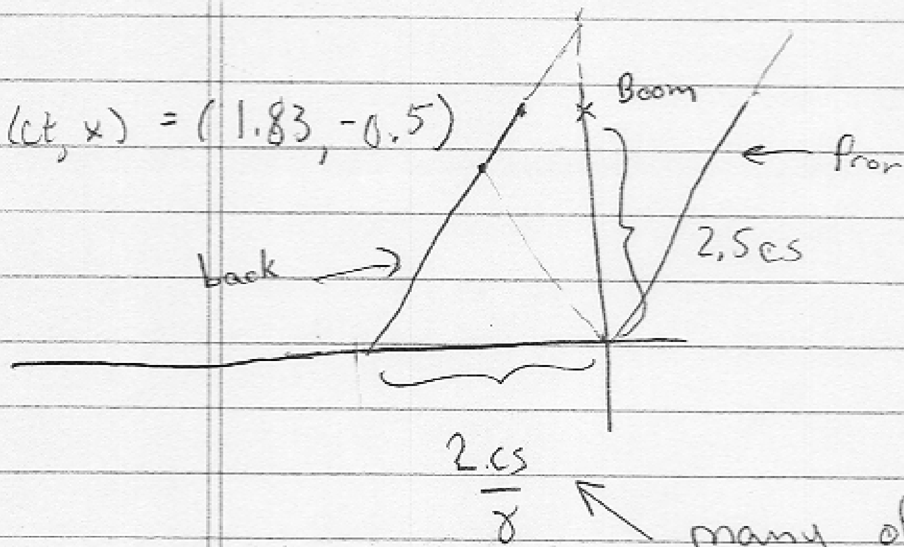
$$x' = -\gamma\beta\, ct + \gamma\, x = -\frac{5}{4}\left(\frac{-3}{5}\right)\left(\frac{8}{3}\right) + \frac{5}{4}(-2cs) = -0.\ cs$$

which agrees with whith EQ A and EQ B.

⑤ Train Picture



The earth picture



many of you got this wrong!

It is worth calculating the answer to part 4 in yet another way (three total)

According to the earth the train has length

$$L = \frac{L_p}{\gamma} = 2\frac{cs}{\gamma}$$

① The back of train moves forward to right with $v = \frac{4}{5}c$ according to earth

② The signal moves backward to left according to earth @ speed $u_s = \frac{3}{11}c$, according to earth

So the earth sees the back and the signal approaching each other at $u_s + v$

So the time they reach each other is

$$t_* = \frac{L}{(u_s+v)} = \frac{L_p}{\gamma(u_s+v)}$$

Which is the same as before.

Solution P2 :

(1) $\Delta N = \dfrac{P}{h\nu} \cdot \dfrac{1\,cm^2}{4\pi(8cm)^2}$

$h\nu = \dfrac{hc}{\lambda}$

$h\nu = \dfrac{1240 eV\ nm}{435 nm}$

$= 2.8 eV$

$\dfrac{\Delta N}{\Delta t} = \dfrac{1.8\times10^{-3} W}{2.8\times 1.6\times10^{-19} J} \times \dfrac{(1.2)^2\ cm^2}{4\pi\ 64\ cm^2}$

$\dfrac{\Delta N}{\Delta t} = 0.719\times10^{13}$ photons /sec

(2) So the current

$I = e\dfrac{\Delta N}{\Delta t} = 1.6\times10^{-19} C \left(0.719\times10^{13}\ \dfrac{1}{s}\right)$

$I = 0.1\times10^{-5}$ Amp

$I \sim 1\ \mu$Amp $\sim$ pretty small

(3) Using the photo effect

$K = h\nu - w$

$K = 2.8eV - 2.1eV \simeq 0.7eV$

So

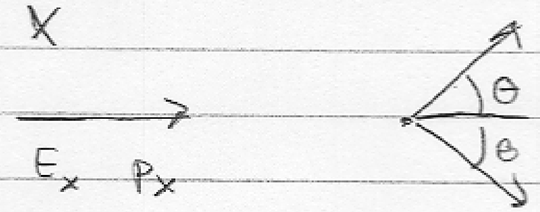
$$K = \frac{1}{2} m v^2 =$$

$$\frac{v}{c} = \sqrt{\frac{2K}{mc^2}} = \sqrt{\frac{2\,(0.7\,eV)}{511{,}000\,eV}} = 1.7 \times 10^{-3}$$

(4) Relativistic corrections are smaller (in percentage terms) by $O\left(\left(\frac{v}{c}\right)^2\right)$. With $\frac{v}{c} \sim 10^{-3}$,

we expect the values computed in part #3 to be corrected by a part in 10^6, or $10^{-4}\,\%$.

P3



① $E_x = 2E_\pi$

$p_x = 2p_\pi \cos\theta$ $\Longrightarrow$ $\dfrac{p_x}{E_x} = \dfrac{p_\pi}{E_\pi}\cos\theta$

Using ; $E_\pi = E_x/2 = \dfrac{825}{2}$ MeV

$cp_\pi = \sqrt{E_\pi^2 - (m_\pi c^2)^2} = 388$ MeV

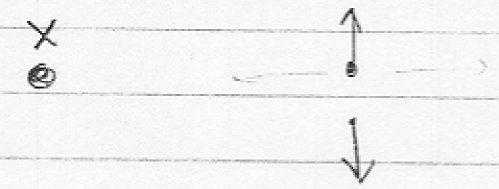
$\cos\theta = \dfrac{E_\pi}{p_\pi}\dfrac{p_x}{E_x} = 0.386$

$\theta \simeq 67^\circ$

② $v_{obs} = V_x = c^2\dfrac{p}{E} = c\left(\dfrac{cp_x}{E_x}\right) = 0.36c$

According to observer A

Before After



Using e-consve

$$E_x = m_x c^2 = 2E_\pi$$

$$\frac{m_x c^2}{2} = E_\pi$$

using

$$(m_x c^2 = \sqrt{E_x^2 - (cp_x)^2} = (825^2 - 300.^2)^{1/2}$$

$$\simeq 768 \text{ meV}$$

So $E_\pi = \frac{m_x c^2}{2} = 384$ MeV

Lets also compute this with Lorentz transformations: (For those who really care!)

Using the transformation rule

$$E' = \gamma E - \gamma\beta\, cp^x$$

$$cp^{x\prime} = -\gamma\beta E + \gamma\, cp^x$$

$$p'^y = p^y$$

E' = energy of π in frame A

E = Energy of π in original frame

p^x = X momentum of π component in original frame

Now from part #2

$$\beta = \frac{cp_X}{E_X} \quad \begin{cases} p_X = 300\ \text{meV} \\ E_X = 825\ \text{meV} \end{cases}$$

Then

$$\gamma\beta = \frac{cp_X}{m_X c^2} \qquad \gamma = \frac{E_X}{m_X c^2}$$

note: $E_\pi = \frac{E_X}{2}$

$p^x_\pi = \frac{p_X}{2}$

from part ① x-momentum of π equal to half total momentum of particle X

So

$$E' = \left(\frac{E_X}{m_X c^2}\right)\frac{E_X}{2} - \left(\frac{cp_X}{m_X c^2}\right)\frac{cp_X}{2}$$

$$E' = \frac{1}{2}\frac{1}{m_X c^2}\underbrace{\left(E_X^2 - (cp_X)^2\right)}_{=(m_X c^2)^2}$$

So

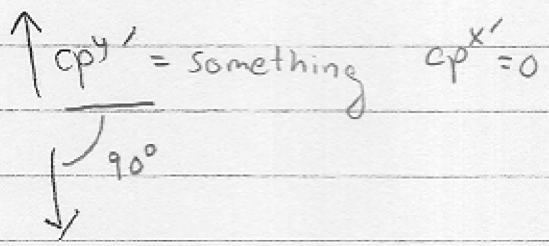
$$E' = \frac{1}{2}\,\frac{(m_{*}c^2)^2}{m_{*}c^2} = \frac{m_{*}c^2}{2}$$

Similarly:

$$cp^{x'} = -\frac{cp_{*}}{m_{*}c^2}\,\frac{E_{*}}{2} + \frac{E_{*}}{m_{*}c^2}\,\frac{cp_{*}}{2}$$

$$cp^{x'} = 0$$

angle is 90°



and

$$cp^{y'} = cp^{y}$$

$$cp^{y'} = cp^{y} = cp\sin\theta$$

$$cp^{y'} = (388\,\mathrm{MeV})\ \sin 67^\circ$$