

Quantity	Symbol	Value
Coulombs Constant	$k_C = 1/4\pi\epsilon_o$	$8.98 \times 10^9 \text{ Nm}^2/\text{C}^2$
Electron Mass	m_e	$9.1 \times 10^{-31} \text{ kg}$
Electron Charge	e	$-1.6 \times 10^{-19} \text{ C}$
Electron Volt	eV	$1.6 \times 10^{-19} \text{ J}$
Permittivity	ϵ_o	$8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}$
Magnetic Permeability	μ_o	$4\pi \times 10^{-7} \text{ N} \cdot \text{A}^2$
Speed of Light	c	$3.0 \times 10^8 \text{ m/s}$
Planck's Constant	h	$6.6 \times 10^{-34} \text{ m}^2\text{kg/s}$

Integrals	Value
$\int_{-\infty}^{\infty} du e^{-\alpha u^2}$	$\sqrt{\frac{\pi}{\alpha}}$
$\int_{-\infty}^{\infty} du u^2 e^{-\alpha u^2}$	$\frac{1}{2\alpha} \sqrt{\frac{\pi}{\alpha}}$
$\int_0^{\infty} du u^n e^{-\alpha u}$	$\frac{n!}{\alpha^{n+1}}$
$\int du \sin^2(\alpha u)$	$\frac{u}{2} - \frac{\sin(2\alpha u)}{4\alpha}$
$\int du \cos^2(\alpha u)$	$\frac{u}{2} + \frac{\sin(2\alpha u)}{4\alpha}$
$\int_{-\frac{1}{2}}^{+\frac{1}{2}} du u^2 \sin^2(n\pi u)$	$\frac{-6+n^2\pi^2}{24n^2\pi^2} \quad n = 2, 4, 6, 8$
$\int_{-\frac{1}{2}}^{+\frac{1}{2}} du u^2 \cos^2(n\pi u)$	$\frac{-6+n^2\pi^2}{24n^2\pi^2} \quad n = 1, 3, 5, 7$
$\int (\cos(\theta))^\alpha \sin(\theta) d\theta$	$\frac{-1}{\alpha+1} (\cos(\theta))^{\alpha+1}$
$\int (\sin(\theta))^\alpha \cos(\theta) d\theta$	$\frac{+1}{\alpha+1} (\sin(\theta))^{\alpha+1}$

Problem 1. You have an electron which moves around a proton forming hydrogen.

1. (Symbol) Starting from Newton's law, the coulomb law, and the Bohr quantization condition for the angular momentum, determine the velocity and radius of the $n - th$ orbit
2. (Symbol) Determine the magnitude of the electric field \mathcal{E} for an electron in the $n - th$ orbit (You can use whatever formulas you brought with you).
3. (Number) For $n = 1$ and $n = 1000$ how large is the electric field numerically in Volts/m. (Hint to save time compute $e\mathcal{E}$ in eV/m and then divide by e using that $1V = 1eV/e$ to get your answer)
 - (a) How does the field compare to a very strong laboratory electric fields of $\mathcal{E} \sim 10^7 V/m$ for $n = 1$ and $n = 1000$
 - (b) Why is the electric field vastly different for $n = 1$ and $n = 1000$

Problem 2.

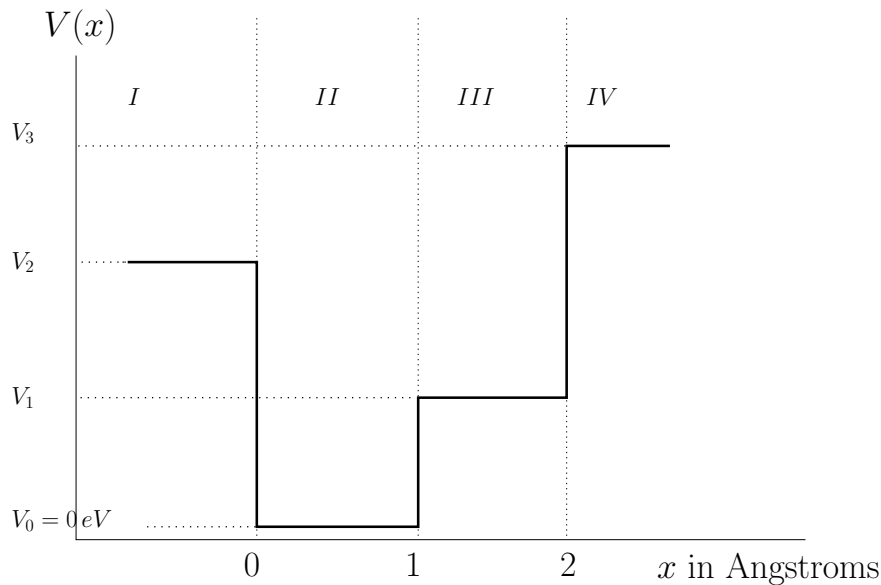
An (unnormalized) electron wave fcn in free space is given by

$$\Psi(x) = e^{ik_2x} - e^{ik_1x}$$

where $k_1 = 6 \text{ \AA}^{-1}$ and $k_2 = 7 \text{ \AA}^{-1}$.

1. (This is just a random question unrelated to the Ψ given above) What is the approximate size an atom? What is the approximate size of a gold nucleus ?
2. Estimate the kinetic energy in eV for the wave function given above.
3. Estimate the uncertainty in the kinetic energy in eV for the wave function given above.
4. Does this electron have enough kinetic energy to (a) knock an electron out of an atom (b) Knock a nucleon out of a nucleus ?
5. Write the the imaginary part of the wave function as a product of sin's and cos' and accurately sketch the result. *Label all axes and give units on the x-axis. Draw all feature of the function to scale. Explain which aspects of your formula control which features of your graph*

Problem 3. Consider an electron in a potential shown below



- For each range of energy $V_0 < E < V_1$, $V_1 < E < V_2$, $V_2 < E < V_3$, and $V_3 < E < \infty$) would you expect continuous values of the energy or discrete allowed values of the energy. Explain very briefly for $V_0 < E < V_1$
- Assuming that the first excited state has energy $V_1 < E < V_2$, sketch the first excited state. Label the regions I,II,III,IV and make the differences between the different regions clear in your the graph.
 - Explain the qualitative differences (exhibited in your graph) between region I and region IV and the differences between region II and region III using the Schrödinger equation.
- In the limit $V_1, V_2, V_3 \rightarrow \infty$ (so that the electron exists only in region II) what is the full time-dependent wave function of the second excited state (Just write it down using the your formula sheet) . Show that the corresponding time independent wave function satisfies the time independent Schrödinger equation.
- Again in the box limit that $V_1, V_2, V_3 \rightarrow \infty$ determine the probability to be between 0.0 and 0.33 \AA for the second excited state. (Hint, graph the probability density)