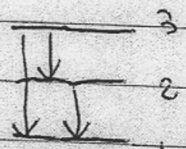
Problem 1



$$\Delta E_{32} = 13.6\,eV\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.88\,eV$$

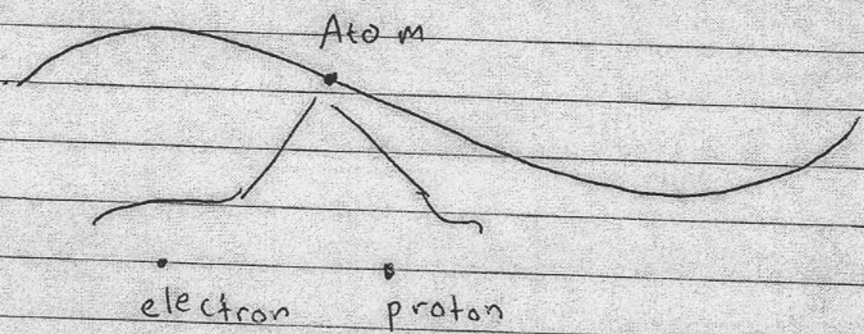
$$\Delta E_{31} = 13.6\,eV\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 12.0\,eV$$

$$\Delta E_{21} = 13.6\,eV\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 10.2\,eV$$

b) $$\lambda = \frac{hc}{E} = \frac{1240\,eV\,nm}{12.0\,eV} = 103\,nm = 1030\,\mathring{A}$$

$$r_{atom} = 0.5\,\mathring{A}$$

$$\lambda_c = \frac{h}{mc} = 0.024\,\mathring{A}$$



(3) $F_c = \frac{mv^2}{r}$

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m^2v^2r^2}{mr^3} \qquad (mvr = n\hbar)$$

$$\alpha \frac{\hbar c}{r^2} = \frac{n^2\hbar^2}{mr^3}$$

$$r = \frac{n^2\hbar^2}{mc\hbar\alpha}$$

$$r = n^2 \frac{\hbar}{mc\alpha}$$

(4) $n\hbar = mvr$

$$\cancel{n}\cancel{\hbar} = \cancel{m}v \frac{n^2\cancel{\hbar}}{\cancel{m}c\alpha}$$

$$\frac{c\alpha}{n} = v$$

(5) $\text{time} = \frac{2\pi r}{v} = \frac{2\pi n^2\hbar/mc\alpha}{c\alpha/n}$

$$\Delta t = n^3 \frac{2\pi \hbar}{mc^2}$$

(6) $r = 0.5 \mathring{A}$

$$v/c = \frac{1}{137}$$

$$t = \frac{2\pi \hbar}{mc^2 \alpha}$$

$$t = \frac{2\pi}{c} \frac{\hbar c}{mc^2 \alpha} = 2\pi \frac{(197\, eV\, nm)}{(511000\, eV \; \frac{1}{137})} \frac{1}{3 \times 10^8\, m/s}$$

$$t = 0.1 \times 10^{-17}\, s$$

Problem 2

(1) $$\frac{1}{2} k a_0^2 = \frac{e^2}{4\pi\varepsilon_0} \frac{1}{a_0}$$

$$k = \frac{e^2}{4\pi\varepsilon_0} \frac{1}{a_0^3} \times 2$$

(2) N has 14 nuckons

$$\frac{m_e}{M_N} = \frac{m_e}{14(m_p)} = \frac{1}{14 \cdot (2000)} = 0.35 \times 10^{-4}$$

(3) To determine the length set

$$\frac{1}{2} k L^2 = \frac{\hbar^2}{2mL^2}$$

$$L^4 = \frac{\hbar^2}{Mk}$$

$$L = \left(\frac{\hbar^2}{Mk} \right)^{1/4}$$

Plugging in

$$L = \left(\frac{\hbar^2}{M \frac{e^2}{4\pi\varepsilon_0 a_0^3}} \right)^{1/4}$$

$$L = \left(\frac{\hbar^{\cancel{2}}}{M \alpha \cancel{\hbar} c \, a_0^{-3}} \right)^{1/4}$$

$$L = \left(\frac{m_e}{M} \frac{\hbar}{m\alpha c} a_0^3 \right)^{1/4}$$

$$L = \left(\frac{m_e}{M} a_0 \, a_0^3 \right)^{1/4}$$

$$L = \left(\frac{m_e}{M} \right)^{1/4} a_0$$

(4) $\frac{L}{a_0} = \left(\frac{m_e}{M} \right)^{1/4} = 0.07$

Problem 3

(1) Hydrogen:

$$\Delta E = 13.6\,eV\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 10.2\,eV = \frac{\hbar^2}{2ma_0^2}\left(\frac{3}{4}\right)$$

Box:

$$\Delta E = \frac{\hbar^2\pi^2}{2mL^2}\left(2^2 - 1^2\right) = \frac{\hbar^2}{2mL^2}\cdot 3$$

So comparing

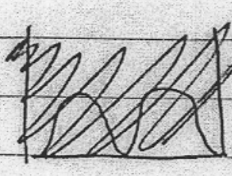
$$\frac{3}{4}\frac{\hbar^2}{2ma_0^2} = \frac{\hbar^2\pi^2}{2mL^2}\cdot 3$$

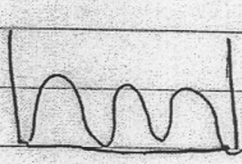
$$L^2 = a_0^2\,\frac{3\pi^2}{3}\,4$$

$$L = 2\pi a_0$$

(2) $$\psi_2(x) = \sqrt{\frac{2}{a}}\cos\left(\frac{3\pi x}{a}\right)$$

$$|\psi_2|^2 = \frac{2}{a}\cos^2\left(\frac{3\pi x}{a}\right)$$





(3) From the Graph

$-\frac{1}{2}$ $-\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{2}$

From the graph we estimate $P \approx \frac{1}{3}$

(4) $\Psi_2 = \sqrt{\frac{2}{a}} \cos\left(\frac{3\pi x}{a}\right) e^{-iEt/\hbar}$

(5) Want to show

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + \cancelto{0}{V(x)}\,\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$$i\hbar \frac{\partial}{\partial t} e^{-iEt/\hbar} = E e^{-iEt/\hbar}$$

So

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right] \Psi(x)\, \cancel{e^{-iEt/\hbar}} = E\, \Psi_n(x)\, \cancel{e^{-iEt/\hbar}}$$

So need to show:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi_n}{\partial x^2} = E\, \Psi_n(x)$$

Then

$$\frac{\partial^2}{\partial x^2} \cos\left(\frac{3\pi x}{a}\right) = -\cos\left(\frac{3\pi x}{a}\right)\left(\frac{3\pi}{a}\right)^2$$

Then

$$-\frac{\hbar^2}{2m}\left(-\cos\left(\frac{3\pi x}{a}\right)\left(\frac{3\pi}{a}\right)^2\right) = E\cos\left(\frac{3\pi x}{a}\right)$$

Is true provided:

$$E = \frac{\hbar^2}{2m}\left(\frac{3\pi}{a}\right)^2$$

(6) $\Psi_2(x,t) = \sqrt{\frac{2}{a}} \cos\left(\frac{3\pi x}{a}\right) e^{-iEt/\hbar}$

$$\Psi_2(x,t) = \sqrt{\frac{2}{a}}\,\frac{1}{2}\left[e^{+i\frac{3\pi x}{a} - iEt/\hbar} + e^{-i\frac{3\pi x}{a} - iEt/\hbar}\right]$$

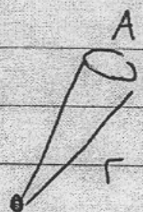
Problem 4

① $\rho = \frac{10^3 \, kg}{m^3} \frac{1}{m_p}$

$\rho = \frac{10^3 \, kg}{m^3} \frac{1}{1.67 \times 10^{-27} \, kg}$

$\rho = 0.6 \times 10^{30} \frac{1}{m^3}$

②



$\Omega_{earth} = \frac{A}{r^2} = \frac{1.1 \times 10^8 \, m^2}{(1.4 \times 10^{11} \, m)^2} = 0.56 \times 10^{-14}$

$A = \pi r_e^2 = 113 \times 10^6 \, m^2 = 1.1 \times 10^8 \, m^2$

$r = ct = (3 \times 10^8 \frac{m}{s})(8 \cdot 60 s) = 1.4 \times 10^{11} \, m$

③ $\cancel{\Delta N_{scatt} = [\Delta t' \mathcal{L}] \frac{d\sigma}{d\Omega} \Delta\Omega}$

~~The integrated Luminosity~~

$\cancel{\Delta t \mathcal{L} = \frac{N_p N_t}{A} = N_p \frac{N_t \Delta x}{A \Delta x} = N \rho \, \Delta x}$

So.

$$\cancel{\Delta N_{scatt} = [N\rho\,\Delta x]\cdot A(1-\cos^2\theta)\,\Delta\Omega}$$

3) To determine the total # scattered backwards we integrate

$$\Delta N_{back} = \sum_{\theta > \pi/2} \Delta N_{scatt}$$

$$= \int_{\pi/2}^{\pi} [N\rho\,\Delta x]\,A\,(1-\cos^2\theta)\;2\pi\sin\theta\,d\theta$$

$$= N\rho\,\Delta x\,A\cdot 2\pi \int_{-\pi/2}^{\pi} (1-\cos^2\theta)\sin\theta\,d\theta$$

$$= \text{---}//\text{---}\left[-\cos\theta + \frac{1}{3}\cos^3\theta\right]_{\pi/2}^{\pi}$$

$$= \text{---}//\text{---}\left[(1-0)+\left(-\frac{1}{3}-0\right)\right]$$

$$\Delta N_{back} = \text{---}//\text{---}\cdot\frac{2}{3}$$

$$\Delta N_{back} = N\rho\,\Delta x\,A\cdot\frac{4\pi}{3}$$

4

$$\frac{\Delta N_{back}}{N} = \rho \, \Delta x \, A \cdot \frac{4\pi}{3}$$

Then:

$$\Delta x = \frac{\Delta N_{back}/N}{\frac{4\pi}{3} \rho A} = \frac{3}{4\pi \rho A} \cdot 1$$

We want $\Delta N_{back}/N = 1$

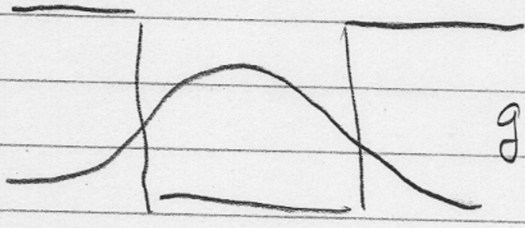
$$\Delta x = \frac{3}{4\pi} \frac{1}{(0.6 \times 10^{30} \frac{1}{m^3})(8.0 \times 10^{-30} m^2)}$$

$$\Delta x = 0.05 \, m = 5 \, cm$$

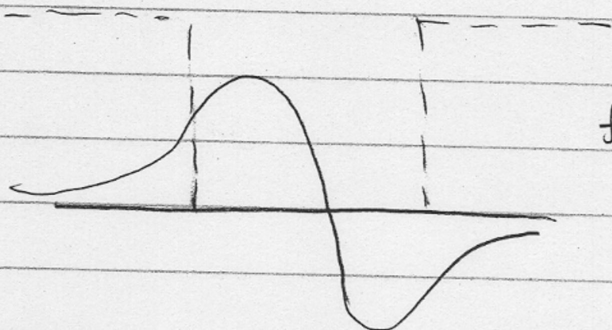
Problem 5

(1) Second Excited

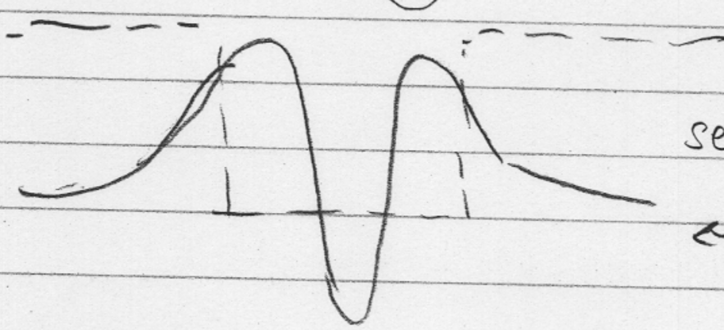
(2)



ground E_1



first excited E_2



second excited E_3

← same as drawn figure

(3) $\lambda \sim 1\,\text{Å}$ in middle region:

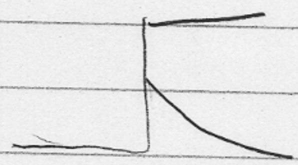
$$K = E - \cancel{V} \simeq \frac{\hbar^2 k^2}{2m} \sim \frac{h^2}{2m\lambda^2} = \frac{(hc)^2}{2(mc^2)\lambda^2}$$

$$p = \hbar k = \frac{h}{\lambda}$$

$$E = \frac{(12400\,eV\,\text{Å})^2}{2(0.5\times10^6\,eV)(1\,\text{Å})^2}$$

$$E \approx \sim 150\,eV$$

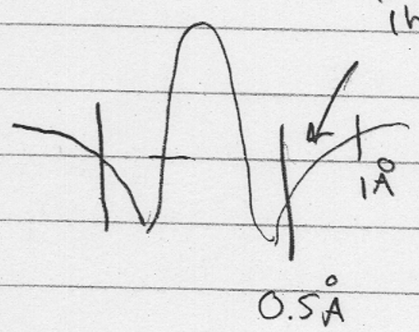
(4) In the classically forbidden region



$$\psi \propto e^{-Kx}$$

$$K = \sqrt{\frac{2m(V-E)}{\hbar^2}}$$

Now from the figure (b) we see that in this reigion ψ decreases by a factor of order:



$$\sim \frac{0.13}{0.5} \sim \frac{1}{4}$$

amount wave decreases from 0.5 Å to 1 Å

So $\frac{1}{4} \sim e^{-K(0.5\,\text{Å})}$

$$K = \frac{1}{0.5\,\text{Å}} \ln 4 \sim 0.7\,\frac{1}{\text{Å}}$$

So

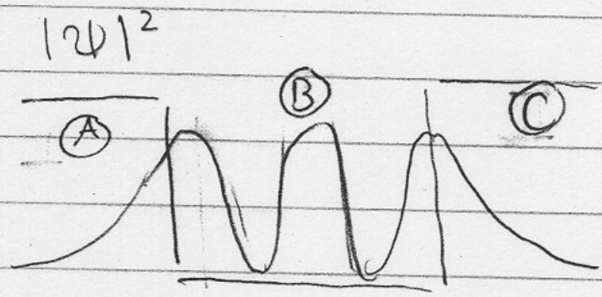
$$K^2 = \frac{2m(V-E)}{\hbar^2} \implies V = E + \frac{\hbar^2K^2}{2m}$$

$$\frac{\hbar^2 \kappa^2}{2m} \simeq 75\,eV$$

So

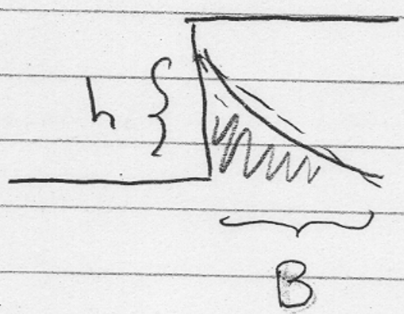
$$V_0 \cong 225\,eV$$

(5) $|\psi|^2$



$I_{(A)} = I_{(c)}$ = area under curve

Approximate curve as triangle:



$$A = \frac{1}{2} h B$$

For B take the penetration depth

$$B = \frac{1}{2K} = 0.7\,\text{Å}$$

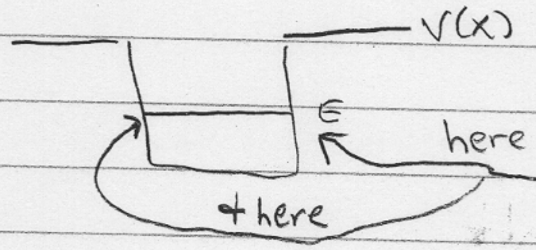
$$h = |\psi|^2 \Big|_{x = 0.5\,\text{Å}} \simeq 1^2 = 1$$

So $I_c \simeq \frac{1}{2} B h \simeq 0.35$ about 1/3

(6)

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}(E-V)\psi$$

There are inflection points (i.e $\psi''(x)=0$) when $E=V$ and $\psi=$



The $E-V$ inflection points occure at $x = \pm 0.5$ Å

The $\psi=0$ inflection points occur at $x \simeq \pm 0.25$ Å