Problem 1

a)

$$F_c = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r}$$

$$L = mvr = n\hbar$$

So,

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{(mvr)^2}{mr^3}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{(n\hbar)^2}{mr^3}$$

and

$$r = n^2\frac{\hbar^2}{m(e^2/4\pi\varepsilon_0)}$$

$$v = \frac{n\hbar}{m r_n} = \frac{\cancel{n}\hbar}{\cancel{m}\,\frac{\hbar^2\, n^{\cancel{2}}}{\cancel{m}e^2/4\pi\varepsilon_0}}$$

$$v_n = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_0\hbar}$$

b) $E_n = \frac{e}{4\pi\varepsilon_0}\frac{1}{r^2}$

$E_n = \frac{e}{4\pi\varepsilon_0 a_0^2 n^4}$ $\quad r_n = n^2 a_0$

c) $E_n = \frac{1}{n^4 e}\left(\frac{e^2}{4\pi\varepsilon_0 a_0}\right)\frac{1}{a_0}$

$E_n = \frac{1}{n^4 e}\left(\frac{27.2\,eV}{0.5\times 10^{-10}\,m}\right)$

$E_n = \frac{1}{n^4}\, 54\times 10^{10}\,\frac{V}{m}$

→ So for $n=1$

$E_1 = 54\times 10^{10}\,\frac{V}{m}$ $\gg$ then lab fields $\sim 10^7$ V/m

$E_{1000} = 0.54\,\frac{V}{m}$ $\ll$ strong lab fieds $\sim 10^7$ V/m

This are very different because $r_{1000} = 10^6 r_1$ and $E \propto \frac{1}{r^2}$ → i.e the radius is much larger

Prob2

(1) $r_{atom} \sim 0.5\ \text{Å}$

$r_{nucleus} \sim 5\ fm \sim 5 \times 10^{-15}\ m$

(2) $KE = \frac{(\hbar k)^2}{2m} = \frac{(\hbar c)^2}{2mc^2} k^2$

$= \frac{(197\ eV\ nm)^2}{2\ (511000 eV)}\ 6.5^2\ \frac{1}{(\frac{1}{10}\ nm)^2}$

$KE \simeq 160 eV$

(3) $dKE = 2 \frac{(\hbar k)}{2m} \hbar\, dk$

$dKE = 2 \frac{(\hbar k)^2}{2m} \frac{dk}{k}$ $dk \sim 0.5$

$k \sim 6$

$dKE \simeq 27 eV$

(4) Now since the binding E is 13.6eV we have suffient energy to rip an electron from an atom

5) $\psi(x) = e^{ik_2x} - e^{+ik_1x}$

let $\bar{k} = \frac{k_1 + k_2}{2}$ $\quad \Delta k = k_2 - \bar{k}$

So

$$\psi(x) = e^{i\bar{k} + \Delta k x} - e^{+i(\bar{k} - \Delta k)x}$$

$$= e^{i\bar{k}x}\left(e^{i\Delta k x} - e^{-i\Delta k x}\right)$$

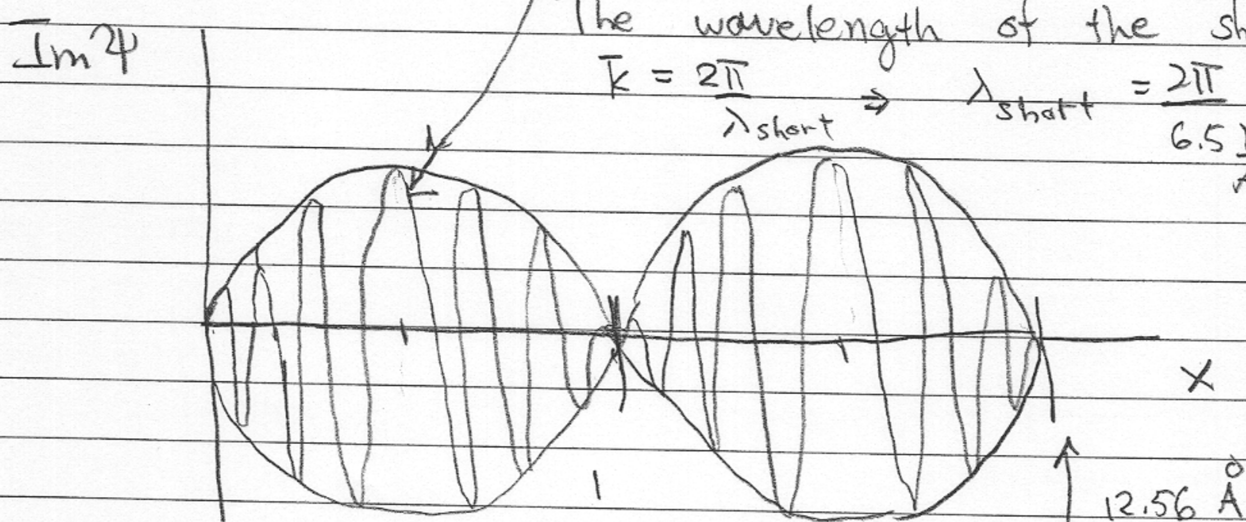
$$\psi(x) = e^{i\bar{k}x}\, 2i \sin(\Delta k x)$$

$$\mathrm{Im}\,\psi(x) = 2\cos(\bar{k}x)\sin(\Delta k x)$$

Then

The wavelength of the short oscillati

$\bar{k} = \frac{2\pi}{\lambda_{short}} \Rightarrow \lambda_{short} = \frac{2\pi}{6.5\,\frac{1}{\mathring{A}}} \simeq 1.$



$$\Delta k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{\Delta k} = \frac{2\pi}{0.5\,\frac{1}{\mathring{A}}} \simeq 12.56$$

Problem 3

① $V_0 < E < V_1 \rightarrow$ discrete

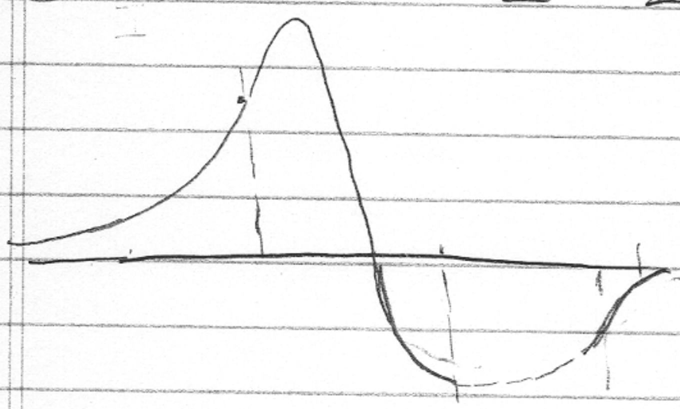
$V_1 < E < V_2 \rightarrow$ discrete

4/4 $V_2 < E < V_3 \rightarrow$ continuous

$V_3 < E < \infty \rightarrow$ continuous

For $V_0 < E < V_0$ the particle bounces back and forth setting up a standing wave

② I II III IV



4/4

a) In the SEQ

2/2 $$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(V-E)\psi$$

Since $V-E$ is larger in region $\overline{IV}$ than I

ψ decays more rapidly

b) Since

2/2 $$\frac{d^2\psi}{dx^2} = \frac{-2m(E-V)\psi}{\hbar^2}$$

a since $E-V$ is larger in II than in III the wavelength is shorter in II.

c) When $V_1, V_2, V_3 \to \infty$

1/4 $$\psi_3 = \sqrt{\frac{2}{a}} \cos\left(\frac{3\pi x}{a}\right) e^{-iE_3 t/\hbar}$$

where $E_3 = \frac{\hbar^2 \pi^2}{2ma^2} 3^2$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\right]\psi = i\hbar \frac{\partial \psi}{\partial t}$$

note $i\hbar\frac{\partial\psi}{\partial t} = i\hbar \frac{\partial}{\partial t} \psi_n(x) e^{-iE_n t/\hbar} = E_n \psi_n(x) e^{-iE_n t/\hbar}$

So further $V=0$

So differentiating

$$\frac{d^2\psi}{dx^2} = \frac{d^2}{dx^2}\psi_n(x) \qquad \psi_n$$

$$\frac{d\psi}{dx} = \sqrt{\frac{2}{a}}\,\frac{d}{dx}\cos\left(\frac{3\pi x}{a}\right)$$

$$\frac{d\psi}{dx} = \sqrt{\frac{2}{a}}\,-\sin\left(\frac{3\pi x}{a}\right)\left(\frac{3\pi}{a}\right)$$

$$\psi'' = \sqrt{\frac{2}{a}}\,(-)\cos\left(\frac{3\pi x}{a}\right)\left(\frac{3\pi}{a}\right)^2 = -\left(\frac{3\pi}{a}\right)^2\psi$$

So

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = i\hbar\frac{\partial\psi}{\partial t}$$

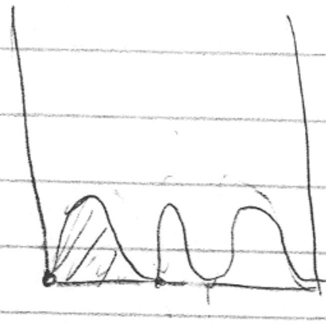
$$\cancel{e^{-iE_nt/\hbar}}\,\frac{\hbar^2}{2m}\frac{\partial^2\psi_n}{\partial x^2} = E_n\psi_n(x)\,\cancel{e^{-iE_nt/\hbar}}$$

So

$$3/4 \quad +\frac{\hbar^2}{2m}+\left(\frac{3\pi}{a}\right)^2\psi = E_n\psi$$

Which is true provided $E_n = \frac{\hbar^2\pi^2}{2ma^2}\,3^2$

(4)



$$\text{Prob} = \text{Area} = \frac{1}{3}$$