Problems: Due Thurs, 9/15/2011

1. (medium. **Classical Relativity and Space Time Diagrams**.) According to an observer on earth a ticking time bomb at $x = 0$ starts ticking at time $t = 0$. Five seconds later the bomb explodes. splitting into two pieces which move at a speed of 30m/s in opposite directions . Another observer (observer B) on an airplane moves at $100m/s$ in the positive x direction relative to earth. Still another observer (observer C) moves at $20m/s$ in the negative x direction relative to earth.

   (a) Draw a cartoon of the process as seen by someone on earth and observers B and C.

   (b) Determine the velocity of the bomb, and the fragments as measured by observer B

   (c) Determine the velocity of the bomb, and the fragments as measured by observer C

   (d) There are two events. The start of the clock and the exploding of the bomb. Determine the space time coordinates of these two events as measured by B

   (e) Determine the space time coordinates of these two events as measured by C.

   (f) There are three objects moving with there own world-lines. The bomb, the right-moving fragment, and the left moving fragment. Determine the equation of motion of these three objects according to earth

   (g) Determine the equation of motion of these three objects according to B.

   (h) Determine the equation of motion of these three objects according to C.

   (i) Draw a space time graph for the earth observer

   (j) Draw a space time graph for this process according to B

   (k) Draw a space time graph for this process according to C

2. (easy. **Basic Relativity and Taylor Series**) An atomic clock is placed in a jet airplane. The clock measures a time interval of 1000 s when the jet moves with a speed of 300 m/s.

   - Derive an approximate expression (i.e. taylor series) for γ for $v/c \ll 1$.
   - Without using a calculator, determine how much longer or shorter a time interval does an identical clock held by an observer on the ground measure.

3. (medium. **Basic Relativity and Taylor Series. Definitely graded**) The velocity of a proton relative to our galaxy is $v_p/c = 1 - (0.5 \times 10^{-20})$, i.e. almost one. Such protons are actually observed.

   (a) A light year is the distance light travels in a year. Express this in units of meters. Our galaxy is approximately 10^5 light years across. How many years does it take for the proton to cross our galaxy.

   (b) For an observer sitting on the proton how long does it take for the galaxy to "pass him by". Give your answer in seconds. You probably will have a difficult time substituting into the formula for γ and should therefore do parts (c) and (d) first

   (c) Show that $(v/c)^2 = 1 - 1/\gamma^2$

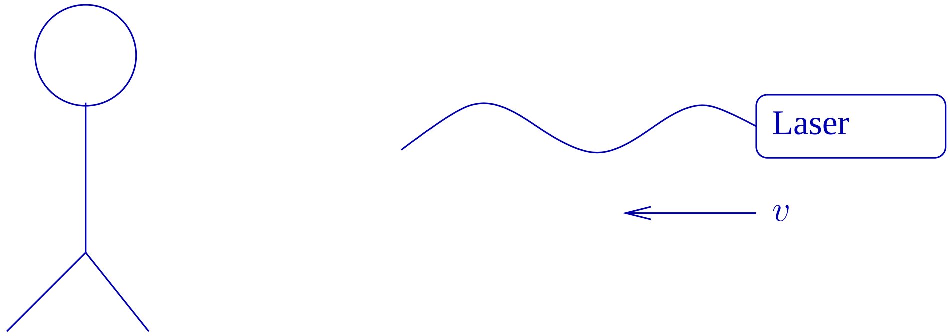
   (d) When velocity it very nearly one γ is very large. $1/\gamma$ is very small. Use Taylor series to show that for v almost one we have

   $$v/c \approx 1 - \frac{1}{2}\frac{1}{\gamma^2} + \ldots$$

   (e) How large is the galaxy in according to the proton? Express your answer in kilometers. Roughly how many kilometers are between New York and California.

4. (easy. **This problem will definitely be graded.**) Spaceship B overtakes spaceship A at relative speed of $0.2c$. Observers in A measure the length of B to be 150 m. (a) What is the proper length of B? How long does it take B to pass a given point on A as measured by observers in (b) in A, and (c) in B.

5. (medium **Doppler shift.**) A laser at rest beam emits light with frequency, $f_o = \frac{c}{\lambda_o}$. However, an observer who is approaching the light source head on sees a higher frequency and someone who is moving away from the source sees a lower frequency. In this problem consider the observer fixed and the laser moving towards the observer with speed v (see below).



(a) At time $t = 0$ a wave train (i.e. the start of a sinusoidal wave) is emitted from the laser as shown below. At what time according to someone on the laser on the does the sinusoidal wave begin to repeat its self, i.e. the next sinusoidal wave train is emitted.

(b) At what time according to someone one standing on the ground does the laser begin to repeat itself.

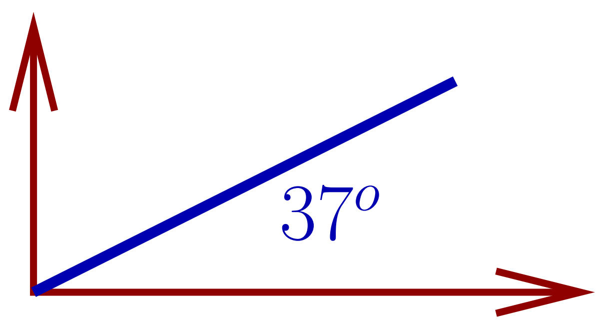
(c) Relative to the ground observer, how far has the laser moved in this time-till-repeat period, and how far has the first wave train moved in this time period. Draw a sketch.

(d) Determine the wavelength of the light as measured by someone on the ground.

(e) Determine the frequency of the light as measured by someone on the ground. Express your answer in terms of the rest frequency f_o. You should find

$$f = f_o \sqrt{\frac{1 + v/c}{1 - v/c}} \tag{1}$$

6. (A little tricky – requires algebra. **Basic Relativity**) A rod at rest makes a definite angle in the x, y plane of 37^o. (Hint a 37^o corresponds to a 3-4-5 triangle.) An observer O' moving rapidly along the x axis, measures the angle to be 45^o rather than 37^o. What was the speed of the observer relative to the rod?



Basic Classical Relativity

1. An observer measures coordinates of events

$$t, x, y, z$$

Another observer moving with velocity v in the x direction sees a different set of coordinates. These coordinates are related the first set by a Gallilean transformation

$$\begin{aligned} t' &= t \quad (2) \\ x' &= x - vt \quad (3) \\ y' &= y \quad (4) \\ z' &= z \quad (5) \\ & \quad (6) \end{aligned}$$

2. The first observer measures the velocity of and object to be (u_x, u_y, u_z) where u_x is the x component of the velocity, etc. Another observer moving with velocity v in the x direction relative to the first observer measures a different velocity (u'_x, u'_y, u'_z) which is related to (u_x, u_y, u_z) by the transformation

$$\begin{aligned} u'_x &= u_x - v \quad (7) \\ u'_y &= u_y \quad (8) \\ u'_z &= u_z \quad (9) \end{aligned}$$

3. The two observers measure the same forces and the same accelerations.

4. For an object moving with constant velocity the equation of motion according to one observer (on earth say) is

$$\Delta x = u\Delta t \qquad \text{or} \qquad x = x_o + u(t - t_o) \quad (10)$$

where x_o is the position at time t_o. For an observer moving with velocity v in the x-direction relative to the first observer (i.e. the jupiterite) the equation of motion is the same

$$x' = x'_o + u'(t' - t'_o) \quad (11)$$

Here, for example $x'_o = x_o - vt$ is the relation between x'_o (the starting position as measured by Jupiter) and x_o (the starting position measured by earth

Basic Special Relativity

1. The speed of light is constant in all reference frames

2. For an observer moving with velocity v relative to a "lab" we use two symbols alot

$$\gamma \equiv \frac{1}{\sqrt{1 - (v/c)^2}} \qquad \beta \equiv \frac{v}{c}$$

3. Moving clocks are time dilated. A time interval $\Delta\tau$ in the rest frame of the clock is measured to be

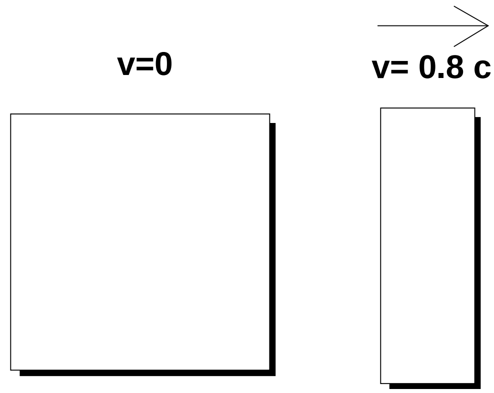
$$\Delta t = \gamma\, \Delta\tau \quad (12)$$

according to an observer moving relative to the clock.

4. An observer moving relative to a ruler stick with rest length L_p will see it length contracted by a factor γ.

$$L = L_p/\gamma \quad (13)$$

The transverse directions are not affected by the motion, i.e. a square becomes a rectangle according to an observer moving quickly with respect to the square.



5. A source emits light waves with frequency f_o. According to an observer moving directly toward the source with speed v the source has a frequency which is blue shifted

$$f = f_o \sqrt{\frac{1 + v/c}{1 - v/c}} \tag{14}$$

For an observer moving away from the source make the replacement $v \to -v$.

Mathematical Relations

1. The Taylor series for any function around a point x_o is

$$f(x) = f(x_o) + f'(x_o)\,\Delta x + \frac{1}{2!} f''(x_o)\,\Delta x^2 + \ldots \tag{15}$$

with $\Delta x = (x - x_o)$.

2. The following Taylor series comes up frequently in relativity

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \ldots \tag{16}$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots \tag{17}$$

3. In relativity the following Taylor series comes up a lot for small v (with respect to c)

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \approx 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 \tag{18}$$