Problems:

$$5.4, 5.6\,\text{from relativity book} \tag{1}$$

1. The wording of problem 5.4 is confusing. The are several things that move along world lines.

   - A is the space-time trajectory or world line of the front of the barn.
   - B is the world line of the the back of the barn.
   - Q is the world line of the front of the pole.
   - P is the world line of the back of the pole.

   Your space time graphs should show these world lines, as observed by both observers. There are several events: you

   - E_1 the front of the pole reaches the front of the barn, i.e. Q coincides with A
   - E_2 the front of the pole reaches the back of the barn, i.e. Q coincides with B
   - E_3 the back of the pole reaches the back of the barn.

   Yours space time graph should show these events.

2. (medium) Two events occur at the same time according to an observer A and are separated by a distance of 1 km along the x axis. What is the time difference between these two events according to an observer B moving with constant velocity along the x axis, given that this B observer measures a spatial separation of 2 km. (Ans: $\beta = \sqrt{3}/2$, $\Delta t = -5.77\mu s$)

3. (hard, important) A rocketship of length 100m, traveling at $v/c = 0.6c$ carries a radio receiver at its nose. A radio pulse is emitted from a stationary space station just as the tail of th rocket passes by.

   (a) How far from the space station is the nose of the rocket at the instant of arrival of the radio signal at the nose?

   (b) By the space-station time, what is the time interval between the arrival of this signal and its emission from the station?

   (c) What is the time interval according to measurements in the rest frame of the rocket?

   (Ans: (a) 80m, (b) 0.66 μs (c) 0.33μs)

4. Consider three galaxies A, B and C. An observer in A measures the velocities of C and B and finds they are moving in opposite directions each with a speed of $0.7c$ relative to him. Thus, according measurements in his frame, the distance between them is increasing at the rate $1.4c$. What is the speed of A observed in B? What is the speed of C observed in B? (Ans: (a) 0.7c (b) 0.94c)

5. (Easy) Use the Lorentz transformations to show that the "invariant space-time interval"

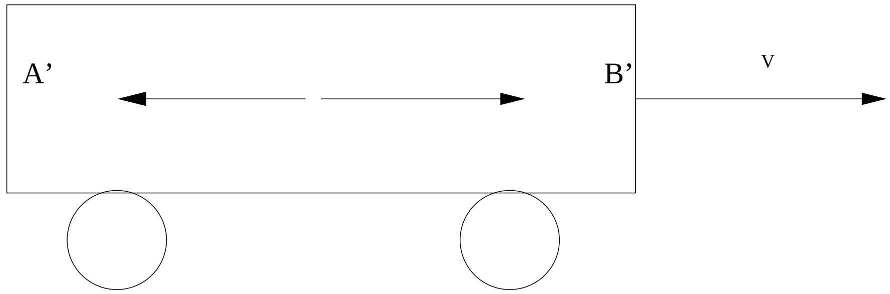
   $$(\Delta s)^2 \equiv (c\Delta t)^2 - (\Delta x)^2$$

   is invariant, i.e. show that

   $$(c\Delta t')^2 - (\Delta x')^2 = (c\Delta t)^2 - (\Delta x)^2$$

   Do not worry about the y and z directions.

6. (medium, **Graded**) A train (frame S') of proper length 3.2 km moves at 0.6c relative to the platform (frame S). At $t = t' = 0$ two light pulses are emitted in opposite directions from the center as shown below. At what times do the pulses reach the ends of the train A' and B' (a) in frame S' and (b) in frame S. (Ans: (a) 5.6 μs and 5.6 μs (b) 2.6 μs and 10.6 μs)



7. (medium, **Graded**) Relative to earth, spaceship A moves at 0.6c and is chasing B whose speed is 0.8c. Spaceship A fires a missile at $0.3c$ relative to itself. (a) Does the missile hit B (b) If the answer to (a) is negative, what is the minimum required speed of the missile relative to A to hit B. (Ans: (a) No (b) 0.38c)

Basic Special Relativity

1. The speed of light is constant in all reference frames
2. For an observer moving with velocity v relative to a "lab" we use two symbols a lot

$$\gamma \equiv \frac{1}{\sqrt{1-(v/c)^2}} \qquad \beta \equiv \frac{v}{c}$$

3. Moving clocks run slow. A time interval $\Delta\tau$ in the rest frame of the clock is measured to be

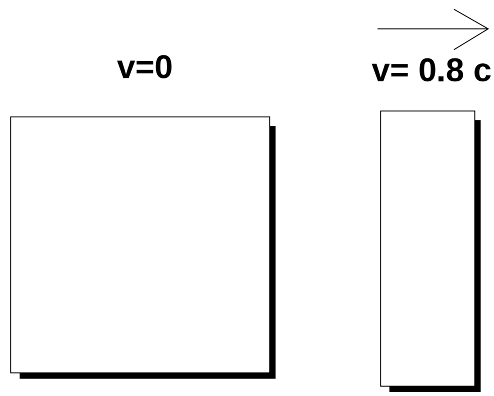
$$\Delta t = \gamma\, \Delta\tau \tag{2}$$

   according to an observer moving relative to the clock.

4. An observer moving relative to a ruler stick with rest length L_p will see it length contracted by a factor γ.

$$L = L_p/\gamma \tag{3}$$

   The transverse directions are not affected by the motion, i.e. a square becomes a rectangle according to an observer moving quickly with respect to the square.



Lorentz Transformations

1. An event at space time point (t, x, y, z) will appear at a different space time point (t', x', y', z') according to an observer moving with velocity v in the positive x direction.

$$\begin{aligned} ct' &= \gamma\,(ct) - \gamma\beta\, x \\ x' &= -\gamma\beta\,(c\,t) + \gamma\, x \\ y' &= y \\ z' &= z \end{aligned} \tag{4}$$

   For an observer moving in the negative x direction the same formula holds with the replacement $\beta \to -\beta$

2. This should be compared to *Classical Galilean* transformations where the space time point (t, x, y, z) appears at a different space point but the same time point (t', x', y', z')

$$\begin{aligned} t' &= t \\ x' &= x - vt \\ y' &= y \\ z' &= z \end{aligned}$$

3. For a given observer in a fixed frame the normal rules of classical physics apply.
4. The Lorentz transformation leaves the following unchanged

$$(\Delta x')^2 - (c\Delta t')^2 = (\Delta x)^2 - (c\Delta t)^2 \tag{5}$$

Other results

1. A space ship moves with velocity $\mathbf{u} = (u_x, u_y, u_z)$ in the "lab" frame. According to an observer moving with speed v in the positive x direction relative to the lab, the spaceship moves with a different velocity $\mathbf{u}'$. $\mathbf{u}'$ is related to $\mathbf{u}$ by

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} \tag{6}$$

$$u'_y = \frac{u_y}{\gamma(1 - u_x v/c^2)} \tag{7}$$

$$u'_z = \frac{u_z}{\gamma(1 - u_y v/c^2)} \tag{8}$$

If the observer is moving to the left then the same formula applies with the substitution $v \to -v$. If the space-ship is moving to the left then u_x is negative in this formula.

2. A source emits light waves with frequency f_o. According to an observer moving directly toward the source with speed v the source has a frequency which is blue shifted (i.e. is higher)

$$f = f_o \sqrt{\frac{1 + v/c}{1 - v/c}} \tag{9}$$

For an observer moving away from the source make the replacement $v \to -v$.

Dynamics

1. The rest energy is the energy of a particle when it is not moving

$$E_o = mc^2 \tag{10}$$