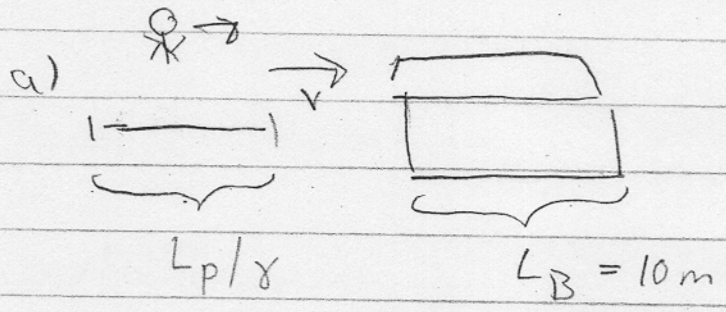
5.4

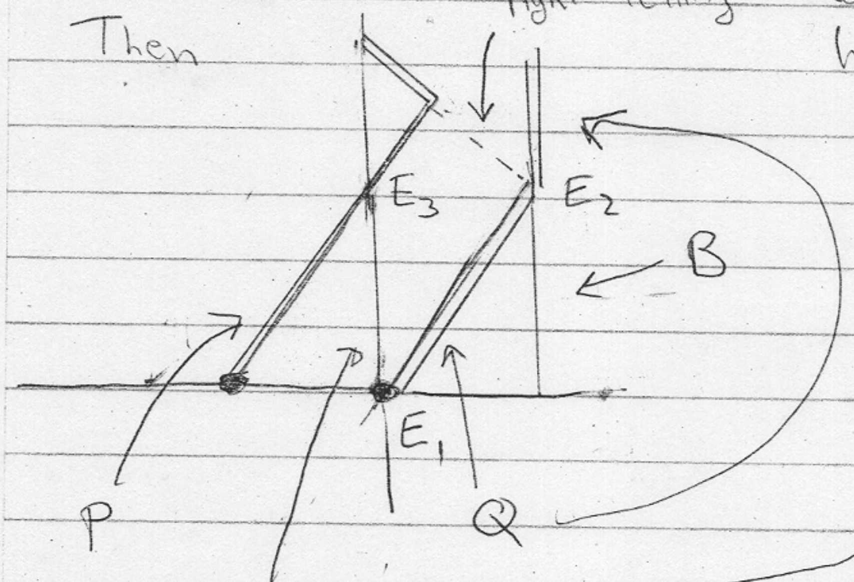
Barn Picture - <u>Discussed in Recitation</u>

a)

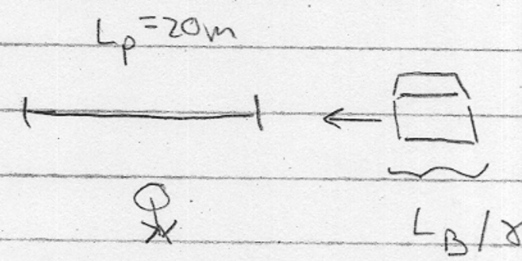


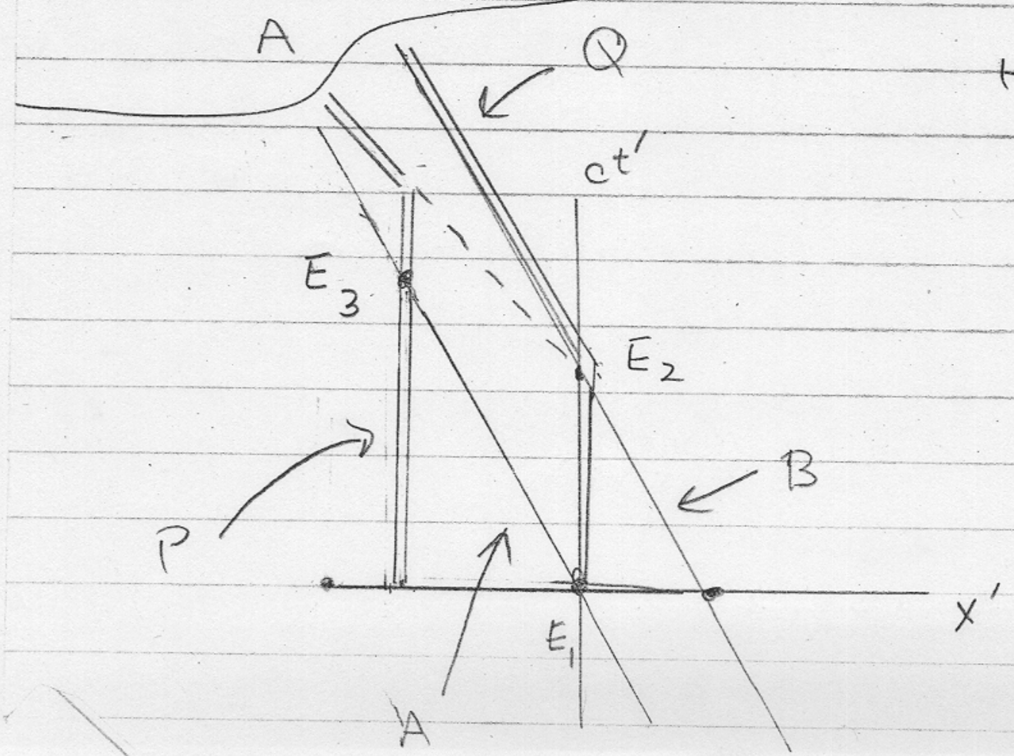
Then

light telling back of pole that the front has hit the back



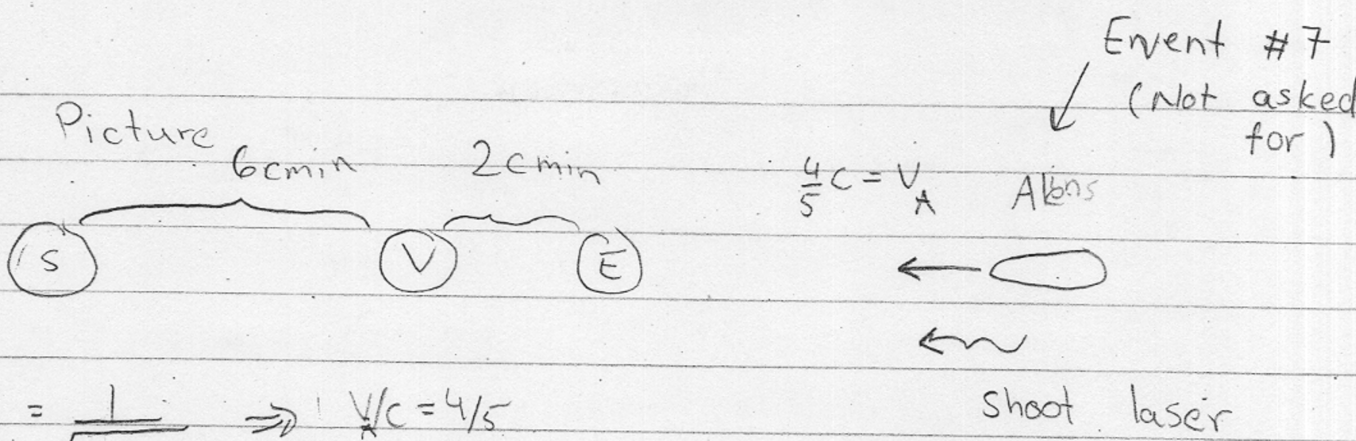
Barn Picture



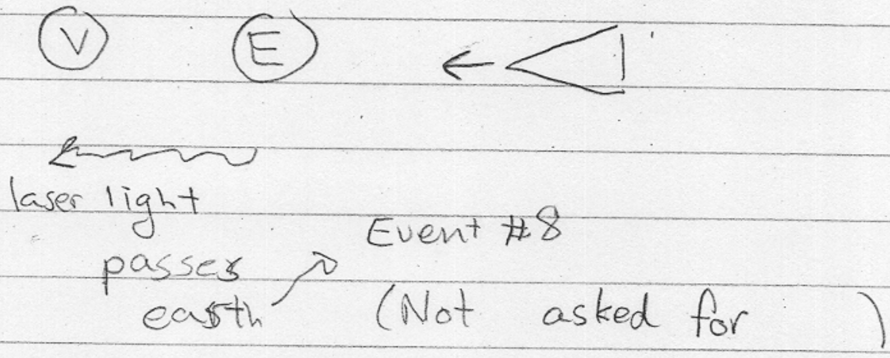


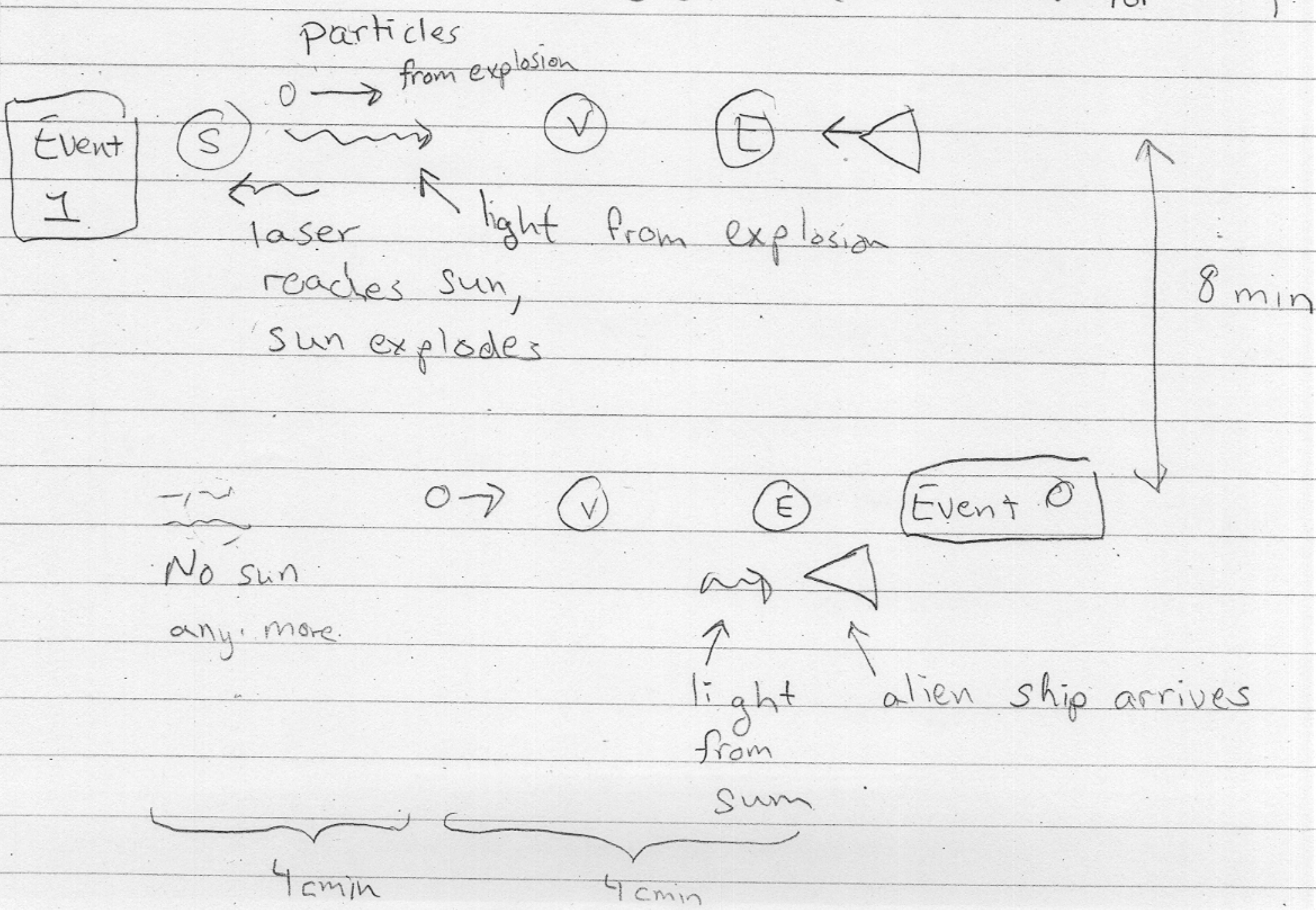
Runner Picture

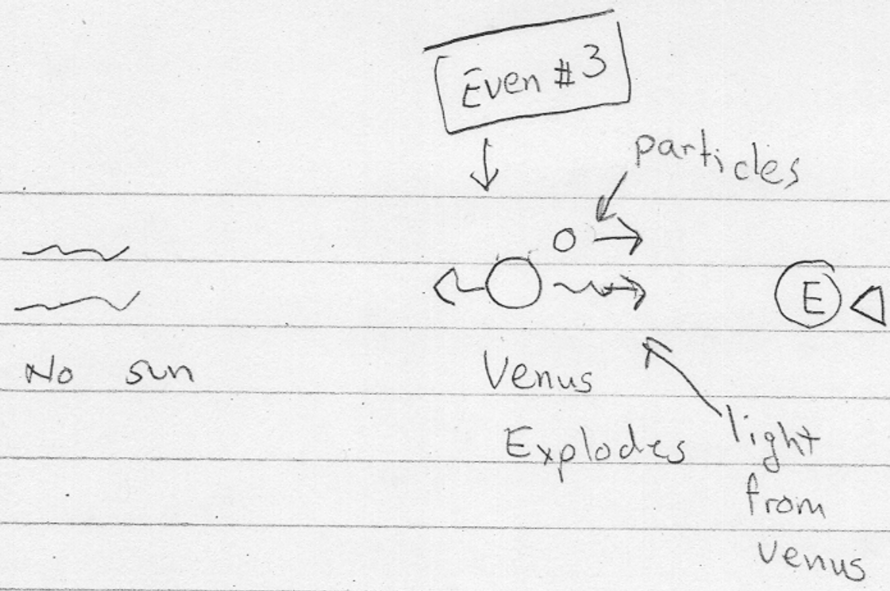
5.6 Picture

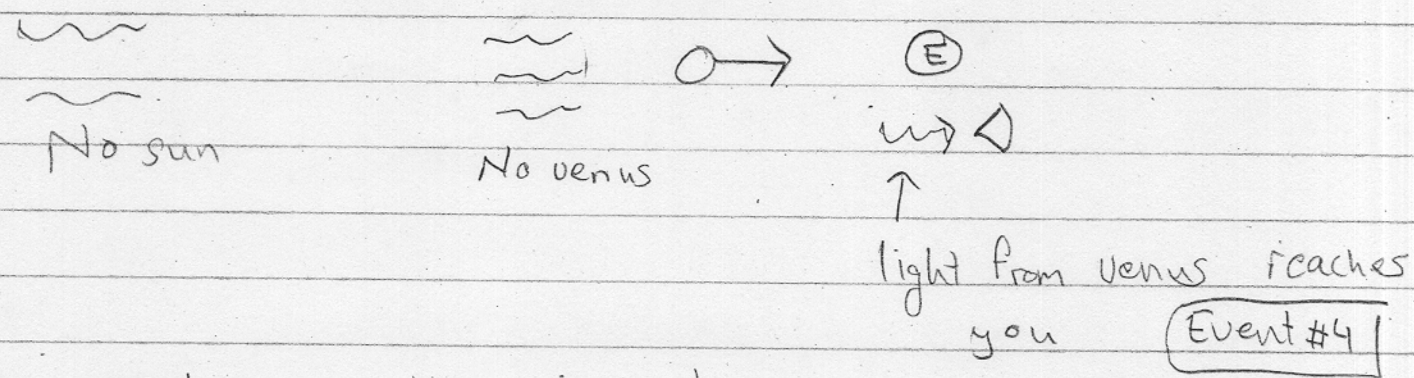


$$\gamma_A = \frac{5}{3} = \frac{1}{\sqrt{1-(V/c)^2}} \Rightarrow V_A/c = 4/5$$

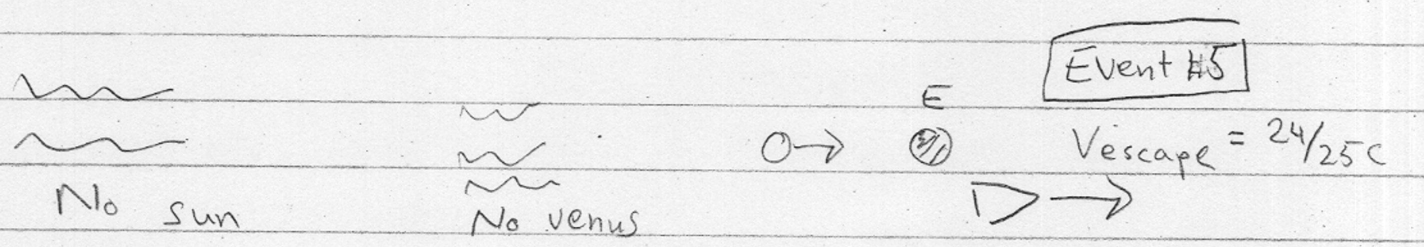




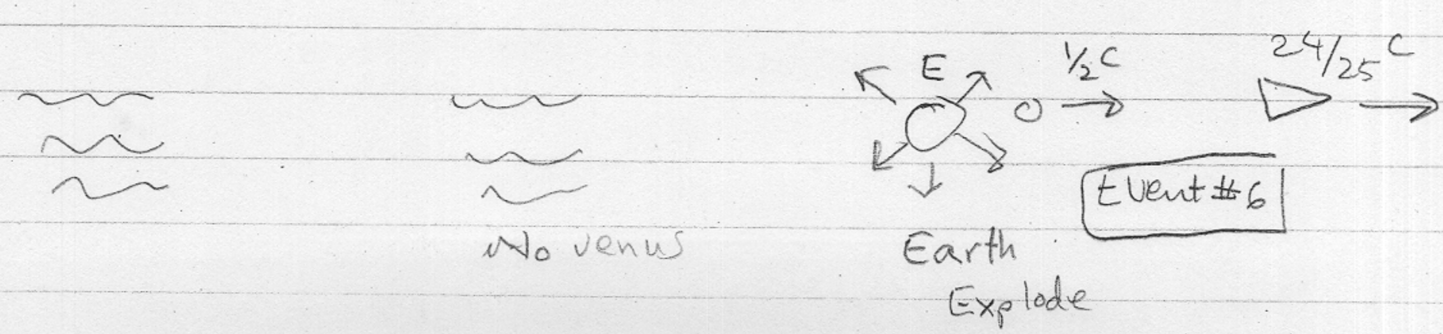




The next question is when the particles arrive before you leave. The answer is no. (see below)

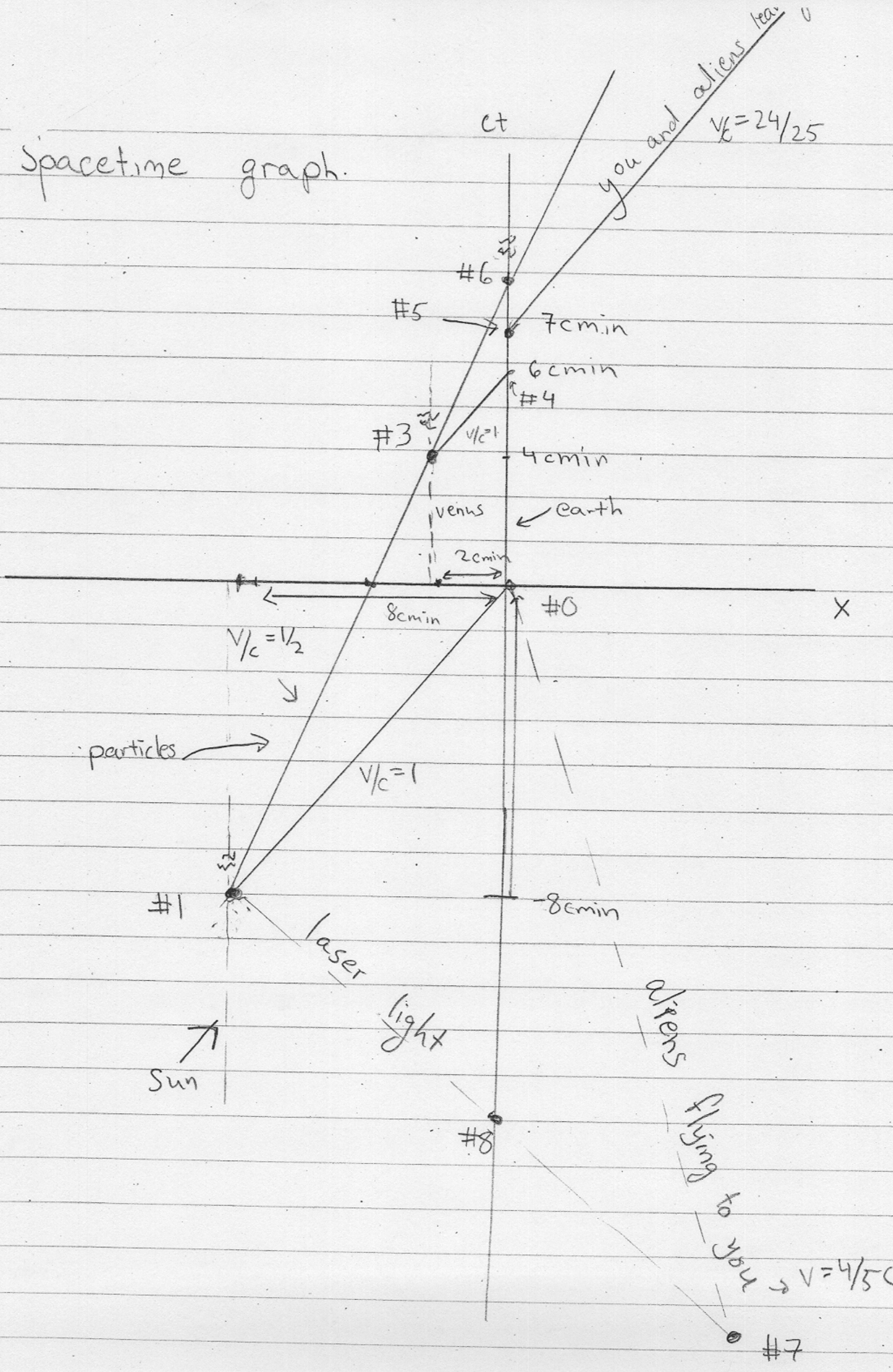


$$\gamma_{escape} = \frac{1}{\sqrt{1-(v/c)^2}} \Rightarrow v/c = \sqrt{1 - 1/\gamma^2} = \frac{24}{25}$$

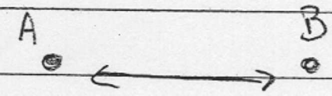


You escape

Spacetime graph.



$\boxed{P2}$



$\Delta x = 1\,\text{km}$

$\Delta t = 0$

Then

$$c\Delta t' = \gamma\, c\Delta t - \gamma\beta\,\Delta x$$
$$\Delta x' = -\gamma\beta\, c\Delta t + \gamma\,\Delta x$$

Substituting $\quad c\Delta t = 0 \qquad \Delta x = 1\,\text{km}$

$$c\Delta t' = -\gamma\beta\,\Delta x$$
$$\Delta x' = \gamma\,\Delta x$$

Now $\quad \Delta x' = 2\,\text{km} \quad$ so $\quad \dfrac{\Delta x'}{\Delta x} = \gamma = \dfrac{(2\,\text{km})}{(1\,\text{km})} = 2$

Then

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \implies \gamma^2 = \frac{1}{1-\beta^2} \quad \text{or} \quad \beta^2 = 1 - \frac{1}{\gamma^2}$$

So $\quad \beta^2 = 1 - \dfrac{1}{2^2} = \dfrac{3}{4} \quad$ or $\quad \beta = \dfrac{\sqrt{3}}{2}$

So the time differenc

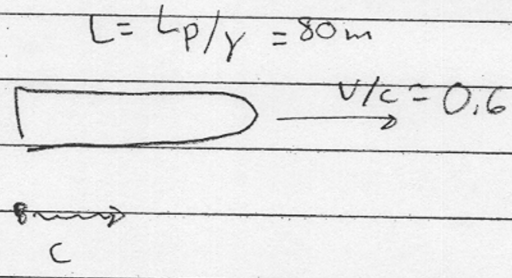
$$c\delta t' = -\gamma\beta\,\Delta x = -(2)\left(\frac{\sqrt{3}}{2}\right)(1\,\text{km}) = -\sqrt{3}\,\text{km}$$

Then

$$\Delta t' = -\frac{\sqrt{3}\ \text{km}}{c} = -5.77 \times 10^{-6}\ \mu s$$

This may sound fast but easily within reach of micro-processor's which have a clock speed of several GHz. The minus means B happens before A.

$\boxed{P3}$



(a) First Lets determin the Length measured on earth

$$L = \frac{L_p}{\gamma} \qquad \gamma = \frac{1}{\sqrt{1-(3/5)^2}} = \frac{5}{4}$$

$$L = \frac{100m}{5/4} = 80m$$

To determine the distance the earth sees the trajectory of the nose

$$x = x_0 + ut = 80m + (3/5\,c)t$$

and the trajectory of the light

$$x = ct$$

comparing these two

$$ct = x_0 + ut$$

$t = \frac{x_0}{c-u}$, i.e. normal rules apply if you stick in one frame

$t = 0.66 \times 10^{-6}s$

$x_0 = 80m \qquad u = 3/5\,c$

The distance of the nose from space station

$$x = x_0 + u\left(\frac{x_0}{c-u}\right) = \frac{(c-u)x_0 + ux_0}{(c-u)} = \left(\frac{c}{c-u}\right)x_0$$

$$x = \frac{\cancel{c}}{(1\cancel{c}-3/5\cancel{c})}\cdot x_0 = \frac{5}{2}x_0 = 200m$$

According the rocket:

<u>Warning</u>: The the emission and arrival do not occur at the <u>same</u> space time point in the rocket frame

The formula

$$\Delta t = \gamma\,\Delta\tau$$

applies when the $\Delta\tau$ is measured at the same space point!

Rather one should use the lorentz transformation

Earth

- Space Station sends signal $(ct, x) = (0, 0)$

- Nose receives signal $(ct, x) = (c\, 0.66 \times 10^{-6}\,\mu s, 200m)$

$(ct, x) = (200m, 200m)$

Then the space Ship events are

Space-Ship:

Send: $(ct', x') = (0, 0)$ we treat the back of the space ship as origin

Receive:

$$ct' = \gamma\, ct - \gamma\beta\, x = \left(\frac{5}{4}\right)(200m) - \frac{5}{4} \cdot 3/5 \cdot 200m$$

$$x' = -\gamma\beta\, ct + \gamma\, x = -\frac{5}{4}\frac{3}{5} \cdot (200m) + \frac{5}{4} \cdot 200m$$

$\gamma = 5/4$.

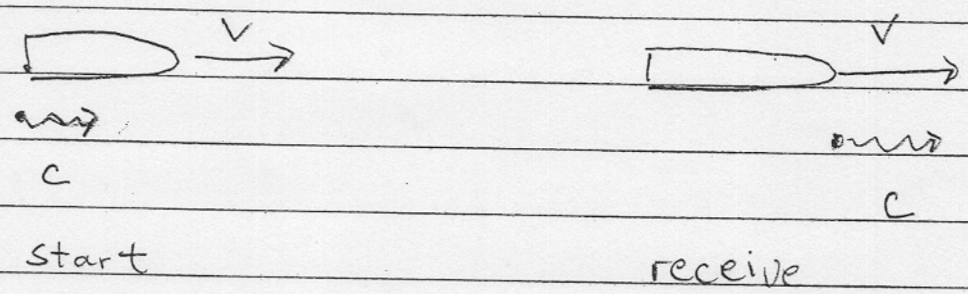
$\beta = 3/5$

$\gamma\beta =$

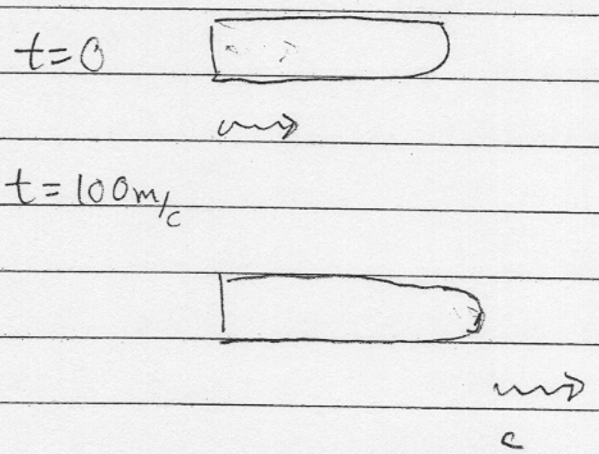
$ct' = 100m$ or $t' = 0.33\,\mu s$

$x' = 100m$

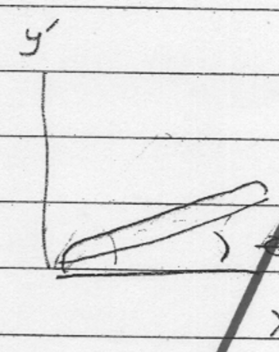
Earth Sees



Space Ship Sees:



4.16



$\theta = \arcsin(3/5) \simeq 37°$

Let us first determine the aspect ratio.

$\cos\theta = \sqrt{1-\sin^2\theta} = \frac{4}{5}$ $\qquad \tan\theta = \frac{3/5}{4/5} = \frac{3}{4}$

Problem 5

$$ct' = \gamma\, ct - \gamma\beta\, x$$

$$x' = -\gamma\beta\, ct + \gamma\, x$$

So

$$(ct')^2 - x'^2 = (\gamma ct - \gamma\beta x)^2 - (-\gamma\beta ct + \gamma x)^2$$

$$= \gamma^2 ct^2 - \gamma^2 \cancel{\beta\, xct} + (\gamma\beta)^2 x^2$$

$$- \left((\gamma\beta)^2 (ct)^2 - \gamma^2 \cancel{\beta\, x\, ct} + \gamma^2 x^2 \right)$$

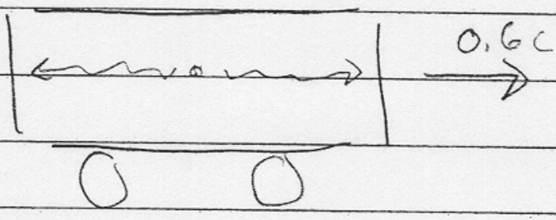
$$= \underbrace{(\gamma^2 - \gamma^2\beta^2)}_{=1}(ct)^2 + \underbrace{(\gamma\beta^2 - \gamma^2)}_{=-1} x^2$$

$$\gamma^2 = \frac{1}{1-\beta^2} \quad \text{so} \quad \gamma^2 - \gamma^2\beta^2 = \gamma^2(1-\beta^2) = \frac{1-\beta^2}{1-\beta^2} = 1$$

So

$$(ct')^2 - (x')^2 = (ct)^2 - x^2$$

Extra problem 6



a) The time is from the train view

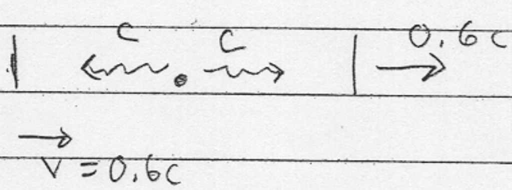
$$t = \frac{d}{c} = \frac{L_p}{c} = \frac{1.6 \times 10^3 \text{ m}}{(3\times10^8 \text{ m/s})} = 5.3\ \mu s$$

$$L_p = \frac{3.2\text{ km}}{2} \qquad \gamma = \frac{1}{\sqrt{1-(3/5)^2}} = \frac{5}{4}$$

$L_p = 1.6$ km

The time is the same for A′ and B′

b) In S we have



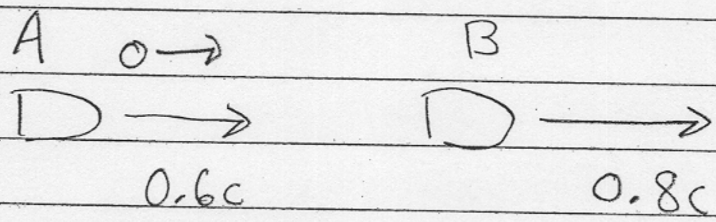
A′ happens first: at a time of

$$t' = \frac{d'}{v+c} = \frac{L_p}{\gamma(v+c)} = \frac{1.6\text{ km}}{(1.25)(0.6c + 1c)} = 2.6\ \mu s$$

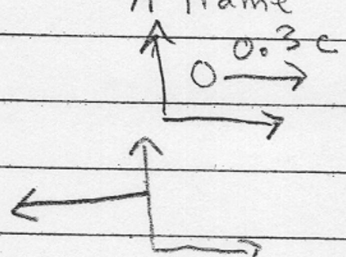
B happens next

$$t' = \frac{d'}{(c-v)} = \frac{1.6\,\text{km}}{(1c - 0.6c)(1.25)} \simeq 10.66\,\mu s$$

Extra Problem 7



Let us first determine the velocity of Bullet relative to earth



$V_{earth} = -0.6c$ relative to A

Bullet Relative to earth

velocity Bullet relative to A

velocity of earth relative to A

$$u_B' = \frac{u_B - V}{(1 - u_B V/c^2)}$$

$$u_B' = \frac{(0.3c) - (-0.6c)}{(1 - (0.3c)(-0.6c)/c^2)} = \frac{0.9c}{(1 + (0.3)(0.6))} = 0.76c$$

So the Bullet does not reach spaceship B.

(b) To find out what speed is necessary we want

$$u_B' = \frac{u_B - V}{(1 - u_B V/c^2)} = 0.8c$$

$$\frac{u_B - (-3/5c)}{1 - u_B(-3/5c)/c^2} = \frac{4}{5}c$$

So: $$\frac{(u_B/c + 3/5)c}{1 + (u_B/c)(3/5)} = \frac{4}{5}c$$

Cancelling c and writing $\beta = u_B/c$

$$\frac{\beta + 3/5}{1 + \beta \cdot 3/5} = \frac{4}{5}$$

Solving $\beta + 3/5 = \frac{4}{5}(1 + \beta \cdot 3/5)$

So

$$\beta = \frac{4/5 - 3/5}{1 - 4/5 \cdot 3/5} = \frac{5}{13}$$

$$u_B = \frac{5}{13}c \simeq 0.38c$$