Problems: Due 9/30

7.10 (**Graded**) (1)

1. (easy) Suppose one photon has an energy of 200 MeV and is traveling along the x axis. Suppose another has an energy of 100 MeV and is traveling along the y axis. (a) What is the total energy of this system? (b) If a single particle had this same energy and moemntum, what would be its mass? (c) In what direction would it be traveling? (d) With what speed?

2. (medium,**Graded**) A particle as observed in a certain reference frame (i.e. observer A) has a total energy of 5 GeV and a momentum of 3 GeV/c (i.e. cp which has the dimension of energy, is equal to 3 GeV)
   (a) What is the energy in the frame in which its momentum is equal to 4 GeV/c, i.e. an observer B moving with velocity v relative to A measures a momentum of 4 GeV/c.
   (b) What is its rest mass in GeV/c^2 and what is the ratio between this mass and the proton mass?
   (c) What is the relative velocity of the two frames?

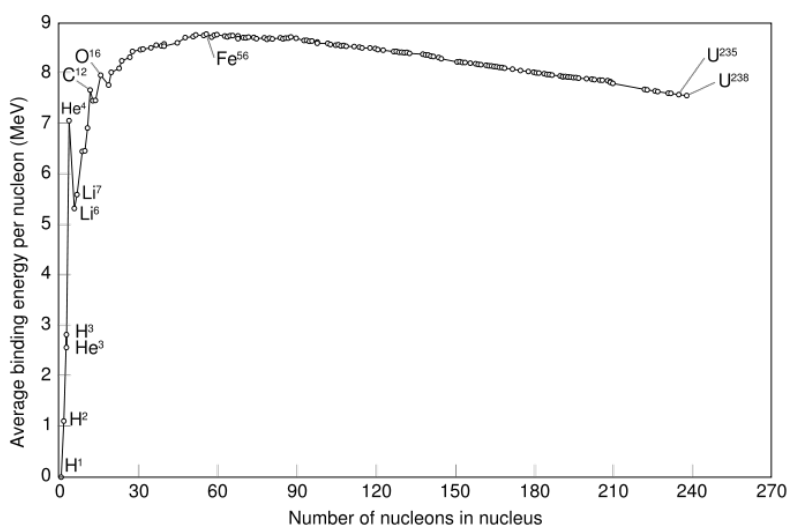
3. (**medium**) A neutral π meson (π^o) decays into two γ rays (and nothing else). If a π^o (whose rest mass is 135 MeV is moving with a kinetic energy of 1 GeV:
   (a) What are the energies of the γ rays if the decay process causes them to be emitted in opposite directions along the pion's original direction.
   (b) What angle is formed between the two γ rays if they are emitted at equal angles to the direction of the pions motion?

4. (easy) The total energy of an electron is 50 MeV determine $\beta = u/c$

5. (easy) An electron moves at $0.998\,c$ determine its (a) kinetic energy (units MeV) . (b) its linear momentum (units MeV/c) (c) Energy (units MeV).

6. (medium) At what speed is is the kinetic energy of a particle 1% higher than its classical value. (Hint: expand γ to $(v/c)^4$ using taylor series)

7. (medium) This problem is designed to help you understand binding energy and in the process teach you about fusion and fission. The following graph shows the binding energy per nucleon of different nuclei.



   (a) First understand this graph. For example oxygen-16 ${}^{16}_{8}O$ has 8 protons and 8 neutrons for a total of 16 nucleons. Read from the graph the binding energy per nucleon and multiply by the total number of nucleons to determine the total binding energy of ^{16}O. This is the energy that you would gain by taking 8 protons and 8 neutrons from far apart and combining them into an ^{16}O nucleus.

(b) Triton ${}^3_1$H has one proton and two neutrons for a total of three nucleons. How many proton, neutrons and nucleons does ${}^7_3$Li have.

(c) Fusion is the process by which lighter nuclei are combined together to make heavier nuclei and releasing energy. Use this graph to estimate the energy required to released. Consider the reaction fusion of deuteron with a proton which is important in the sun

$$ {}^2_1\mathrm{H} + {}^1_1\mathrm{H} \to {}^3_2\mathrm{He} + \mathrm{Energy} $$

Estimate from the graph how much energy is released during this step. (Hint: imagine separating all the protons and neutrons in the initial state out to a a great distance. Calculate the energy you must expend for this procedure. Then reaasembling them as ${}^3_2$He and calculate the energy you gain from this procedure.)

(d) Estimate from the graph how much energy would you gain our lose by fusing two ^{56}Fe nuclei?

(e) Consider the fission reaction

$$ n + {}^{235}_{92}U \to {}^{140}_{54}\mathrm{Kr} + {}^{94}_{38}\mathrm{Sr} + 2n + \underbrace{Q}_{\text{Energy}} \tag{2} $$

Show that the number of protons and neutrons are the same on both sides of the equation. Estimate from the graph the energy released in this process

(f) Explain why it is only profitable to fuse light elements and fission heavy elements.

Answers

1. (a) 300 MeV (b) 200 MeV (c) 27^o (d) 0.74 c: 2. (a) 5.62 GeV (b) Ratio $\simeq 4$ (c) B moves to the left relative to A, $|v| = 0.19c$; 3. (a) 1.31 GeV and 0.004 GeV (b) $2\theta \simeq 14^o$ 4. $\beta \simeq 1 - 0.52 \times 10^{-4}$ 5. (a) 7.5 MeV (b) 8.06 MeV (c) 8.08 MeV 6. $\beta \simeq 0.12c$ 7. (c) $\simeq 5.5$ MeV (d) costs approximately 44 MeV (e) $\simeq 81$ MeV

Dynamics

1. The momentum of a particle moving with velocity $\mathbf{u}$ is

$$ \mathbf{p} \equiv \gamma m \mathbf{u} \qquad \text{with} \qquad \gamma \equiv \frac{1}{\sqrt{1 - u^2/c^2}} \tag{3} $$

For a massless particle $u = c$ and $E = c|\mathbf{p}|$.

2. The energy of a particle moving with velocity $\mathbf{u}$ is

$$ E \equiv \gamma m c^2 \qquad \text{with} \qquad \gamma \equiv \frac{1}{\sqrt{1 - u^2/c^2}} \tag{4} $$

3. The rest energy is the energy of a particle when it is not moving, and the kinetic energy is the energy minus the rest energy

$$ E_{\text{rest}} = mc^2 \qquad K = \gamma mc^2 - mc^2 = E - mc^2 \tag{5} $$

4. The energy and momentum and velocity given above are related by the formulas

$$ E^2 = (cp)^2 + (mc^2)^2 \qquad \mathbf{u} = c^2 \frac{\mathbf{p}}{E} \tag{6} $$

For a photon we have $E = cp$ and $u = c$ (mass is zero)

5. Total energy and momentum (the sum of the energy's and momenta of all the particles) are always conserved before and after the collision

$$ E^{\text{before}}_{\text{tot}} = E^{\text{after}}_{\text{tot}} \qquad \mathbf{p}^{\text{before}}_{\text{tot}} = \mathbf{p}^{\text{after}}_{\text{tot}} \tag{7} $$

6. If the energy and momentum $(E, c\mathbf{p})$ of a particle according to one observer is known, then according to an observer moving to the right with speed v, the energy and momentum of the particle is $(E', c\,\mathbf{p}')$ with

$$
\begin{aligned}
E' &= \gamma\, E - \gamma\beta\, (cp_x) && (8)\\
cp'_x &= -\gamma\beta\, E + \gamma\, (cp_x) && (9)\\
cp'_y &= cp_y && (10)\\
cp'_z &= cp_z && (11)
\end{aligned}
$$

with $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ and $\beta = v/c$. Of course for a left moving observer we have $v \to -v$. We multiply c times momentum so that $c\,p$ has the same units of energy. The change of frames mixes energy and momenta.