\[ a) \quad E_a = K + mc^2 \]
\[ E_b = m_c^2 \]
\[ E_c = E_a + E_b = K + 2mc^2 \]

\[ b) \quad c_P_a = E_a^2 - mc^2 \]
\[ = (K + mc^2)^2 - mc^2 \]
\[ c_P_a = K(K + 2mc^2) \]
\[ c_P_c = c_P_a = K(K + 2mc^2) \]

\[ c) \quad m_c^2 c^2 = E_c^2 - (c_P_c)^2 \]
\[ = (K + 2mc^2)^2 - (K + 2mc^2)K \]
\[ = 2mc^2K + (2mc^2)^2 \]
\[ (m_c^2)^2 = (2mc^2)^2(1 + \frac{K}{2mc^2}) \]

\[ d) \quad \text{non-rel:} \quad m_c^2 = 2mc^2 \quad k \ll 1 \quad \text{There is no limit} \]
\[ \text{rel:} \quad (m_c^2)^2 \approx 2mc^2K \Rightarrow m_c^2 \approx \sqrt{2mc^2K} \]
\[ E_{\text{tot}} = E_1 + E_2 = 200 \text{ MeV} + 100 \text{ MeV} = E \]

\[ p_{\text{tot}} = p_1 + p_2 = 200 \text{ MeV}/c \]

\[ p_y = p_3 + p_5 = 100 \text{ MeV}/c \]

Note we have used \( E = cp \) so for the photon moving in the \( x \) direction

\[ p = \frac{E}{c} = 200 \text{ MeV}/c \]

\[ |p_{\text{tot}}| = \sqrt{(p_{x\text{tot}})^2 + (p_{y\text{tot}})^2} = \sqrt{(200)^2 + (100)^2} \]

\[ p \approx 223 \text{ MeV}/c \]

Its mass is:

\[ E^2 = (mc^2)^2 + (cp)^2 \]

\[ mc^2 = \sqrt{E^2 - (cp)^2} \approx 200 \text{ MeV} \]

\[ m = 200 \text{ MeV}/c^2 \]

Direction

\[ \tan \theta = \frac{p_3}{p_x} = \frac{100 \text{ MeV}/c}{200 \text{ MeV}/c} = \frac{1}{2} \]

\[ \theta \approx 27^\circ \]
Then Speed

\[ \frac{v}{c} = \frac{E}{mc^2} = \frac{0.723 \text{ MeV}}{300 \text{ MeV}} = 0.74 \]

\[ v = 0.74c \]

First let's determine its mass:

\[ E^2 = (mc^2)^2 + (cp)^2 \]

\[ mc^2 = \sqrt{E^2 - (cp)^2} = \sqrt{5^2 - 3^2} = 9 \text{ GeV} \]

\[ m = \frac{4 \text{ GeV}}{c^2} \]

\[ 1 \text{ GeV/c}^2 = 1.07 \text{ amu} \]

\[ m = 4.29 \text{ amu} \]

Part b

\[ m = 9.29 \text{ amu} \]

\[ E = \sqrt{(mc^2)^2 + (cp)^2} = \sqrt{(4 \text{ GeV})^2 + (4 \text{ GeV})^2} = 5.62 \text{ GeV} \]
The velocity in the first frame

\[ u = \frac{c (cp)}{E} = \frac{c (3 \text{GeV})}{5.6 \text{GeV}} = 0.6c \]

\[ u' = \frac{c (cp')}{E} = \frac{c (4 \text{GeV})}{5.62 \text{GeV}} = 0.711c \]

The picture is the first frame measures

\[ u = 0.6c \]

Then an observer moving to the left with relative speed \( v \) measures

\[ u' = 0.711c \]

Then

\[ u' = \frac{u - v}{\sqrt{1 - \frac{uv}{c^2}}} \quad v < 0 \]

\[ u' = \frac{u + |v|}{\sqrt{1 + \frac{|v|}{c^2}}} \]

Solving for \( |v| \)

\[ u' + u' |v|/c^2 = u + |v| \]

\[ u' = - |v| - uu'/c^2 |v| \]
or \[ v' = \frac{u' - u}{1 - \frac{uu'}{c^2}} = \frac{0.7u1c - 0.6c}{1 - (0.7u1c)(0.6c)/c^2} \]

\[ |v'| = 0.19c \]

\[ P(\gamma) \]

\[ E = m_0 c^2 \]

\[ \gamma \]

\[ a) \quad E_f = E + m_0 c^2 = E + m_0 c^2 \]

\[ P_f = P_1 + P_2 = \frac{E}{c} \quad \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \]

The velocity \( \gamma \)

\[ \frac{u}{c} = \frac{c(\gamma E)}{E_f} = \frac{E}{E_f} = \frac{E}{E + m_0 c^2} \]

\[ (\gamma^2 - 1) \]

\[ b) \quad m_0 u_1 = \frac{4}{5}c, \quad m_0 u_2 = 0 \]

\[ E_f = \gamma_1 m_0 c^2 + \gamma_2 m_0 c^2 \]

\[ \gamma_1 = \frac{1}{\sqrt{1 - (\gamma E)^2}} \]

\[ E_f = \frac{5m_0 c^2 + m_0 c^2}{3} = \frac{8m_0 c^2}{3} \]

\[ P_f = P_1 + P_2 \]
Then
\[ p = \sqrt{\frac{8}{3} m_0 c^2} \cdot (\frac{1}{2} c) = \frac{4 m_0 c}{3} \]

The final velocity
\[ u_f = \frac{c^2 p_f}{E_f} = \frac{c^2}{\frac{8}{3} m_0 c^2} = \frac{1}{2} c \]
\[ m_f = \sqrt{E_f^2 - (c p_f)^2} = \sqrt{\left(\frac{8}{3} m_0 c^2\right)^2 - \left(\frac{4}{3} m_0 c^2\right)^2} = \sqrt{\frac{4 \sqrt{3} m_0 c^2}{3}} \]

**Problem 3**

\[ K = 1 \text{GeV} \]
\[ \frac{E_2 p_2}{E_1 p_1} \]

- The energy of the pion is \( E_\pi = K + m_0 c^2 = 1.135 \text{ GeV} \)
- The momentum of the pion is
\[ E^2 = (c p)^2 + (m c^2)^2 \]
\[ c p = \sqrt{E^2 - (m c^2)^2} \approx 1.127 \text{ GeV} \]

Energy and momentum gives:
\[ E_\pi = E_1 + E_2 \]
\[ P_\pi = P_1 - P_2 \]
\[ P_2 = \frac{E_2}{c} \]
for photons
\[ E_{\pi} = E_1 + E_2 \]
\[ C_{\pi} = C_{\pi_1} - C_{\pi_2} = E_1 - E_2 \]

Solving for \( E_1 \) and \( E_2 \)

\[ \frac{E_{\pi} + C_{\pi}}{2} = E_1 = 1.135 + 1.127 \text{ GeV} = \boxed{1.131 \text{ GeV}} = E_1 \]

\[ \frac{E_{\pi} - C_{\pi}}{2} = E_2 = 1.135 - 1.127 \text{ GeV} = 0.004 \text{ GeV} = \boxed{4.40 \text{ MeV}} = E_2 \]

\[(b)\]

\[ \pi^0 \rightarrow \gamma + \gamma \]

The symmetry dictates that the two photons have equal momenta:

\[ E_{\pi} = E + E \]

\[ P_{\pi} = p_{\gamma} \cos \theta + p_{\gamma} \cos \theta \]

\[ \theta = \rho \sin \theta - \rho \sin \theta \]
Then:

\[ E_\pi = 2E \]

\[ \theta \approx 6.9^\circ \] — The Book quotes \( 2\theta \approx 14^\circ \)

\[ 7.1 \]

Energy and momentum:

1. \[ E_K = E_1 + m_\pi c^2 \]

2. \[ p_K = p_1 + p_2 \]

Now count: two equations (1) \& (2) and

3. \[ E_1^2 = (cp_1)^2 + (m_\pi c^2)^2 \]

4. \[ E_1^2 = (cp_1)^2 + (m_\pi c^2)^2 \]

Unknowns:

\[ E_2, E_\pi, c_\pi, \text{ and } p_\pi \]

Four equations (1), (2), (3), (4)
Problem 4

\( E = 50 \text{ MeV} \)

\[ m = \frac{0.511 \text{ MeV}}{c^2} \]

\[ \gamma = \frac{E}{m} = \frac{50 \text{ MeV}}{0.511 \text{ MeV}} = 97.84 \]

\[ \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \]

\[ \left(\frac{v}{c}\right)^2 = 1 - \frac{1}{\beta^2} \quad \beta = 1 - \frac{1}{2\gamma^2} \]

\[ \beta = 0.998 \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}} = 15.8 \]

\[ \gamma = \frac{E}{mc^2} \]

\[ P = \gamma mc \]

\[ E = (15.8)(0.511 \text{ MeV}) \]

\[ E = 8.08 \text{ MeV} \]

\[ K = E - mc^2 \]

\[ K = (8.08 \text{ MeV}) - (0.511 \text{ MeV}) = 7.5 \text{ MeV} \]

\[ \rho = \frac{8.0674 \text{ MeV}}{c} \]
\[ K = \gamma mc^2 - mc^2 \]

\[ \gamma = \frac{1}{\sqrt{1 - \beta^2}} \]

\[ \gamma \approx 1 + \frac{1}{2} \beta^2 - \frac{1}{8} \beta^4 \]

\[ K = (1 + \frac{1}{2} \beta^2 + \frac{3}{8} \beta^4) mc^2 - mc^2 \]

\[ K = \frac{1}{2} \frac{v^2}{c^2} mc^2 + \frac{3}{8} \beta^4 mc^2 \]

\[ K = \frac{1}{2} mv^2 + \frac{3}{8} mv^2 \beta^2 \]

\[ \%_{\text{err}} = \left( \frac{K - \frac{1}{2} mv^2}{\frac{1}{2} mv^2} \right) = \% \text{ deviation of } K \text{ from } \frac{1}{2} mv^2 \]

\[ \%_{\text{err}} = \frac{3}{8} \frac{mv^2 \beta^2}{\frac{1}{2} mv^2} = \frac{3}{4} \beta^4 \approx 0.01 \]

\[ \beta = 0.12 \]
\[ (\Delta s)^2 = (c \Delta t)^2 - (\Delta x)^2 \]

\[ (\Delta s)^2 = (0)^2 - (1m)^2 = -1m^2 \]

d\)

\[ E' = \gamma E - \gamma \beta cp \]

\[ cp' = -\gamma \beta E + \gamma cp \]

\[ (E')^2 - (cp')^2 = (\gamma E - \gamma \beta cp)^2 - (-\gamma \beta E + \gamma cp)^2 \]

\[ = \gamma^2 E^2 - 2\gamma^2 \beta Ecp + \gamma^2 \beta^2 (cp)^2 \]

\[ - \gamma^2 \beta^2 E^2 + 2\gamma^2 \beta Ecp - \gamma^2 (cp)^2 \]

\[ (E')^2 - (cp')^2 = E^2 - (cp)^2 \]

**Problem 7**

a) Binding Energy \( \approx 16 \times \overline{BE} \approx 16 \times 8 \text{ MeV} \)

\( \overline{BE} = 128 \text{ MeV} \)

b) \(^7_3\text{Li}\) 3 protons and 4 neutrons
(c) $^2\text{H} + ^1\text{H} \rightarrow ^3\text{He} + \text{energy}$

Then we have the energy required to tear apart the $^2\text{H}$ + $^1\text{H}$ into 2 protons and two neutrons is the binding energy of $^2\text{H}$.

$$BE\text{ of }^2\text{H} = 2 \times (5 \text{ MeV}) = 10 \text{ MeV}$$

The energy gained by assembling the two protons and neutron into $^3\text{He}$ is

$$3 \times (2.5 \text{ MeV}) = 7.5 \text{ MeV}$$

So the net energy released is

$$\approx 5.5 \text{ MeV}$$

(d) By fusing two $^{56}\text{Fe}$ nuclei, the energy balance is:

$$\text{mass of } (2 \times 56 \text{ protons} + \text{neutrons}) - \text{ (Binding Energy of } ^{56}\text{Fe}) \rightarrow \text{mass of } (2 \times 56 \text{ protons} + \text{neutrons}) + \text{ (Binding Energy of } ^{56}\text{Fe})$$

$$- 112 \text{ protons} + \text{neutron} + \text{Energy Released}$$
Then for fission you move down the

Thus for the following process you move down the x-axis and

Since $Q = \Delta BE$ the energy released is positive

See part e.