Problems:

\[ E^{2.1}, E^{2.2} \text{(Graded)}, E^{2.15} \text{(Graded)}, E^{2.16}, E^{2.19}, E^{2.24}, E^{2.10}, E^{2.11}, E^{2.13} \] (1)

1. On problem E2.19 be sure to explain carefully what happens at \( \theta = 0 \) and \( \theta = \pi \)

2. (Ultra Easy) We often will use a slightly different notation as the course progresses. We define

\[
\hbar \equiv \frac{\hbar}{2\pi}
\] (2)

The angular frequency and wave number are

\[
\omega \equiv 2\pi \nu \quad k \equiv \frac{2\pi}{\lambda}
\] (3)

We use these quantities because it is easier to write a wave as

\[
\cos(\kappa x - \omega t) \quad \text{rather than} \quad \cos(\frac{2\pi}{\lambda}x - 2\pi ft)
\] (4)

Show the following relations (a) \( \hbar c = 197 \text{ eV nm} \) (b) \( E = h\omega \) (c) \( \omega = c k \) (d) \( p = \hbar k \) (e) The book also uses \( \kappa \equiv \frac{1}{\lambda} \). This notation is not widely used. Show that \( k = 2\pi\kappa \)

3. From the first practice test Problem 3, parts (1)–(3)

Answers – See back of book

\[ E^{2.1} : 5390 \text{ Å, } E^{2.11} : (b) 6 \times 10^{19} \text{ Hz, } E^{2.13} : \lambda = \text{compton wavelength} = \hbar/m_e c \]

Quantum Nature of Light

1. Light comes in discrete units called photons. The energy and momentum of the photon is related to the frequency (given symbol \( \nu \) or sometimes \( f \)).

\[
E = h\nu \quad c p = E \quad c = \lambda \nu
\]

2. \( h \) is planck’s constant and is

\[
hc = 1240 \text{ eV nm}
\]

3. The intensity of the light is the energy moving across a screen per area per time

\[
I = \left\langle \frac{E}{A\Delta t} \right\rangle
\] (5)

The brackets \( \langle \rangle \) denote averages over a sufficiently long time or large area so that many photons are involved.

4. For monochromatic (one frequency) light this is

\[
I = h\nu \left\langle \frac{N}{A\Delta t} \right\rangle
\] (6)

\( N/A\Delta t \) is the number of photons per area per time.

5. If for a detector of area \( A \) and time resolution \( \Delta t \) the number of photons \( N \) crossing the detector is very large (compared to one) then classical electromagnetism is a good approximation and we have that the intensity is the average of the poynting vector

\[
I = \left\langle \frac{\mathbf{E} \times \mathbf{B}}{\mu_0} \right\rangle
\] (7)

\[
= \sqrt{\frac{\varepsilon_0}{\mu_0}} \left\langle \mathbf{E}^2 \right\rangle
\] (8)

where \( \mathbf{E} \) and \( \mathbf{B} \) are the electric and magnetic fields respectively.
TABLE I: Work function for various metals

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<th>Material</th>
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6. **Photoelectric Effect and Compton Scattering**

1. The photo-electric effect is when photons tear away an electron from a metal. The kinetic energy of the electron after it is ripped away is

\[ K_e = h\nu - w_o \quad (9) \]

Here \( w_o \) is the work function of the metal given and is the energy required to tear the electron away from the atom. By applying a retarding voltage \( V_o \) we can stop the flow of electrons from the metal. When the voltage is

\[ |e|V_o = K_e \quad (10) \]

the electrons will not have enough energy to make it to the collector and current will stop.

2. X rays are produced by colliding electrons into a metal foil:

- When then electrons decelerate they radiate many different frequencies. The total power \( P \) emitted by the electron is related to the deceleration \( a \) by the Larmour formula which is stated without proof

\[ P = \frac{2}{3\pi\epsilon_0} \frac{e^2a^2}{c^3} \]

- The shortest wavelength of light produced during the deceleration process is when all of the electrons kinetic energy gets converted into a single photon. If the electrons initial kinetic energy is \( K \)

\[ K = \frac{hc}{\lambda_{\text{min}}} \quad (11) \]

3. The wavelength of an X-ray may be determined with Bragg’s crystal. The condition for constructive interference is

\[ 2d\sin(\theta) = n\lambda \quad n = 1, 2, 3, \ldots \quad (12) \]
4. The Compton process is the scattering of light (X-rays) on electrons nearly at rest. The following formula results (after algebra) from energy and momentum conservation and \( E = hf \)

\[
\lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta)
\]  

To determine the electron momentum and direction one would write down relativistic energy and momentum conservation and use the above formula. Energy and momentum conservation and use the above formula to determine

5. The Compton wavelength is a measure of the size of the “fuzziness” of the electron

\[
\lambda_C = \frac{h}{mc} = 0.0024 \text{nm}
\]  

![Diagram of Compton scattering](image-url)