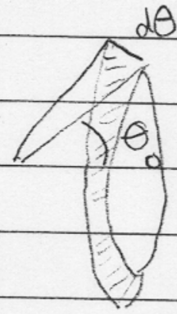
Problem 4.8

$$dN = [\Delta t \mathcal{L}] \frac{d\sigma}{d\Omega} \, d\Omega$$

$$dN_\theta = \left[N_\alpha \rho_{Au} t_{foil} \right] \cdot \frac{D^2}{16 \sin^4(\theta/2)} \cdot 2\pi \sin\theta \, d\theta$$



Then dN is the number scattered into a ring of size $d\theta$. To find the total number which scatter with angle greater than θ_0

$$N = \sum dN_\theta = \int_{\theta_0}^{\pi} \left[N_\alpha \rho_{Au} t_{foil} \right] \frac{D^2}{16 \sin^4 \theta/2} \, 2\pi \sin\theta \, d\theta$$

$$\int \frac{\sin\theta}{\sin^4(\theta/2)} = \int \frac{\sin(2\theta/2)}{\sin^4(\theta/2)} = \int \frac{2 \cancel{\sin\theta/2} \cos\theta/2}{\sin^{\cancel{4}\,3}\theta/2} = \frac{-2}{\sin^2\theta/2}$$

So

$$N = [N_\alpha \rho_{Au} t_{foil}] \frac{D^2}{16} \cdot 2\pi \left[\frac{-2}{\sin^2\theta/2} \right]_{\theta_0}^{\pi}$$

$$= [\,] \frac{D^2}{16} \cdot 4\pi \left[-1 + \frac{1}{\sin^2\theta_0/2} \right]$$

$$N = [\,] \frac{D^2 \pi}{4} \left[\frac{\overbrace{1 - \sin^2\theta_0/2}^{\cos^2\theta_0/2}}{\sin^2\theta_0/2} \right] = [\,] \frac{\pi D^2}{4} \cot^2(\theta_0/2)$$

with $D = \dfrac{Z z_\alpha e^2}{4\pi\varepsilon_0 \left(\frac{1}{2} m v^2\right)}$

$$D^2 = 4 \left(\frac{Z z_\alpha e^2}{4\pi\varepsilon_0 m v^2} \right)^2 \Rightarrow \frac{D^2}{4} = \left(\frac{Z z_\alpha e^2}{4\pi\varepsilon_0 m v^2} \right)^2$$

So

$$N = [N_\alpha \rho_{Au} t_{foil}] \left(\frac{Z z_\alpha e^2}{4\pi\varepsilon_0 m v^2} \right)^2 \cdot \pi \cos^2(\theta_0/2) \quad \checkmark$$

4.9

$$N_{\theta>60} = [N_\alpha \rho t] \frac{\pi D^2}{4} \cot^2(60°/2)$$

$$\cot^2 60/2 = \frac{\cos^2 30}{\sin^2 30} = \frac{(\sqrt{3}/2)^2}{(1/2)^2} = 3$$

$$D = \alpha \frac{Z \cdot z_p \hbar c}{(\frac{1}{2}mv^2)} = \frac{1}{137} \cdot \frac{79 \cdot 1 \cdot 197 \text{ MeV fm}}{6 \text{ MeV}} = 18.9 \text{ fm}$$

$$D^2 = 358 \text{ fm}^2 = 3.58 \text{ barns} = 3.58 \times 10^{-24} \text{ cm}^2$$

$$\frac{N_{\theta>60}}{N_\alpha} = 2 \times 10^{-5} = \text{fraction with } \theta > 60°$$

$$t_{foil} = \left(\frac{N_{\theta>60°}}{N_\alpha}\right) \frac{1}{\rho_{Au} \frac{\pi D^2}{4} \cdot 3}$$

$(\cot 60/2)^2$

ρ_{Au} from next problem $\rho_{Au} = 0.58 \times 10^{23} \frac{1}{\text{cm}^3}$

$$t_{foil} = (2 \times 10^{-5}) \frac{1}{0.58 \times 10^{23} \frac{1}{\text{cm}^3} \cdot \left(\frac{3\pi}{4}\right) 3.58 \times 10^{-24} \text{ cm}^2}$$

$t_{foil} = 4 \times 10^{-5}$ cm $= 0.4\ \mu$m ~ "tenths of a micron"

4.10

The number of particles which are scattered into a dector which spans a solid angle $d\Omega$ is

$$dN = [N_\alpha \rho_{Au} t_{foil}] \frac{D^2}{16 \sin^4(\theta/2)} d\Omega$$

Units

$$[N_\alpha \rho_{Au} t_{foil}] = \frac{\#}{\text{area}}$$

$$\left[\frac{D^2}{16 \sin^4(\theta/2)}\right] = \text{area}$$

$d\Omega$ = unit less

A unit of area which is commonly used is

barns $= 10^{-24}\ \text{cm}^2$

$$\rho_{Au} = \frac{\text{Number}}{\text{Volume}} = \frac{1}{m_{Au}} \frac{\text{mass}}{\text{Volume}}$$

mass of a gold nulceus

1 Avagradro's # of protons weighs $\simeq$ 1 gram

1 Avagardo's of gold $\simeq$ 197 g

So

$$m_{Au} = \frac{197\,g}{6 \times 10^{23}}$$

$$\rho_{Au} = \frac{19.3\ g/cm^3}{\frac{197\,g}{6 \times 10^{23}}} = 0.58 \times 10^{23} \frac{1}{cm^3}$$

$$t_{foil} = 0.1\ \mu m = 1 \times 10^{-5}\ cm$$

$$N_\alpha = \frac{10^4}{\text{Sec}} \times \underbrace{3600\,s}_{1\,\text{hour}}$$

$$\left[N_\alpha \rho_{Au} t_{foil}\right] = 0.211 \times 10^{26} \frac{1}{cm^2} = \boxed{21.1 \frac{1}{\text{barn}}}$$

The distance of closest appracch

$$D = \frac{Z_{Au} z_\alpha e^2}{4\pi\varepsilon_0 (\frac{1}{2} m v^2)} = \alpha \frac{Z_{Au} z_\alpha \hbar c}{\frac{1}{2} m v^2}$$

$$D = \frac{1}{137} \frac{79 \cdot 2 \cdot (197\ \cancel{MeV}\ fm)}{5.3\ \cancel{MeV}}$$

$D = 42.86$ fm about 4x larger than the Au nucleus radius

$$D^2 = 1836\ fm^2 \approx \boxed{18.3 \text{ barns} = D^2}$$

$1 fm^2 = 0.01$ barn

$$d\Omega = \frac{dA}{r^2} = \frac{1\ cm^2}{(10\ cm)^2} = \boxed{0.01 = d\Omega}$$

Now:

$$dN = \underbrace{\cancel{\quad}}_{[N_\alpha \rho_{Au} t_{foil}]} \times \frac{D^2}{16} \frac{1}{\sin^4\theta/2} d\Omega \quad \text{for}$$

$$dN = \left(21.1 \frac{1}{\text{barn}}\right)(18.3 \text{ barn}) \frac{1}{16} \frac{1}{\sin^4(10^\circ/2)} \cdot 0.01$$

$dN_{10^\circ} = 4180$ at 10°

At 45°

$$dN_{45^\circ} = dN_{10^\circ} \cdot \frac{\sin^4(10^\circ/2)}{\sin^4(45^\circ/2)} = 4180 \cdot \frac{\sin^4(10^\circ/2)}{\sin^4(45^\circ/2)} \simeq 11$$

Problem 4.11

$N \propto \rho Z^2$

where ρ is the number density

$d \equiv$ is the mass denisity, 19.3 g/cm³ for Au, and 8.9 g/cm³ for copper

$\rho \propto \frac{d}{A}$ where A is the atomic number, 197 for Au and 63.6 for Cu. A is proportional to the mass of nucleus $\rho = \frac{d}{\text{mass of nucleus}} \propto d/A$

So

$N \propto \frac{d Z^2}{A}$

constant #

$N = K \frac{d Z^2}{A}$ so $\frac{N_{cu}}{N_{Au}} = \frac{d_{cu} (Z_{cu}/Z_{Au})^2}{d_{Au} (A_{cu}/A_{Au})}$

$$Z_{Au} \sqrt{\frac{N_{cu}}{N_{Au}} \frac{d_{Au}}{d_{cu}} \frac{A_{cu}}{A_{Au}}} = Z_{cu}$$

So with $N_{Au} = 4180$ and $N_{cu} = 820$ we have:

$$Z_{cu} = (79) \left(\frac{820}{4180} \cdot \frac{19.3\,g/cm^3}{8.9\,g/cm^3} \cdot \frac{63.6}{197} \right)^{1/2}$$

$Z_{cu} = 29$

Agrees w periodic table

① $\Delta t \mathcal{L} = \frac{N_p N_t}{A}$

$= N_p \left(\frac{N_t}{A \Delta x}\right) \Delta x$

$\Delta t \mathcal{L} = N_p \rho_{Au} t_{foil}$ and $\mathcal{L} = \frac{\Delta N_p}{\Delta t} \rho_{Au} t_{foil}$

② $\sigma_{Backard} = \frac{N_{backwards}}{\Delta t \mathcal{L}}$

$$\sigma_{Backward} = \frac{\cancel{[N_\alpha \rho t]}\ \pi \left(\frac{Zze^2}{mv^2}\right)^2 \cdot \cot(90^\circ/2)}{\cancel{[N_\alpha \rho t_{foil}]}}$$

$$\sigma_{back} = \frac{\pi}{4}\left(\frac{Zze^2}{\frac{1}{2}mv^2}\right)^2 \cdot 1$$

$$\sigma_{back} = \frac{\pi D^2}{4}$$

③ $\frac{dN}{dt} = \int \frac{dN}{dt\, d\Omega}\, d\Omega = \int_0^\pi N_0 (1 + \cos^2\theta)\, 2\pi \sin\theta\, d\theta$

$= N_0\, 2\pi \int_0^\pi \sin\theta\, d\theta + N_0\, 2\pi \int_0^\pi \cos^2\theta\ \mathrm{si}$ ... d

$= N_0 \left[2\pi \cdot 2\right] + N_0 \frac{2\pi}{3}\left(-\cos^3\theta\right)\Big|_0^\pi$

$$\frac{dN}{dt} = 4\pi N_o + \frac{4\pi N_o}{3}$$

a) $$\frac{dN}{dt} = N_o \cdot \frac{4}{3} \cdot 4\pi = 0.33 \times 10^{21}$$

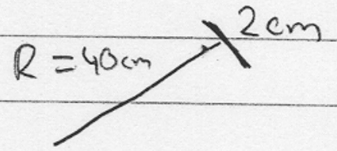
b) $$\frac{dN}{dt\,d\Omega} = \frac{\frac{dN}{dt}}{\Omega_{sphere}} = \frac{0.33 \times 10^{21}}{4\pi} = 0.266 \times 10^{20} \frac{1}{s}$$

Problem 4

units ster-radian

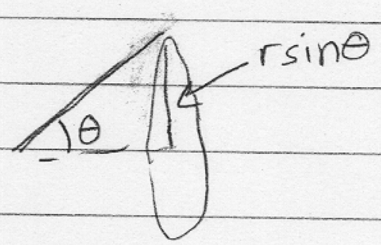
a) $\Delta\Omega = \dfrac{A}{r^2} = \dfrac{4\,cm^2}{(40\,cm)^2} = 0.0025\,str$ but its unitless

b) $\Delta\Theta = \dfrac{2\,cm}{40\,cm} = \dfrac{1}{20}\,rad = 2.86^\circ$

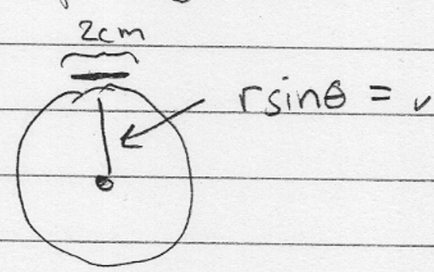


For $\Delta\phi$

side view



top view



$$\Delta\phi = \frac{l}{R\sin\theta} = \frac{2cm}{40\,cm\,\sin 30^\circ} = \frac{1}{10}\,rad = 5.72^\circ$$

c) The Power scattered = 3mW, each photon ha

$$E_{photon} = h\nu = \frac{hc}{\lambda} = \frac{1240\,eV\,nm}{422.7\,nm} = 2.93\,eV$$

So the Number scattered is

$$\frac{dN}{dt}_{scatt} = \frac{3\,mW}{2.93\,eV}$$

$$= 6.39 \times 10^{15}\ 1/s$$

So since it is spread out uniformly over the whole sphere

$$\Omega = \frac{4\pi r^3}{r^2} = 4\pi$$

$$\frac{dN}{dt\,d\Omega} = 0.51 \times 10^{15}\ 1/s$$

d) The number into the detector per time

$$\frac{\Delta N}{dt} = \frac{dN}{dt\,d\Omega}\ d\Omega$$

$$= 0.51 \times 10^{15}\ \frac{1}{s} \cdot (0.0025\ str)$$

$$\frac{dN}{dt} = 1.27 \times 10^{12}\ \frac{1}{s}$$