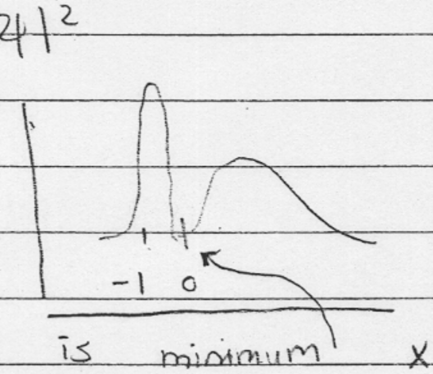
5.2)



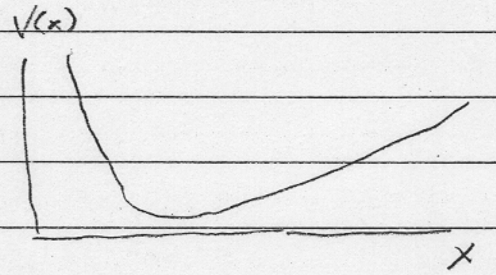
This is the probality distr

a) minus 1 →

b) 0 because $|\psi|^2$ is minimum

c) positive values - because the integral is larger to the right

d)



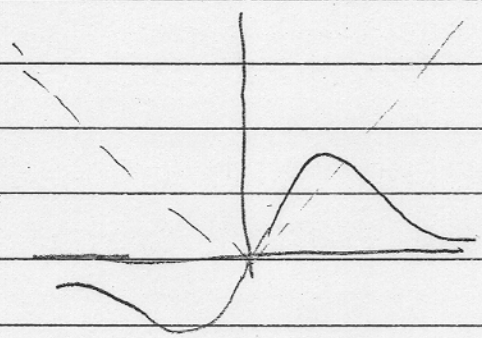
→ This is like a simple harmonic oscillator but stiffer to left than to right

e) The first excited state – the wave fcn has one kink or node

5.22 $V = C|x|$

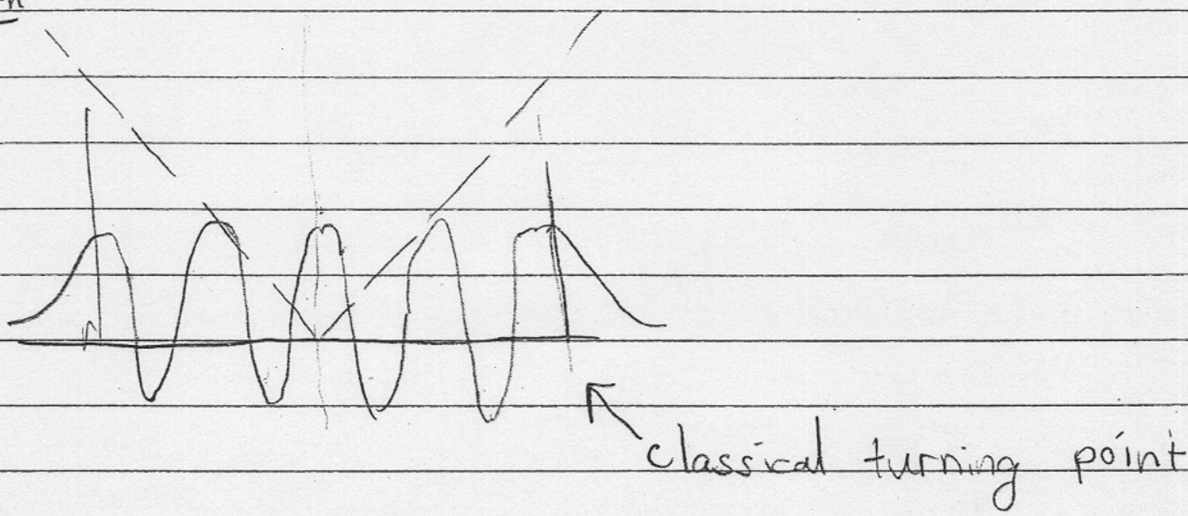
So

a)

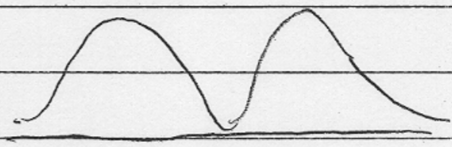


first excited state since it has one kink

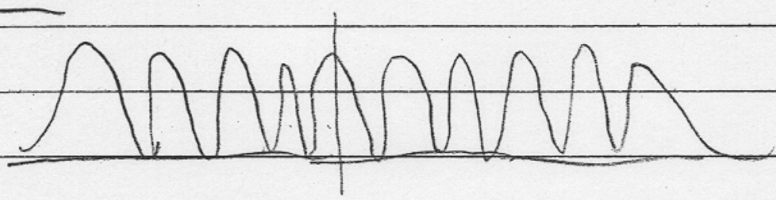
10th



b) 1st



10th



c) My part (c)

When $E = V$ the particle turns around

So $E = C|x_*|$

$x_* = \frac{E}{C}$ ← Thus as

d) My part (d)

Assume size L: Then,

$p \sim \frac{\hbar}{L}$ $K \sim \frac{p^2}{2m} \sim \frac{\hbar^2}{2mL^2}$

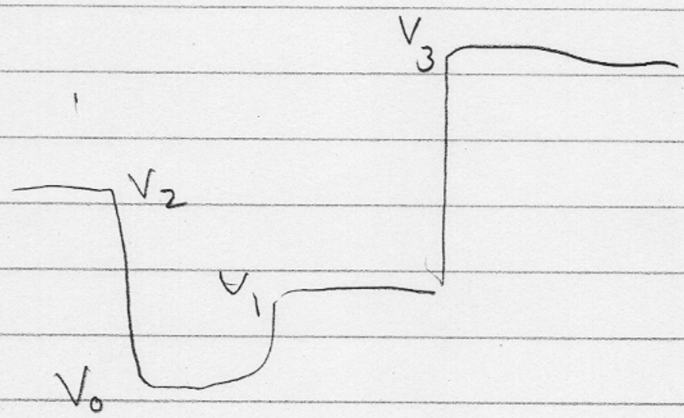
$PE \sim CL$

So the ground state wave fcn has

$$\frac{\hbar^2}{2mL^2} \sim CL$$

$$\left(\frac{\hbar^2}{2mC}\right)^{1/3} \sim L$$

5.23



a) $E < V_0$ no solutions allowed

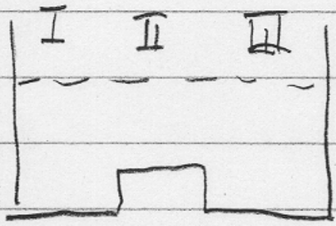
b) $V_0 < E < V_1$ Bound states bouncing back & forth: } discrete

c) $V_1 < E < V_2$ Bound " " back & forth } discrete

d) $V_2 < E < V_3$ continuous, no bouncing back + forth

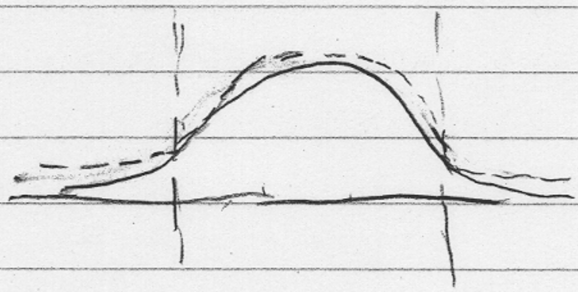
e) $E > V_3$ no bouncing, continuous energies

5.25



The Bump will reduce the curvature of the wave around the bump Since $E - V$ is smaller with bump than without.

Consequently the energy must be slightly highe So that the curvature in I, + III will be larger.



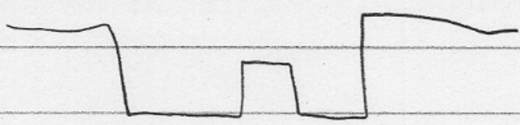
--- modified wave fcn

—— orginal wave fcn

$$\delta E = \overline{\delta V} > 0$$

5.30 This potential

a)



Why? Because around $x = 0$ $\psi < 0$ and $\frac{d^2\psi}{dx^2} < 0$

continued

From Schrödinger

$$\frac{d^2\psi}{dx^2} = \frac{-2m}{\hbar^2}(E-V)\psi$$

So the only way to have $\psi < 0$ and $\frac{d^2\psi}{dx^2} <$

is to have $E-V$ less than zero.

This makes the situation like this

b) The wave function should be symmetric or anti-symmetric:

It should have as few kinks as possible
This leaves two possibilities

Symmetric:

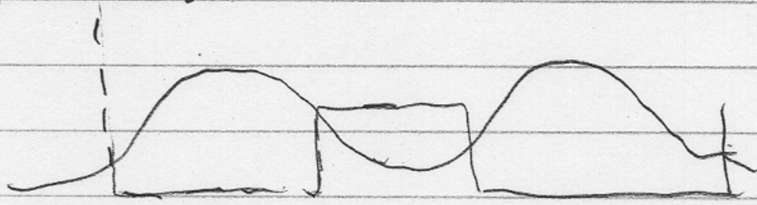


Fig 1

And anti-symmetric:

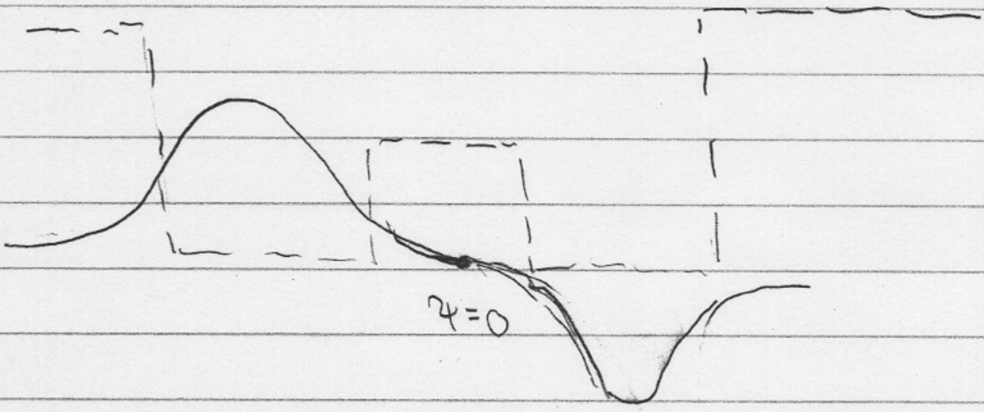


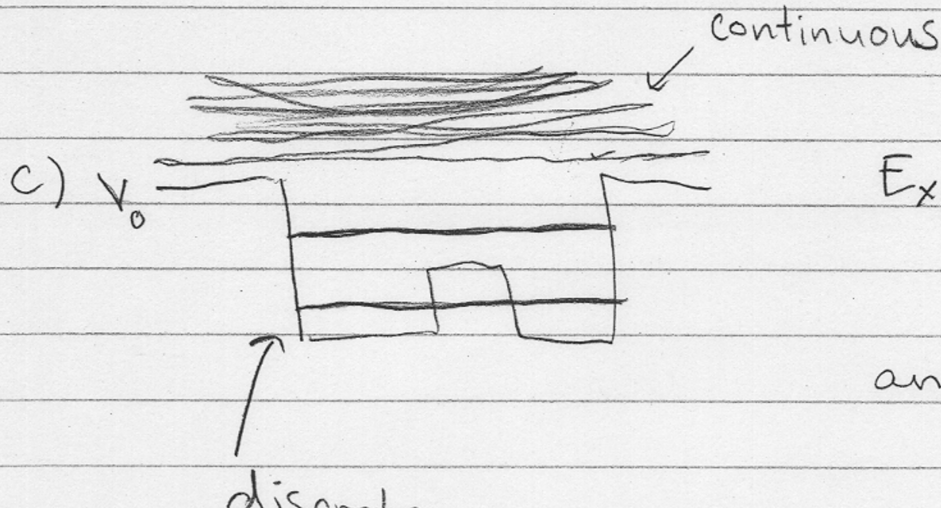
Fig 2

So since it is clear after problem 3 (on homework that

$\overline{\delta V}$ is smaller for anti-symmetric case

So the ground state E_1 is the one in Fig 2

c)

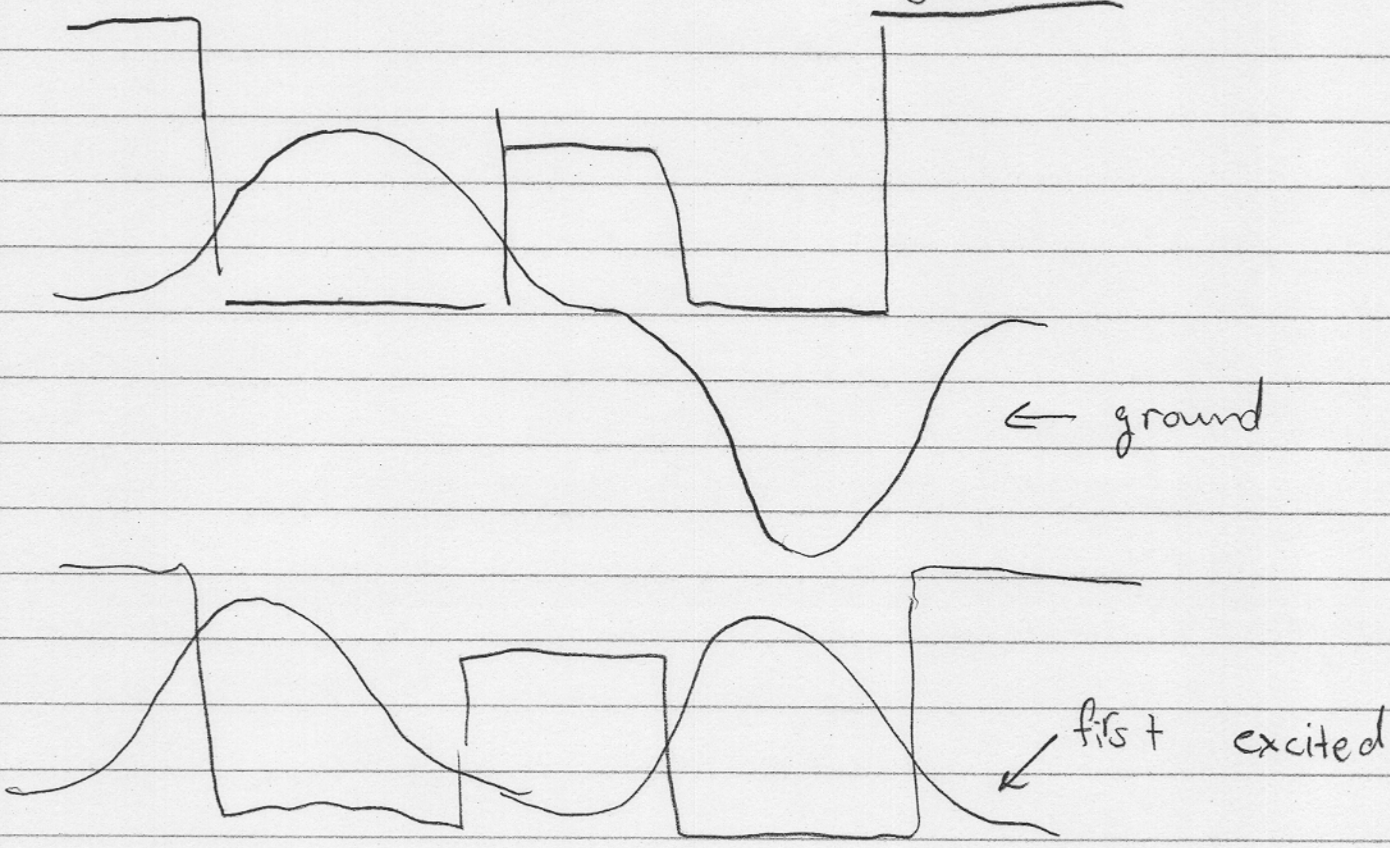


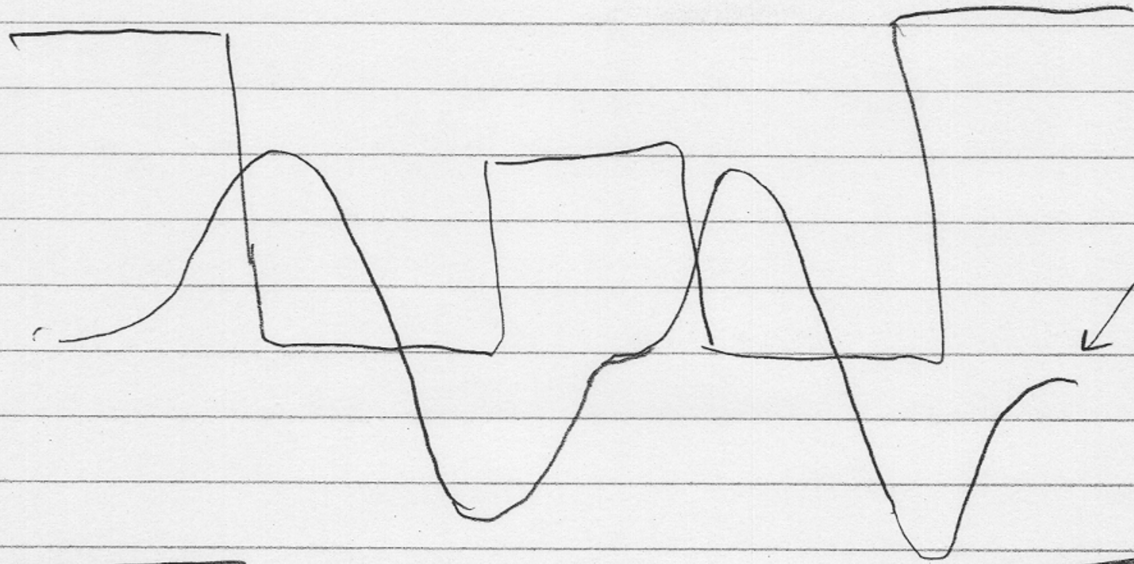
Expects discrete E for $E < V_0$

and continuous for $E > V_0$

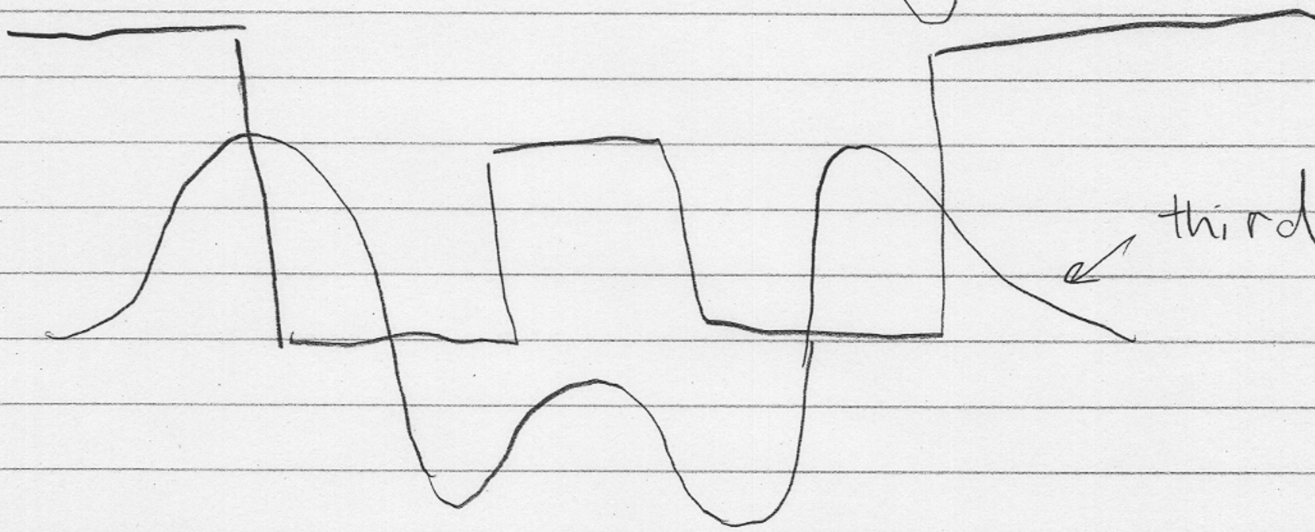
d) The symmetric case (Shown in Fig 1) would have slightly more energy than fig 2 and would be the second excited state

e) This corresponds to the third excited state (or the fourth eigen function).



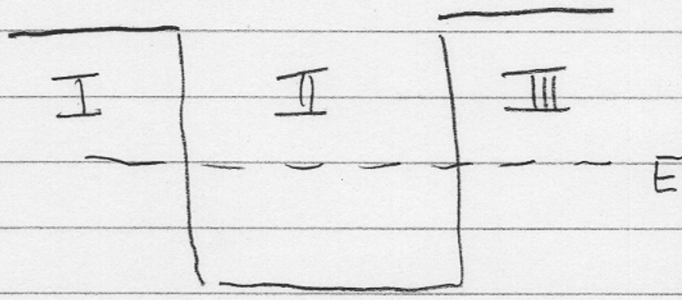


second excited



third

5.27



$$\frac{d^2\psi}{dx^2} = -\frac{2m(E-V)}{\hbar^2}\psi$$

Since:

$$\frac{d^2\psi}{dx^2} = \underbrace{\frac{2m(V-E)}{\hbar^2}}\psi$$

in region III

this is positive $\equiv K^2 = \frac{2m(V-E)}{\hbar^2}$

$$\frac{d^2\psi}{dx^2} = +K^2\psi$$

this, $\psi = Be^{Kx} + Ae^{-Kx}$, is the general solution to this equation.

Requiring, $B = 0$, is needed so $\psi \to 0$ as $x \to +\infty$

So $\psi = Ae^{-Kx}$ $\quad K = \sqrt{\frac{2m(V-E)}{\hbar^2}}$

Problem 5

$$|\psi(x)|^2 = A^2 e^{-2K \cdot x}$$

$$|\psi(a/2)|^2 = A^2 e^{-2K a/2}$$

So

$$|\psi(x)|^2 = |\psi(a/2)|^2 e^{-2K(x-a/2)}$$

So we see that if $x - a/2 = \frac{1}{2K}$

$|\psi(x)|^2$ is $e^{-1} \times |\psi(a/2)|^2$

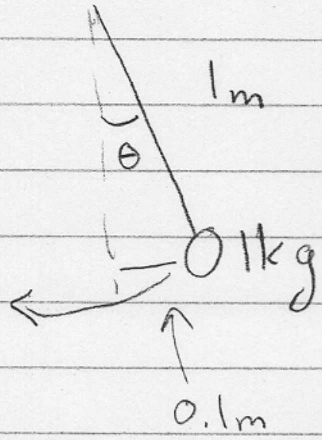
This means

$$D = \frac{1}{2K} = \frac{1}{2}\left(\sqrt{\frac{2m(V-E)}{\hbar^2}}\right)^{-1}$$

$$D = \frac{1}{2}\left(\sqrt{\frac{2mc^2(V-E)}{(\hbar c)^2}}\right)^{-1} = \frac{1}{2}\left(\frac{2(0.5\,MeV)(1eV)}{(1970eV\,Å)^2}\right)^{-1}$$

$$D = 0.97\,Å$$

6.32

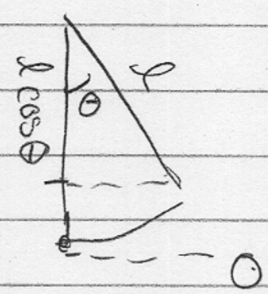


a) $\omega_0 = \sqrt{\frac{g}{l}} = \left(\frac{9.8\,m/s^2}{1m}\right)^{1/2} = 3.13\ 1/s$

b) The amplitude $\Delta x = 0.1m$

$$\sin\theta_{max} \simeq \theta_{max} \simeq \frac{\Delta x}{L} = \frac{0.1m}{1m} = 0.1$$

So $E = PE\Big|_{\text{at } \theta_{max}}$



$$= mgl\,(1 - \cos\theta_m)$$

$$= mgl\left(1 - \left(1 - \frac{\theta_m^2}{2!}\right)\right)$$

$$E = \frac{1}{2} mgl\,\theta_m^2$$

$$= \frac{1}{2} mgl\left(\frac{\Delta x}{l}\right)^2 = \frac{1}{2} mg\Delta x\left(\frac{\Delta x}{l}\right)$$

So

$$\boxed{E = 0.049\,J}$$

Then:

c) $E = (n + \frac{1}{2})\hbar\omega_0$

$$\frac{E}{\hbar\omega_0} = \frac{0.049\,J}{(1\times10^{-34}\,J\cdot s\ \ 3.13\,\frac{1}{s})}$$

$$(n + \tfrac{1}{2}) = 10^{32} \times 1.5$$

$$n \simeq 10^{32} \times 1.5$$

d) $\hbar\omega_0 = 1\times10^{-34}\,J\cdot s\ \ 3.13\,\frac{1}{s}$

$$\hbar\omega_0 \simeq 3\times10^{-34}\,J$$

e) $\lambda = \frac{h}{p} \longrightarrow \frac{p^2}{2m} \simeq K \simeq E$ at bottom

$$p = \sqrt{2mE}$$

So $\frac{\lambda}{2}$ = spacing between bumps in prob dist

Then

$$\text{spacing} = \frac{h}{2\sqrt{2mE}} = \frac{6.6\times10^{-34}\,J\cdot s}{2\times\sqrt{2\times1\,kg\cdot0.049\,J}}$$

$$\boxed{\text{Spacing} \simeq 1.0\times10^{-33}\,m}$$

Problem 2

Since $V(-x) = V(x)$, the prob to be on the left = prob to be on right

we have

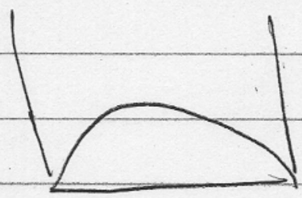
$$|\psi(-x)|^2 dx = |\psi(x)|^2 dx$$

So

$$\boxed{\overline{\psi(x) = \pm\psi(x)}}$$

Prob 3

Near center



$$\psi_n(x) = \sqrt{\frac{2}{a}} \begin{cases} \cos(n\pi x) \\ \sin(n\pi x) \end{cases}$$

for x small

we have

$$\psi_n(x) \simeq \sqrt{\frac{2}{a}} \begin{cases} 1 - \left(\frac{n\pi x}{a}\right)^2/2 \\ \left(\frac{n\pi x}{a}\right) \end{cases}$$

So

$$|\psi|^2 \simeq \begin{cases} \frac{2}{a} \left(1 - \left(\frac{n\pi x}{a}\right)^2/2\right)^2 \\ \frac{2}{a} \left(\frac{n\pi x}{a}\right)^2 \end{cases} \simeq \frac{2}{a} \begin{cases} 1 - \left(\frac{n\pi x}{a}\right)^2 & n \text{ odd} \\ \left(\frac{n\pi x}{a}\right)^2 & n \text{ even} \end{cases}$$

b) Then using

$$\delta E_n = \int_{-\Delta x}^{\Delta x} dx \; \psi_n^*(x) \; V_0 \; \psi_n(x)$$

$$= V_0 \int_{-\Delta x}^{\Delta x} |\psi|^2 \, dx$$

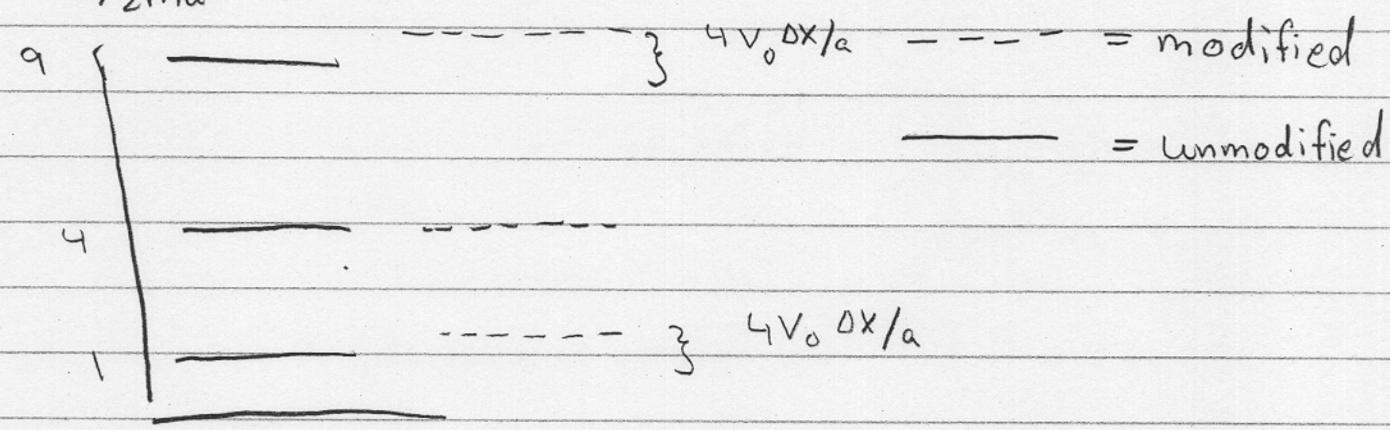
$$= V_0 \int_{-\Delta x}^{\Delta x} dx \begin{cases} \frac{2}{a} \left(1 - \left(\frac{n\pi x}{a}\right)^2\right) \\ \frac{2}{a} \left(\frac{n\pi x}{a}\right)^2 \end{cases}$$

$$= \begin{cases} 4V_0 \frac{\Delta x}{a} - 2V_0 \frac{\Delta x}{a} \left(\frac{n\pi}{a}\right)^2 \frac{2\Delta x^3}{3} \\ \frac{2V_0}{a} \left(\frac{n\pi}{a}\right)^2 \frac{2\Delta x^3}{3} \end{cases}$$

So

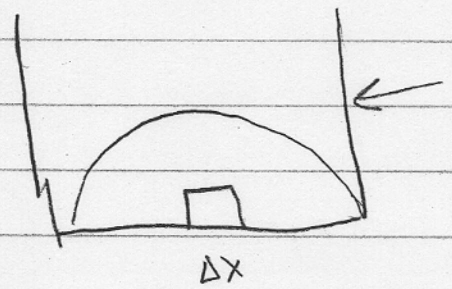
$$\delta E_n = \begin{cases} \dfrac{4V_0\Delta x}{a}\left(1 - \dfrac{1}{3}\left(\dfrac{n\pi\Delta x}{a}\right)^2\right) \\[2ex] \dfrac{4V_0}{3}\dfrac{\Delta x}{a}\left(\dfrac{n\pi\Delta x}{a}\right)^2 \end{cases}$$

units $\hbar^2\pi^2/2ma^2$

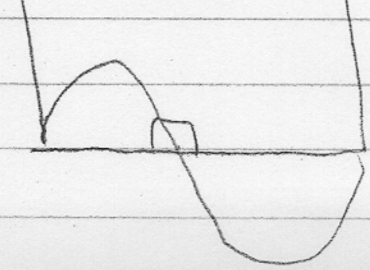


here
even^ means $\psi(-x) = \psi(x)$
not n

c)

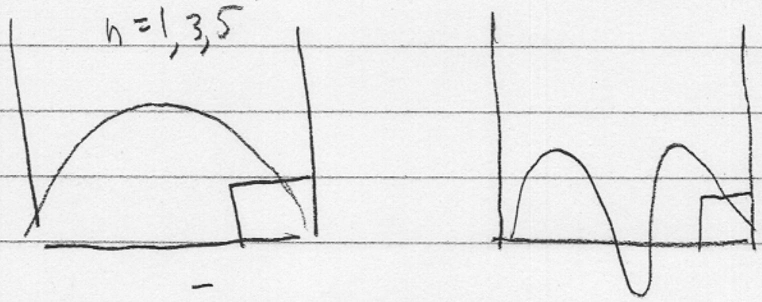


even fcns $n=1,3,5$: The (wave fcn)^2 is largest where the excess potential is maximum

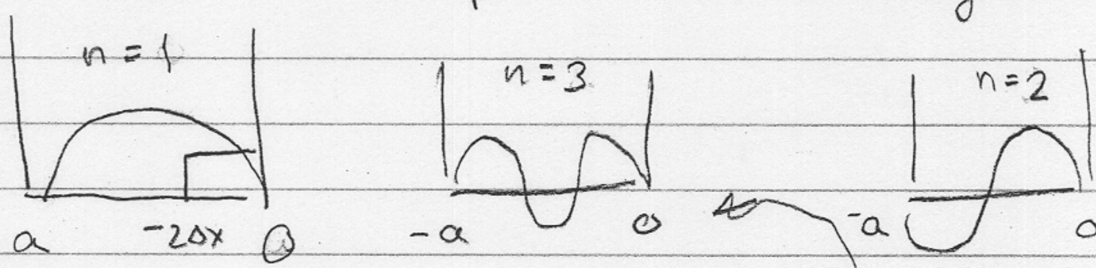


odd fcns $n=2,4,6$: The (wave-fcn)^2 is smallest $\simeq 0$ where the excess potential δV is maximum

If the perturbation is at the edge



Then the even and odd modes are affected the same way, since in both cases $\psi \simeq 0$ near end. To calculate this we shift our origin to put zero at right



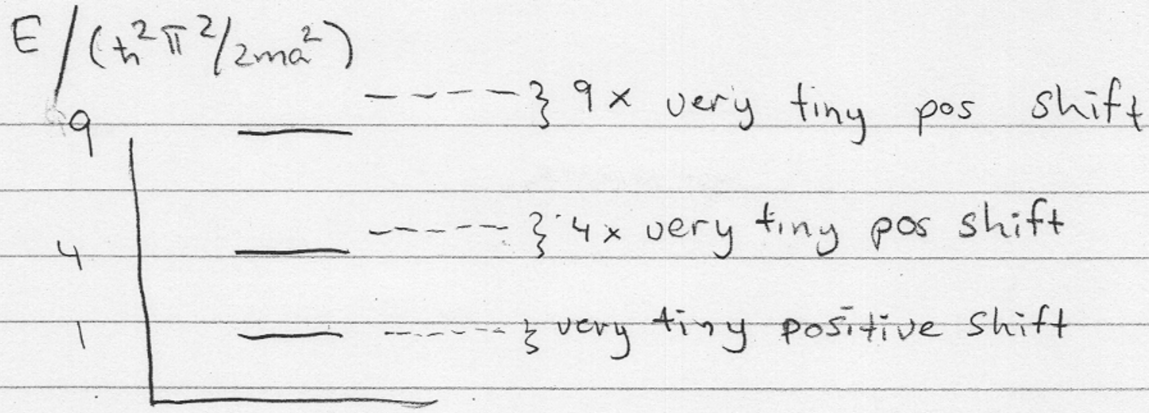
$$\psi_n = \left\{ -\sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \right. \qquad n = 1, 2, 3, 4$$

Near $x = 0$

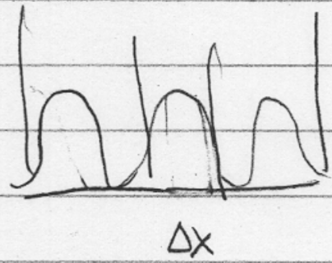
$$\psi_n(x) \simeq -\sqrt{\frac{2}{a}} \, \frac{n\pi x}{a}$$

$$|\psi|^2 \simeq \frac{2}{a}\left(\frac{n\pi x}{a}\right)^2$$

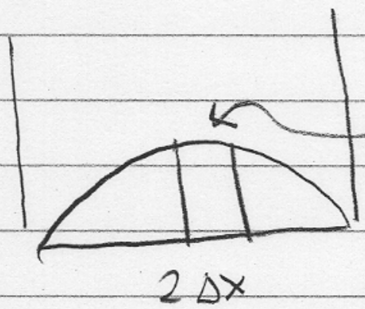
$$\delta E = \int_{-2\Delta x}^{\infty} V_0 \, |\psi_0|^2 = V_0 \int_{-2\Delta x}^{0} \frac{2}{a}\left(\frac{n\pi x}{a}\right)^2 = \frac{2}{3} V_0 \frac{(n\pi)^2 (2\Delta x}{a^3}$$



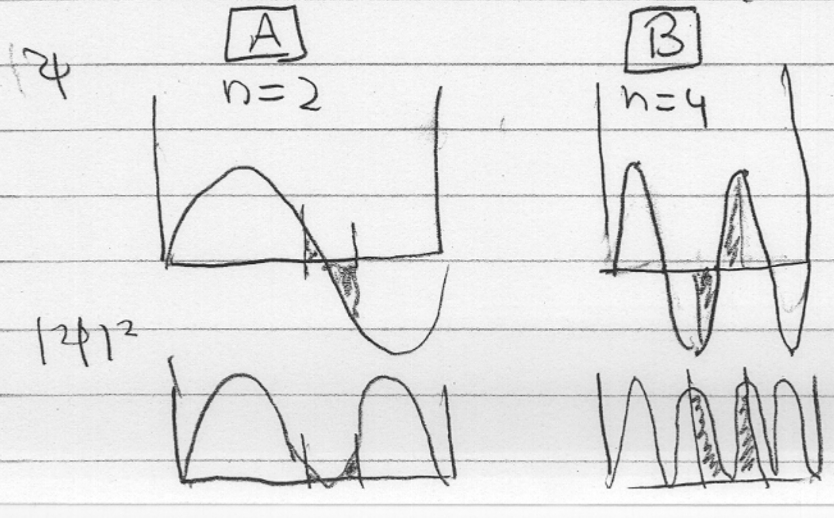
d) The perturbations for odd n decrease with n, because the shorter the wavelength, the less is $|\psi|^2$ on average in the region of δV



i.e. this has somewhat smaller $|\psi|^2$ than this



For odd the situation is reversed:



Case B hase larger overlap with the potential δV than A