

PHY 252 Lab 7: Scattering

Spring 2007

Introduction

The purpose of this laboratory is to show how certain characteristics of an object can be determined from the distribution of particles (or radiation) scattered from the object. In particular, we will use the scattering of a beam of laser light to indirectly measure the radius of several Plexiglas cylinders. The method is similar in principle to that used in particle beam experiments to determine the size and shape of target nuclei.

Apparatus

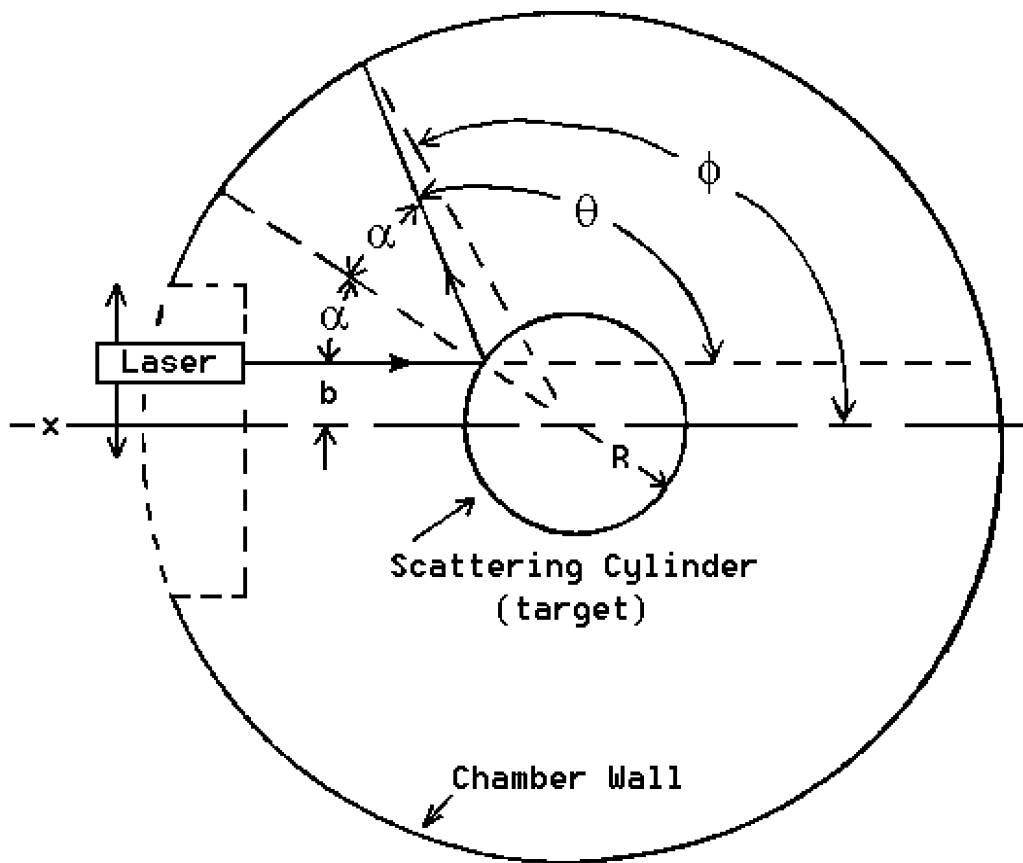


Figure 1. Scattering table, with laser beam and target cylinder.

Measurement

Imagine a well-collimated beam of light striking a polished circular cylinder as shown in the Figure. R is the cylinder radius, and b is the "impact parameter", i.e. the distance between beam and the parallel line passing through the center of the target. After striking the target, the beam is

reflected such that the angle of reflection \mathbf{a} equals the angle of incidence. From geometry it follows that $\sin \mathbf{a} = b/R$. Defining the scattering angle $\mathbf{q} = \mathbf{p} - 2\mathbf{a}$, we see that $\mathbf{a} = (\mathbf{p} - \mathbf{q})/2$. Thus:

$$\sin \mathbf{a} = \sin \frac{\mathbf{p} - \mathbf{q}}{2} = \cos \frac{\mathbf{q}}{2} = \frac{b}{R},$$

so that

$$b = R \cos(\mathbf{q}/2). \quad (1)$$

Suppose b is changed by a small amount db (infinitesimal notation). This will cause a corresponding change $d\mathbf{q}$ in the scattering angle which can be found by differentiating Eq. (1):

$$db = -\frac{R}{2} \sin(\mathbf{q}/2) d\mathbf{q}. \quad (2)$$

If this experiment is performed dN times, while keeping the laser beam aligned so that the impact parameter is between b and $(b + db)$, one will measure dN scattering events in the angular interval between \mathbf{q} and $(\mathbf{q} + d\mathbf{q})$. Thus the *number of scattering events per unit angle* is then

$$\frac{dN}{d\mathbf{q}} = \frac{R}{2} \sin\left(\frac{\mathbf{q}}{2}\right) \frac{dN}{db}, \quad (3)$$

where we used Eq. (2) and dropped the minus sign as we are interested only in the absolute values of the quantities concerned, i.e. the scattering rate. Here dN/db can be regarded as the incident *intensity*. If, for example, one were to perform four scattering measurement for each 0.1 cm step in b , then $dN/db = 4/0.1 = 40$ events per centimeter (units: cm^{-1}). Experimentally, $dN/d\mathbf{q}$ is approximated by marking off the chamber wall in 10° increments, and then counting how many scattering events end up in each angular interval, or “bin” when b is varied. Thus $dN/d\mathbf{q} \approx \Delta N / \Delta \mathbf{q}$, where angle is in radians, as in all equations above.

Analysis

For each of three cylinders, plot $dN/d\mathbf{q}$ on the y -axis and $\sin(\mathbf{q}/2)$ along x . If equation (3) holds, you should find a linear relationship with slope equal to $\frac{1}{2} R(dN/db)$, from which you can determine R . Note that the angular scale on the chamber wall actually measures angle \mathbf{f} , not \mathbf{q} (see Figure). For R small compared to the chamber radius $d\mathbf{q} \approx d\mathbf{f}$, but otherwise you will have to calculate \mathbf{q} from \mathbf{f} (such a derivation will need knowledge of both R and the chamber radius r . This may seem like cheating, but it can be done *iteratively*. Also, in scattering experiments the nuclear diameter is negligibly small compared to the size of the detector.

For each cylinder compare thus measured value of R (with its calculated errors!) with the actual radius of the cylinder.