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ESS 079

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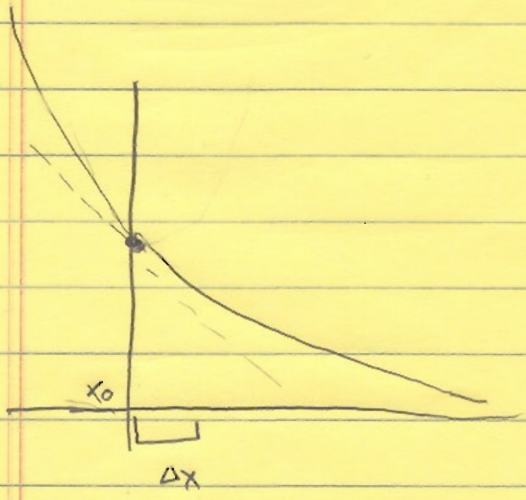
Functions



① $f(x) \simeq f(x_0)$ $\qquad \Delta x \simeq x - x_0$

② $f(x) \simeq f(x_0) + (\text{Slope})\,\Delta x$

$f \simeq f(x_0) + f'(x_0)\,\Delta x$

③ $f = f(x_0) + f'(x_0)\,\Delta x + f''(x_0)\dfrac{(\Delta x)^2}{2!} + \ldots$

Answer

$$f(x) = f(x_0) + f'(x_0)\,\Delta x + f''(x_0)\frac{(\Delta x)^2}{2!} + f'''(x_0)\frac{(\Delta x)^3}{3!}$$

Example

$f(x) = (1+x)^{\alpha}$ for x very small x

① $f(0) = 1$

② $f'(x) = \alpha(1+x)^{\alpha-1} \Rightarrow f'(0) = \alpha$

$f(x) \simeq 1 + \alpha x$

③ $f''(x) = \alpha(\alpha-1)(1+x)^{\alpha-2} \Rightarrow f''(0) = \alpha(\alpha-1)$

$$f(x) \simeq 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \dots$$

Example:

$$\gamma = \frac{1}{\sqrt{1-(V/c)^2}} = \left[1-(V/c)^2\right]^{-1/2}$$ take $V \ll c$

$x = -\left(\frac{V}{c}\right)^2 \ll 1$ $\qquad \alpha = -\frac{1}{2}$

$$\gamma \simeq 1 \overbrace{-\frac{1}{2}}^{\alpha} \overbrace{\left(-\left(\frac{V}{c}\right)^2\right)}^{x} + \overbrace{\frac{-\frac{1}{2}\left(-\frac{3}{2}\right)}{2!}}^{\alpha(\alpha-1)} \overbrace{\left(\left(\frac{-V}{c}\right)^2\right)^2}^{x^2}$$

$$\gamma \simeq 1 + \frac{1}{2}\left(\frac{V}{c}\right)^2 + \frac{3}{8}\left(\frac{V}{c}\right)^4$$

Find Power Series

$$f = \frac{1}{e^{p/T} - 1} \qquad \text{for } \frac{p}{T} \ll 1$$

$x = p/T$

To linear Order

$$e^{p/T} \simeq 1 + p/T$$

$$f \simeq \frac{1}{1 + p/T - 1} \simeq \frac{1}{\frac{p}{T}} \simeq \frac{1}{x} \leftarrow \text{large}$$

Quadratic

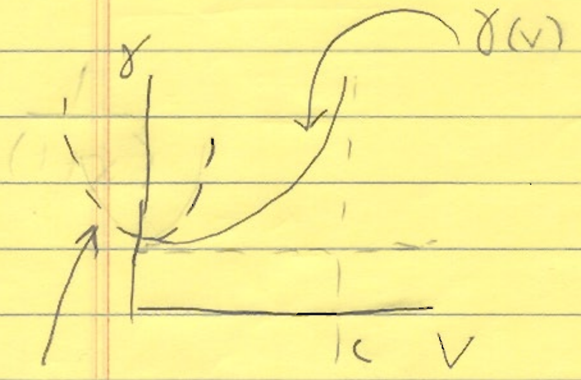
$$e^{p/T} \simeq 1 + p/T + \frac{(p/T)^2}{2!} + O\left(p/T\right)^3$$

$$\frac{1}{\left(1 + \frac{p}{T} + \frac{(p/T)^2}{2!} - 1\right)} \simeq \frac{1}{\frac{p}{T} + \frac{1}{2}\left(\frac{p}{T}\right)^2} = \frac{1}{\left(\frac{p}{T}\right)} \frac{1}{\left(1 + \frac{1}{2}\frac{p}{T}\right)}$$

$$\simeq \frac{1}{\frac{p}{T}}\left(1 - \frac{1}{2}\frac{p}{T}\right)$$

$$\simeq \frac{1}{p/T} - \frac{1}{2}$$

Work out



$$\gamma \simeq 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 + O\left(\left(\frac{v}{c}\right)^4\right)$$

Now look at:

$$\sqrt{1 - v/c} \simeq 1 - \frac{1}{2}\left(\frac{v}{c}\right) + O\left(\left(\frac{v}{c}\right)^2\right)$$

$$\frac{1}{\sqrt{1+\frac{v}{c}}} \simeq 1 - \frac{1}{2}\left(\frac{v}{c}\right) + O\left(\frac{v}{c}\right)^2$$

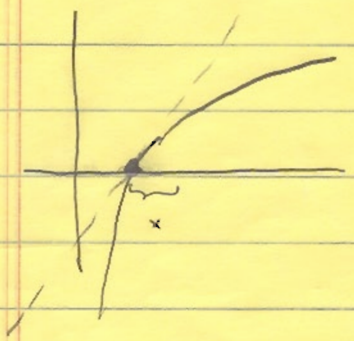
$$\sqrt{\frac{1 - v/c}{1 + v/c}} \simeq \left(1 - \frac{1}{2}\frac{v}{c}\right)\left(1 - \frac{1}{2}\frac{v}{c}\right)$$

$$\simeq 1 - \frac{v}{c} + O\left(\left(\frac{v}{c}\right)^2\right)$$

Integration:

$$\frac{1}{1+x} \simeq 1 - x + x^2 + O(x^3)$$

$$\int_0^x \frac{dx'}{1+x'} = \log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + O(x^4)$$



Can integrate power series

More complicated Situations:

$$f(p) \simeq \frac{1}{e^{p/T} - 1} \qquad p \gg T$$

Exponentials:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$ (Class problem)

Find Power Series

$$f = \frac{1}{e^{p/T} - 1} \qquad \text{for } \frac{p}{T} \ll 1$$

$x = p/T$

To linear Order

$$e^{p/T} \simeq 1 + p/T$$

$$f \simeq \frac{1}{1 + p/T - 1} \simeq \frac{1}{\frac{p}{T}} \simeq \frac{1}{x} \leftarrow \text{large}$$

Quadratic

$$e^{p/T} \simeq 1 + p/T + \frac{(p/T)^2}{2!} + O\left(p/T\right)^3$$

$$\frac{1}{\left(1 + \frac{p}{T} + \frac{(p/T)^2}{2!} - 1\right)} \simeq \frac{1}{\frac{p}{T} + \frac{1}{2}\left(\frac{p}{T}\right)^2} = \frac{1}{\left(\frac{p}{T}\right)} \frac{1}{\left(1 + \frac{1}{2}\frac{p}{T}\right)}$$

$$\simeq \frac{1}{\frac{p}{T}}\left(1 - \frac{1}{2}\frac{p}{T}\right)$$

$$\simeq \frac{1}{p/T} - \frac{1}{2}$$

Mastery

★ $\cos(\pi e^{-P/T})$ $\qquad \frac{P}{T} \ll 1$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$\cos(\pi + x) \simeq -1 + \frac{x^2}{2} - \frac{x^4}{4!}$$

$$\cos\left(\pi\left(1 - \frac{P}{T}\right)\right) \simeq \cos\left(\pi - \frac{\pi P}{T}\right)$$

$$\simeq -1 + \frac{1}{2}\left(-\frac{\pi P}{T}\right)^2$$