Quantum Harmonic Oscillator: Wavefunctions

The Schrödinger equation for a harmonic oscillator may be solved to give the wavefunctions illustrated below.

For the simple harmonic oscillator (the spring) the potential is

\[ V = \frac{1}{2} k x^2 \]  

(1)

and the classical oscillation frequency is

\[ \omega_c = \sqrt{\frac{k}{m}} \quad \omega_o = 2\pi f \]  

(2)

We used the uncertainty principle to estimate that the particle at the bottom of the well oscillates over a length scale

\[ L = \left( \frac{k^2}{mk} \right)^{1/4} \]  

(3)

The lowest energies are

\[ E_n = \left( \frac{1}{2} + n \right) \hbar \omega_o \quad n = 0, 1, 2, 3 \ldots \]  

(4)

The lowest wave functions are

\[ \Psi_0 = \left( \frac{1}{\sqrt{\pi L}} \right)^{1/2} e^{-y^2/2} \]  

(5)

\[ \Psi_1 = \left( \frac{1}{\sqrt{\pi L}} \right)^{1/2} \sqrt{2yc} e^{-y^2/2} \]  

(6)

\[ \Psi_2 = \left( \frac{1}{\sqrt{\pi L}} \right)^{1/2} \frac{1}{\sqrt{2}} (2y^2 - 1)c e^{-y^2/2} \]  

(7)

where

\[ y = \frac{x}{L} \]  

(8)