Quantity	**Symbol**	**Value**
Coulombs Constant	$\frac{1}{4\pi\epsilon_o}$	$8.98 \times 10^9\,\mathrm{Nm^2/C^2}$
Electron Mass	m_e	$9.1 \times 10^{-31}\,\mathrm{kg}$
Proton Mass	m_p	$1.67 \times 10^{-27}\mathrm{kg}$
Electron Charge	e	$-1.6 \times 10^{-19}\,\mathrm{C}$
Electron Volt	eV	$1.6 \times 10^{-19}\,\mathrm{J}$
Permitivity	ϵ_o	$8.85 \times 10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{Nm^2}}$
Magnetic Permeability	μ_o	$4\pi \times 10^{-7}\,\mathrm{N \cdot A^2}$
Speed of Light	c	$3.0 \times 10^8\,\mathrm{m/s}$
Planck's Constant	h	$6.6 \times 10^{-34}\mathrm{m^2kg/s}$
Planck's Constant/2π	$\hbar$	$1.05 \times 10^{-34}\mathrm{m^2kg/s}$

Integrals	**Value**
$\int_{-\infty}^{\infty} du\, e^{-\alpha u^2}$	$\sqrt{\frac{\pi}{\alpha}}$
$\int_{-\infty}^{\infty} du\, u^2 e^{-\alpha u^2}$	$\frac{1}{2\alpha}\sqrt{\frac{\pi}{\alpha}}$
$\int_0^{\infty} du\, u^n e^{-\alpha u}$	$\frac{n!}{\alpha^{n+1}}$
$\int du\, \sin^2(\alpha u)$	$\frac{u}{2} - \frac{\sin(2\alpha u)}{4\alpha}$
$\int du\, \cos^2(\alpha u)$	$\frac{u}{2} + \frac{\sin(2\alpha u)}{4\alpha}$
$\int_{-\frac{1}{2}}^{+\frac{1}{2}} duu^2 \sin^2(n\pi u)$	$\frac{-6+n^2\pi^2}{24n^2\pi^2}$ $\quad n = 2,4,6,8$
$\int_{-\frac{1}{2}}^{+\frac{1}{2}} duu^2 \cos^2(n\pi u)$	$\frac{-6+n^2\pi^2}{24n^2\pi^2}$ $\quad n = 1,3,5,7$
$\int (\cos(\theta))^{\alpha} \sin(\theta) d\theta$	$\frac{-1}{\alpha+1} (\cos(\theta))^{\alpha+1}$
$\int (\sin(\theta))^{\alpha} \cos(\theta) d\theta$	$\frac{+1}{\alpha+1} (\sin(\theta))^{\alpha+1}$

n	ℓ	m	$\Phi_m(\varphi)$	$\Theta_{lm}(\theta)$	$R_{nl}(r)$	Ψ_{nlm}
1	0	0	1	1	$\frac{1}{\sqrt{\pi a_o^3}} e^{-r/a_o}$	$\frac{1}{\sqrt{\pi a_o^3}} e^{-r/a_o}$
2	0	0	1	1	$\frac{1}{\sqrt{32\pi a_o^3}} \left(2 - \frac{r}{a_o}\right) e^{-r/2a_o}$	$\frac{1}{\sqrt{32\pi a_o^3}} \left(2 - \frac{r}{a_o}\right) e^{-r/2a_o}$
2	1	0	1	$\sqrt{3}\cos(\theta)$	$\frac{1}{\sqrt{96\pi a_o^3}} \frac{r}{a_o} e^{-r/2a_o}$	$\frac{1}{\sqrt{32\pi a_o^3}} \frac{r}{a_o} e^{-r/2a_o} \cos(\theta)$
2	1	± 1	$e^{\pm i\varphi}$	$\sqrt{\frac{3}{2}}\sin(\theta)$	$\frac{1}{\sqrt{96\pi a_o^3}} \frac{r}{a_o} e^{-r/2a_o}$	$\frac{1}{\sqrt{64\pi a_o^3}} \frac{r}{a_o} e^{-r/2a_o} \sin(\theta) e^{\pm i\varphi}$

For $n = 3$ we will quote the R_{nl} only. For the necessary table of Θ_{lm} and Φ_m see Eq. 3. Here

$$\rho \equiv \frac{2r}{3a_o} \tag{1}$$

n	ℓ	$R_{n\ell}$
3	0	$\sqrt{\frac{40}{(4\pi)6!}} \left(\frac{2}{3a_o}\right)^{3/2} \frac{1}{2}\left(6 - 6\rho + \rho^2\right) e^{-\rho/2}$
3	1	$\sqrt{\frac{5}{(4\pi)6!}} \left(\frac{2}{3a_o}\right)^{3/2} (4 - \rho)\ \rho\, e^{-\rho/2}$
3	2	$\sqrt{\frac{1}{(4\pi)6!}} \left(\frac{2}{3a_o}\right)^{3/2} \rho^2\, e^{-\rho/2}$

(2)

and

ℓ	m	$\Phi_m(\phi)$	$\Theta_{lm}(\theta)$
0	0	1	1
1	0	1	$\sqrt{3}\cos(\theta)$
1	± 1	$e^{\pm i\phi}$	$\sqrt{\frac{3}{2}}\sin(\theta)$
2	0	1	$\frac{\sqrt{5}}{4}(3\cos(2\theta) + 1)$
2	± 1	$e^{\pm i\phi}$	$\sqrt{\frac{15}{2}}\sin(\theta)\cos(\theta)$
2	± 2	$e^{\pm i2\phi}$	$\frac{1}{2}\sqrt{\frac{15}{2}}\sin^2(\theta)$

(3)

For potential $V = \frac{1}{2} k x^2$ the lowest wave functions

$$\begin{aligned}
\Psi_0 &= \left(\frac{1}{\sqrt{\pi}L}\right)^{1/2} e^{-y^2/2} \\
\Psi_1 &= \left(\frac{1}{\sqrt{\pi}L}\right)^{1/2} \sqrt{2} y e^{-y^2/2} \\
\Psi_2 &= \left(\frac{1}{\sqrt{\pi}L}\right)^{1/2} \frac{1}{\sqrt{2}}(2y^2 - 1) e^{-y^2/2}
\end{aligned}$$

where

$$y \equiv \frac{x}{L} \qquad L = \sqrt{\frac{\hbar}{M\omega_o}} \tag{4}$$