One of

Newton's Great idea

• The moon is \textit{falling} around the earth

\[ \Delta x = v_0 t \]
\[ \Delta y = \frac{1}{2} a_0 t^2 \]

• See Slides

• If this picture is going to work, then the amount you move forward

\[ \Delta x = V \Delta t \]

and the amount you fall

\[ \Delta y = \frac{1}{2} a_0 t^2 \]

must be related to the curvature of the circular arc -- see slides

\[ a = \frac{V^2}{R} \]

\[ a \rightarrow \text{acceleration of moon falling around the earth} \]

And the object's speed \( V_m \)
From this analysis

\[ a_m = \frac{v_m^2}{R_{Em}} = \left( \frac{2\pi R_{Em}}{27 \text{ days}} \right)^2 \]

\[ a_m = \frac{1}{g} \frac{1}{3700} \text{ acceleration due on earth} \]

Newton anticipates that whatever "force" causes the acceleration of the moon must decrease like \( \frac{1}{r^2} \)

\[ \left( \frac{R_{Em}}{R_E} \right)^2 = \frac{1}{3600} \]

\[ \text{smaller accel} \]

\[ \text{larger accel} \]
Newton's Laws

1. Read Law

- A body in motion remains in motion (and projectiles)

2. Newton's Second Law

\[ F = ma \Rightarrow a = \frac{F}{m} \]

- A force causes acceleration in the direction in which it acts (again projectiles).
- The acceleration is inversely proportional to mass.

3. Newton's Third Law

- For every action there is an equal and opposite reaction (force

\[ \Rightarrow \text{Often subtle to use in practice} \]
Gravitation

- Force Decreases like $\frac{1}{r^2}$

- $F = ma$, but $a = \frac{F}{m}$ on earth is independent of mass (Galileo)

So consistency says that $F \propto m$

Thus, $F_{\text{Earth}} \propto \frac{m_{\text{book}}}{r^2}$

- However, if the earth pulls on the book, book pulls on earth

$F_{\text{Earth}} \propto \frac{m_{\text{book}}}{r^2}$

$F_{\text{Book}} \propto \frac{m_{\text{Earth}}}{r^2}$

- The const is the Gravitational const $= G$

\[
F = G \frac{m_{\text{Earth}} m_{\text{Book}}}{r^2}
\]

$G = 6.6 \times 10^{-11} \, \text{m}^3/\text{kg} \cdot \text{s}^{-2}$
Kepler and the third law of Newton:

\[ \frac{1}{2} a d y^2 \, \frac{G m_0 m_p}{r^2} = m_p a \]

\[ G \frac{m_0 m_p}{R^2} = \frac{m_p R^2}{r^2} \]

Now \( v = \frac{2 \pi R}{T} \)

So plugging in \( v \) and a bit of algebra:

\[ G \frac{m_0 m_p}{R^2} = m_p \left( \frac{2 \pi R}{T^2} \right) R \]

\[ \rightarrow \text{ algebra} \]

\[ \left( \frac{G m_0}{4 \pi^2} \right) T^2 = R^3 \]

Relation between period \( T \) and radius \( R \). \( M_0 \) is the mass of the Sun.

If \[ F \propto \frac{m_1 m_2}{R^{2.5}}, \] then find \( T \propto R^{3.5} \).
Thus Kepler's laws give strong constraints on the motion of the planets.
List all the sorts of corrections you can think of that change Kepler's simple rules.