All objects with mass have a Schwarzschild Radius, its just that the object is usually much bigger than $R_{sch}$.

For the sun $G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2$

$$R_{sch \text{ for sun}} = \frac{2GM}{c^2}$$

$G = 2 \times 10^{30} \text{ kg m}^3/\text{kg s}^2$

$c = 3 \times 10^8 \text{ m/s}$

$$R_{sch} = 3 \text{ km}$$

Compare this to the radius of Sun $R_O \approx 700,000 \text{ km}$
Forming Black Holes

Now how are black holes formed?
Need to compress the mass into a very small point.

Star

\[ \text{pressure due to nuclear burning} \]

If the mass is sufficiently large, then as the nuclear fuel runs out, the star will undergo gravitational collapse, driving all of the material into a small radius.
The orbital radius of these stars is $R \sim 1000 \text{AU} \sim 0.01 \text{ light years}$

We will look at the elliptic orbits of a few stars close to Sagittarius A* (in our galaxy) contain a supermassive black hole.
• Look at the change in the image as a function of time (years) and look in different frequency bands (x-rays)

Sagittarius A* (in our galaxy) contains a supermassive black hole
Note: 2 right days = 350 AU

From the orbital period and radius of the star, use Kepler's laws to find the mass of the SgrA*.

Orbits of six stars around the Sagittarius A* in the center of our galaxy.
Sagittarius A*

- The period of orbit of S2 around Sagittarius A* is 15.2 years, the eccentricity of the elliptical orbit is $e = 0.88$. The average radius (the semi-major axis) is $980 \pm 20$ AU

→ Calculate the mass of Sagittarius A*, calculate the distance of closest approach of S2 to the center of mass (Sgr A*,

→ Calculate the Schwarzschild Radius of Sagitarius A
Sagittarius A* 

- The period of orbit of S2 around Sagittarius A* is 15.2 years, the eccentricity of the elliptical orbit is $e = 0.88$, the average radius (the semi-major axis) is $980 \pm 20$ AU.

\[ \text{Calculate the mass of Sagittarius A*; calculate the distance of closest approach of S2 to the center of mass (SGr A*).} \]

\[ \text{Calculate the Schwarzschild radius of Sagittarius A.} \]
Solution

\[ \frac{GM}{(2\pi)^2} \frac{T^2}{a^3} \]

\[ M = \frac{(2\pi)^2}{G} \frac{a^3}{T^2} \]

\[ a = 980 \text{ AU} = 1.4 \times 10^{14} \text{ m} \]

\[ T = 15.2 \text{ y} = 3.1 \times 10^7 \text{ s} \]

\[ G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2 \]

Find

\[ M = \frac{8.2}{\text{AU}} \times 10^{36} \text{ kg} \]

Note

\[ M_0 = \text{mass of sun} = 2 \times 10^{30} \text{ kg} \]

Or

\[ \frac{M}{M_0} = 4.1 \times 10^6 \]

i.e. the mass of SgrA* is 4.1 million solar masses.
So 

\[
\begin{align*}
R_p & = a (1 - e) = 980 \text{ AU (0.12)} \\
\text{Compare} & \\
R_{sch} & = \frac{2 G m}{c^2} = 0.08 \text{ AU}
\end{align*}
\]

\[R_{sch} \approx 0.08 \text{ AU}\]

**Summary:** there is a very massive object with a radius \( R < 117.6 \text{ AU} \).

The Schwarzschild Radius is 0.08 AU.

Picture:
Description of Gravity in Terms of Curvature

\[ \Delta y = \frac{1}{2} gt^2 \]

So instead of describing the motion of objects with the \( t, y \) coordinate system, we can use the \( t', y' \) coordinate's
\[ t' = t \]
\[ y' = y - \frac{1}{2} gt^2 \]
This is the coordinate system of the free falling observer.
In this coordinate system the apple has no forces and its motion is fixed
\[ m a' = 0 \Rightarrow y' = \text{constant} = L \]

If I know the free fall coords, I know effect of gravity.

Now recognize that we must choose a different coordinate system for every radius.

\[ t' = t \]
\[ y' = y - \frac{1}{2} g'(R) t^2 \]

\[ g'(r) = \frac{G M_\odot}{R^2} \]
(see slide)

So we can phrase the problem of finding the forces of gravity as finding how the coordinates of free falling observers change in space and time.
Change from point to point

problem of finding how the coordinates of free-falling observers
can view the problem of determining the forces of gravity, as a

Large Curvature
High Gravity

Small Curvature
Low Gravity

radius
Newton's Law

determines the coordinates of free fall observers

\[ a(r) = \frac{GM}{r^2} \]

acceleration caused by mass

Einstein:

\[ G_{\mu\nu} = 8\pi G T_{\mu\nu} \]

"Curvature" of free falling observers

how the local coordinates of free falling observers are changing from point to point