I. PARALLAX

A. Cassini and Parallax (1672)

- Cassini set out to measure the distance of the Solar system. Recall that all of the ratios of distances between the planets were known. Thus, once *one* distance was known the others could be determined. The closest planet is Mars and Cassini set out to measure this in 1672

- The method is based on parallax. To understand parallax put out your finger $L = 2\,\mathrm{ft}$ from your nose. You can determine L from angle measurements and the distance between your eyes d

  - Look at your finger with one eye closed and then the other. The angle of your finger changes as you close one eye and then the other. See the figure below.
  - With your the left eye you see your finger at a clockwise (to right) angle angle of

    $$+\theta \simeq \frac{d/2}{L} \,.$$

    For the right eye you see your finger in a counter clocwise (to left) direction with angle

    $$\theta \simeq -\frac{d/2}{L}$$

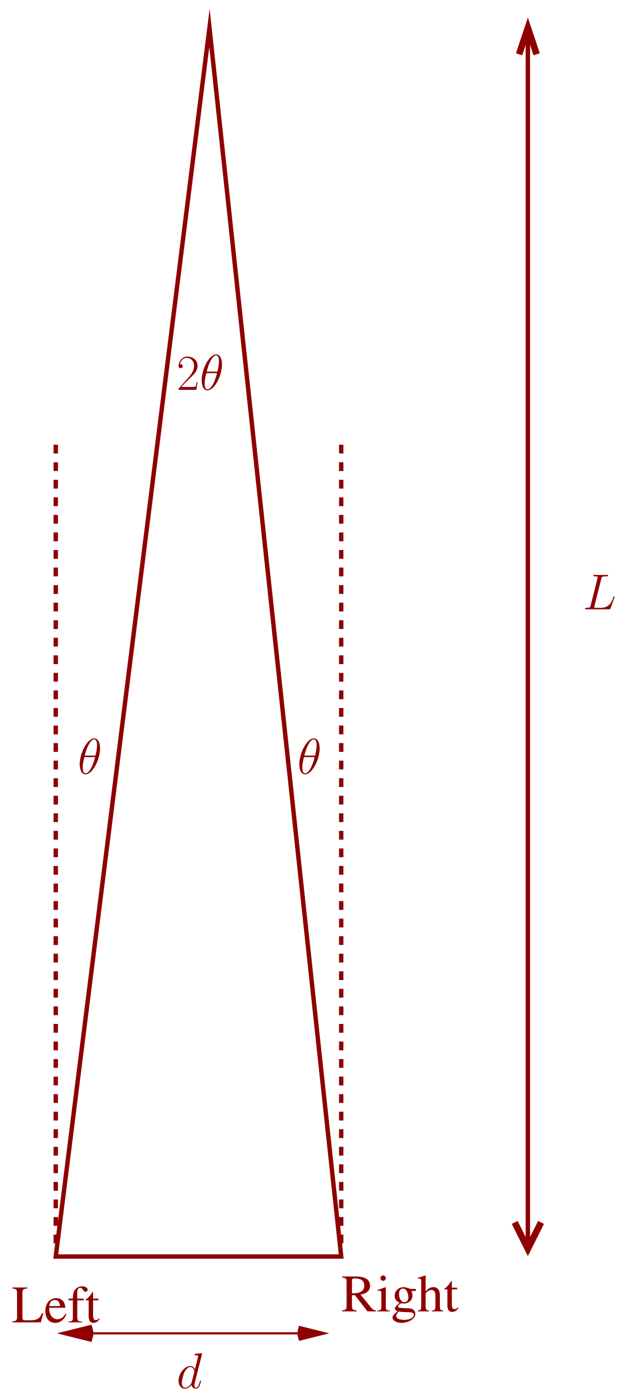
  - Then the sum of the the two measurements is

    $$2\theta \simeq \frac{d}{L}$$

    and thus we find

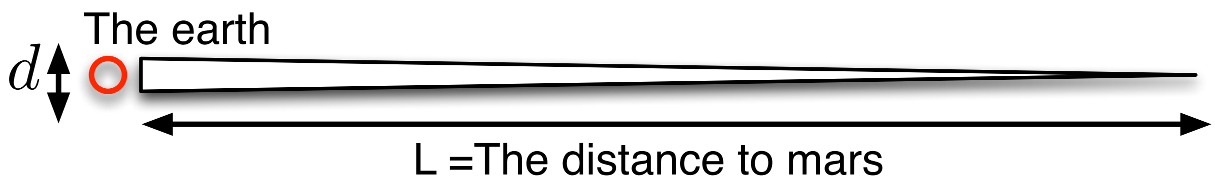
    $$L \simeq \frac{d}{2\theta}$$

    Thus by measuring angles and knowing the distance between your eyes you can determine the distance.



2θ
θ
θ
L
Left
Right
d

- Now Cassini used the same method to measure the distance to Mars by using the two "Eyes" on the earth.

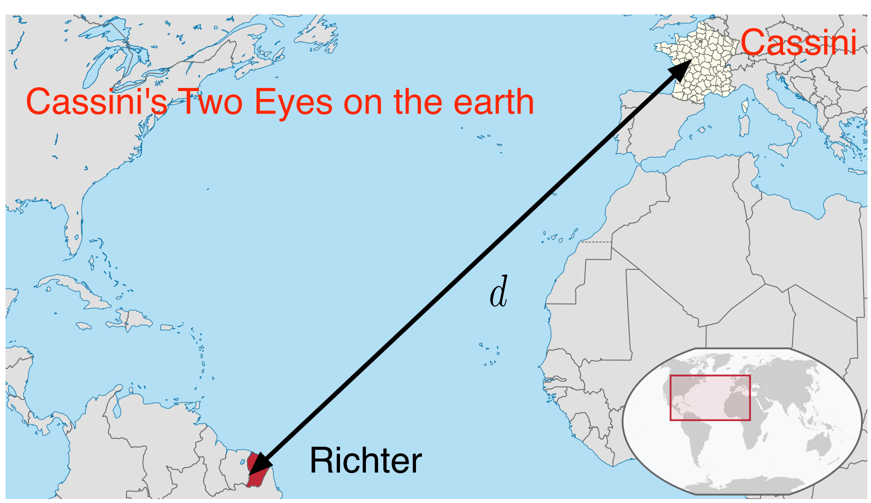


This is not that easy because in the picure above the earth is about *200 times* too large. The actual angle is

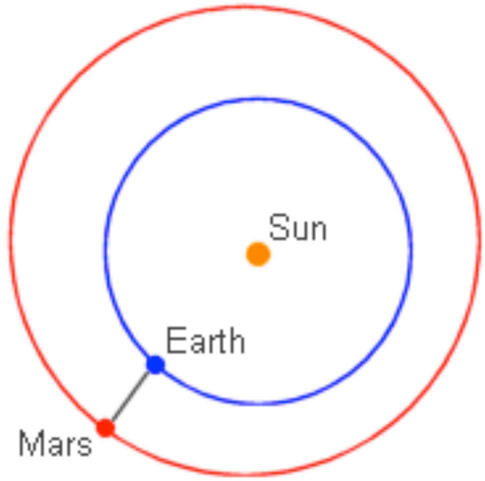
$$2\theta = \frac{d}{L} \sim \frac{12000, \mathrm{km}}{5.4\,10^7\mathrm{km}} \sim 20\,\mathrm{arcsec} \tag{1.1}$$

- Then, in order to get a reasonable measurement we need d as large as possible and L as small as possible

  – To make d as large as possible, Cassini (Richter) sent his assistant to the other end of the earth (French Guiana). (see slide)



  – To make L as small as possible, Cassini waited until Mars was in opposition with the earth as shown below. This happens once every $\sim$ three years



Mars at Opposition

  - In addition, the time at which mars is measured must be precisely synched between Paris and Guiana. This is because the earth is spinning and the angle is changing throughout the night. While it may not look with your naked eye that the planets are moving, with a strong telescope this motion becomes increasingly rapid. Cassini and Richter used the orbits of Jupiters moons to sync the clocks in Paris and Guiana.

- After Richter returned Paris, the two men compared notes and concluded determined the distance to mars and, from this the distance to the sun:

$$1\,\mathrm{AU} = 21,700 \, \text{in units of earth radii}$$

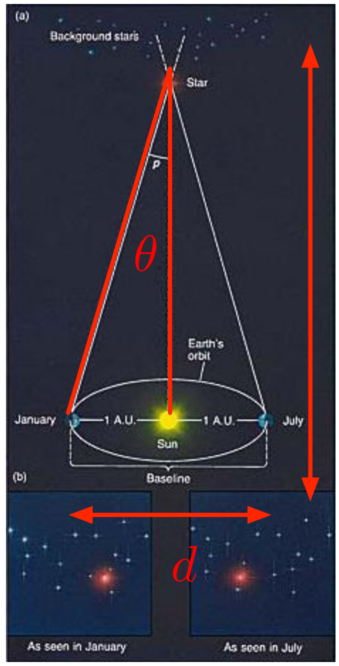
  This can be compared to the currently accepted value

$$1\,\mathrm{AU} = 23,455 \, \text{in units of earth radii}$$

B. Stellar Parallax

In much the same way the paralax method can be used to determine the distance to the nearest stars. This was first done by Bessel ~ 1835 and independently by Henderson. By this point it was noticed that certain stars are not fixed but move (by small amounts) in relation to some stars which seem fixed. This is known as the proper motion of the stars, and is of order arcseconds/year for the largest cases. Bessel reasoned that such stars are probably the closest, using a telescope that he designed himself he was able to use parallax to measure the distance to 61 Cygni.

The setup is like this.



$$L = \frac{1AU}{\theta}$$

Theta =1 arc sec then L=1 parsec

Bessel measured the angle to 61 Cygni to be 0.3 arcsec

Now d (the baseline of the triangle) is twice the radius of the orbit of the earth $2AU$. Incidentally "one parsec" corresponds to one arcsecond of angle with a baseline of $1AU$,

Then, Bessel measured the angle to 61 Cygni with sufficient precision throughout the year, measuring an angle of $\theta = 0.3\,$arcseconds. Then he determined the distance to the nearest star. Using

$$\theta = \frac{d/2}{L} \tag{1.2}$$

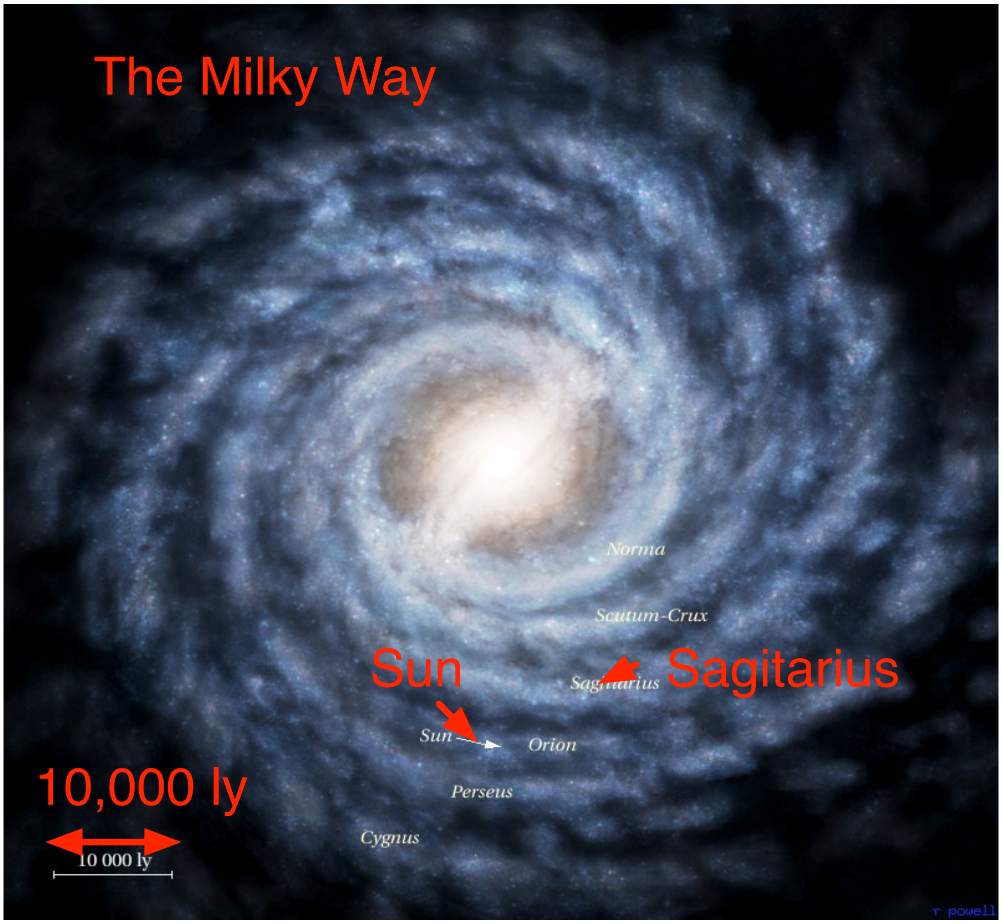
he measured

$$L \simeq \frac{1AU}{0.3arcsec} \simeq 3.4\,\text{parsec} \simeq 11\,\text{lightyears} \tag{1.3}$$

Incidentally "one parsec" corresponds to one arcsecond of angle θ with a baseline of $d/2 = 1\,$AU.

Only a few stars can be measured in this way. The situation improved with the launching of the European Space Agency satellite Hipparcos, which used the method of parallax to measure up to distances of 300 light years. The Gaia mission will measure angles with an accuracy of 10 microarcseconds and determine the distances of up to 10000 light years.

Just to put these in perspecive lets put back up the Milky way map.



C. In Class Problem

See here. Solution: In twenty years the star moves 150 arcsecs. The star is 6 light-years away so the this distance is

$$\Delta x = 0.00436\,\mathrm{c} - \mathrm{year}$$

The time is $\Delta t = 20\,\mathrm{years}$. So

$$v = \frac{\Delta x}{\Delta t} = 65\,\mathrm{km/s}$$

II. THE SPEED OF LIGHT

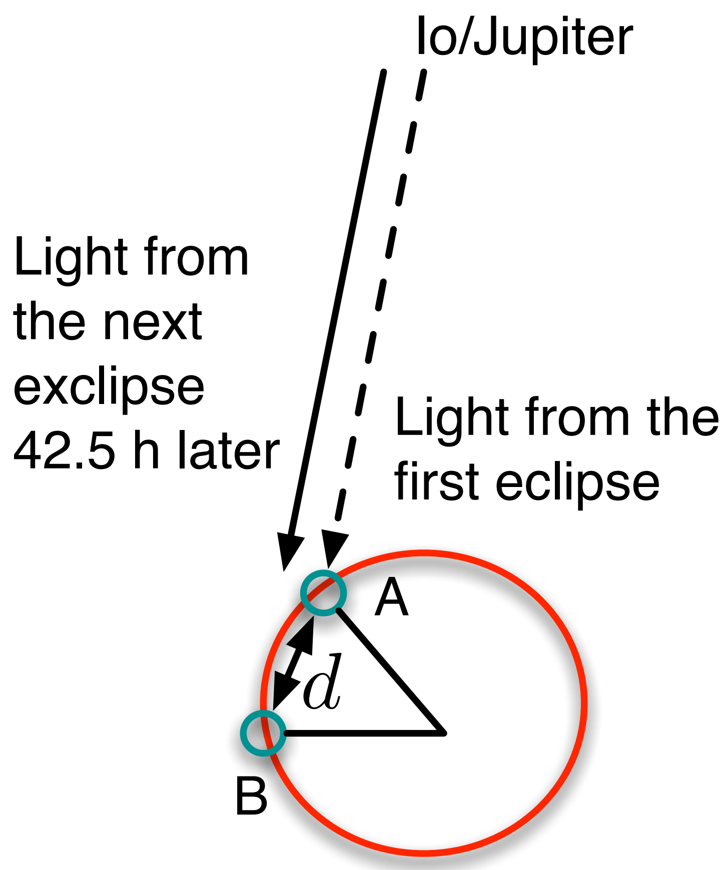
Shortly after the distances of our solar system were known. It was possible to measure the speed of light using astronomical data.

A. Ole Roemer measures the speed 1676

- Recall that Gallileo in the Starry Mesenger saw the orbits of Jupiters moons. Indeed every 42.5 hours the moon Io is ecclipsed by Jupter (i.e. Io travels behind this large planet).

- When the earth is moving away from Jupiter, the *apparant* period is slightly longer than $42.5\,h$. (The apparant period is the time between the observations of the light from of Io's Eclipses.) This ccan be seen from the figure below:

  – Suppose that the light from Io is seen at earth at point A (See figure below). Then 42.5 hourse later the light from the next eclipse is at the same place. However, the in the intervening time, the earth has moved to B. Thus, it takes just a little longer before the second eclipse is seen on earth. This extra bit is the time it takes light to move from A to B.



- A similar arguement shows that when the earth is approaching Jupiter, the apparant period of Io's Eclipses is slightly shorter than 42.5 h.

- Lets estimate how big this effect is. In a time of $42.5h$ the earth has moved a distance

$$d = v_{\rm earth} \times 42.5\,{\rm h} \sim 4.6 \times 10^6 {\rm km}$$

where we have taken the velocity of the earth to be $v_{\rm earth} \sim 30\,{\rm km}/s$. The extra time per period is

$$\Delta t \text{ per period} \sim d/c \sim 0.25\,\text{minutes}$$

- Thus, as the earth moves from top of the circle to the bottom of the circle over the course of a half a year, It will experiences about 90 such eclipses and so we expect a time difference of order

$$\Delta t_{\text{total}} \sim 22\,\text{minutes}$$

Summary. Roemer used the ~ 10 minute time shifts of the observed eclipses during different times of the year to measure the speed of light. He found

$$c \simeq (2.2 \pm 1) \times 10^8\,\text{m/s}$$

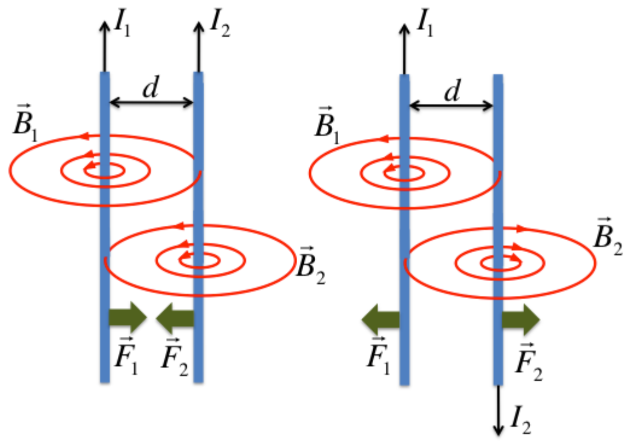
This result was based on observation of Io for more than a decade.

III. LIGHT AND ELECTRICITY AND MAGNETISM (HOW RELATIVITY WAS DISCOVERED)

At the end of the 19th century, the rules of electromagnetism were worked out by a combination of experimental and theoretical work. Important figures are James Clerk Maxwell (1831-1879), and Michael Faraday (1791-1867). For this class it is important to know a couple of things:

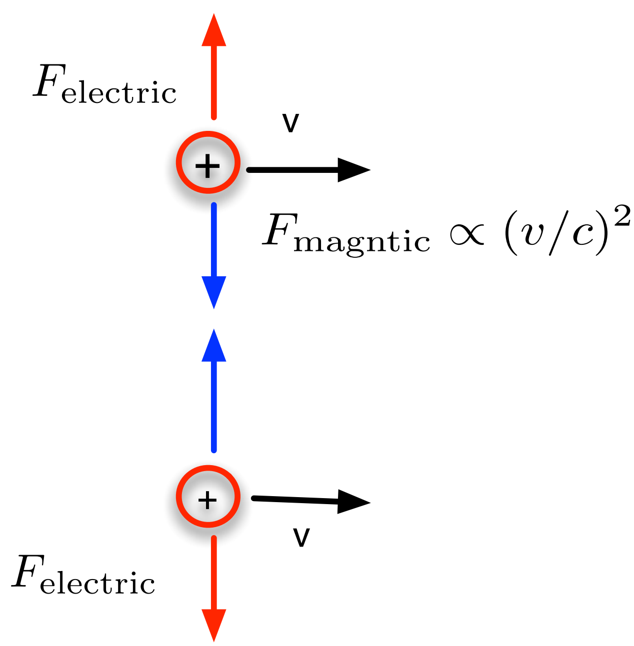
1. Light is a disturbance of electricity and magnetism in much the way sound is a disturbance of air.
2. There is an electric force caused by two charges with the same sign that repel. This is shown below

3. When two wires carry a current in the same direction they attract each other due to the magnetic field that one the currents create. When two wire carry a current in the opposite direction they repel each other



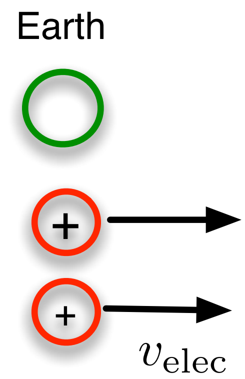
For wires attracting see this You Tube Video

4. Similarly two charges that are moving together with speed v experience both a electric and a magnetic force

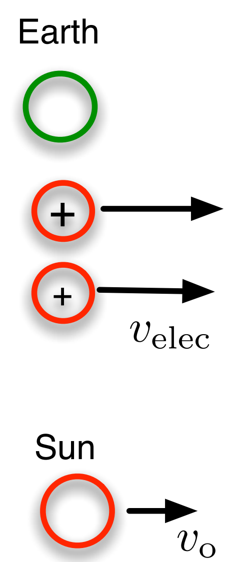


- The magnetic force is typically much smaller than the electric force and is proportional to $(v/c)^2$
- A typical electron speed is $\sim 1/100\,c$. Which is much greater than our speeds.

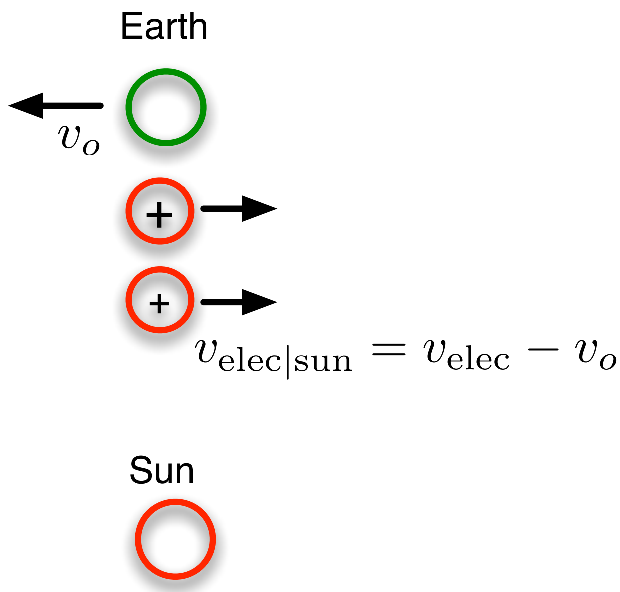
Although the magnetic force is small, it still begs the question *what do you mean by velocity of the electron*. This is the picture from the earth.



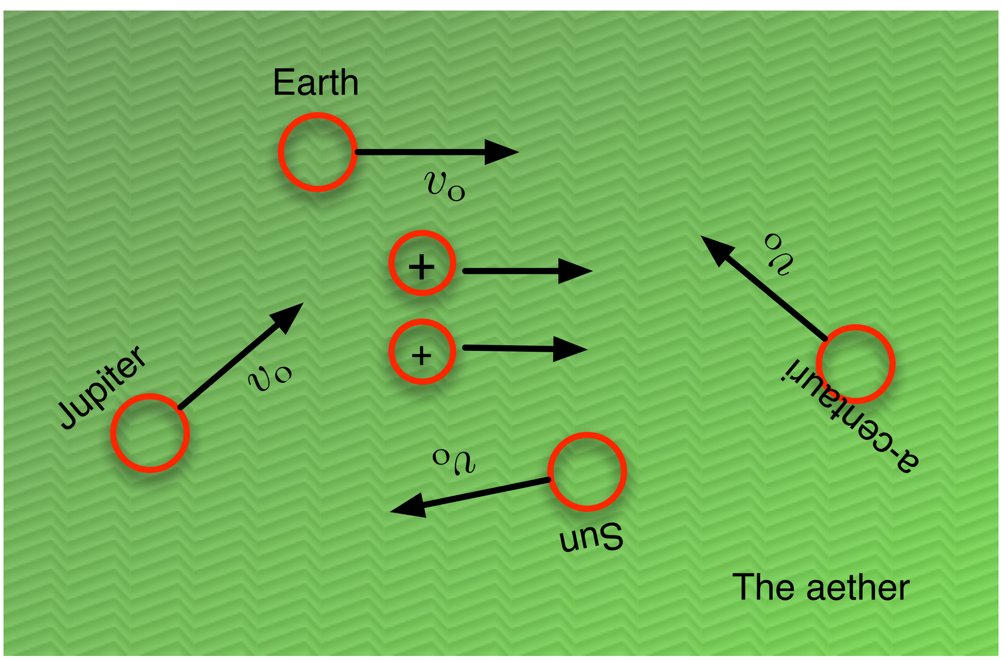
But the sun is nearby and moving



So perhaps we should measure the electron speed relative to the sun $v_{\text{elec|sun}}$. This speed is the difference between the electron seed relative to the earth v_{elec} minus the speed of the earth relative to the sun $v_{\text{elec}} - v_o$.



But then, the sun is moving with respect to $\alpha -$ centauri and Mars and Jupiter etc. In air, we would always measure speeds relative to air. Thus, since light is a wave-like disturbance of electricity and magnetism much in the way that sound is a wave-like disturbance of air, it seemed natural to postulate a light medium, known as "aether", and then the relevant velocities would be measured relative to the aether.



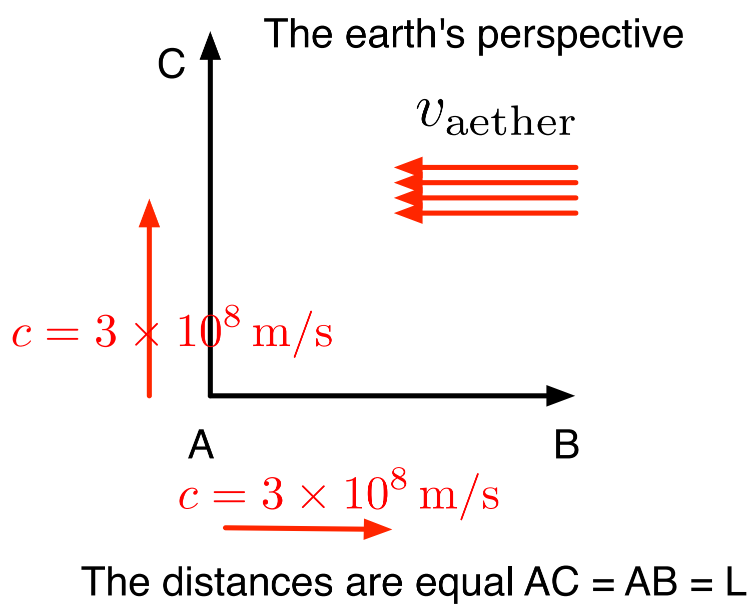
- It important to realize that wether I choose to measure the electron velocity relative to earth or sun is, for many practical purpose, irrelevant, since the magnetic force is small, and the % difference in the electron speed as measured by eath and sun is also small.
- Nevertheless, from a theoretical perspective (circa 1900) it was important to measure the motion of the earth relative to the aether. We will discuss an ingenious measure-

ment which showed that there is no motion of the earth relative to the aether at any time of the year. And, after Einstein explained it all and a few years passed while the world caught up theoretically and experimentally, the aether theory was abandoned.

- **Conclusion of this section.** Abandoning the Aether theory leaves us with a question. Every observer on different planets measures different forces. Different forces means different accelerations. Einstein's answer was that Newton's Law remain intact for each observer separately. Then, each observer then measures with his own clock and ruler stick and observes (essentially) Newtons Laws. However, the the times, distances, and accelerations as measured by the different observers are not the same. However, Einstein explained how using only the measurements of any one of observers, the measureemnts of all other obsevers could be determined. This is why its called relativity.

IV. THE MICHELSON MORLEY EXPERIMENT

The basic idea of the michelson morley experiment is simple and was worked out in homework. It is shown below.

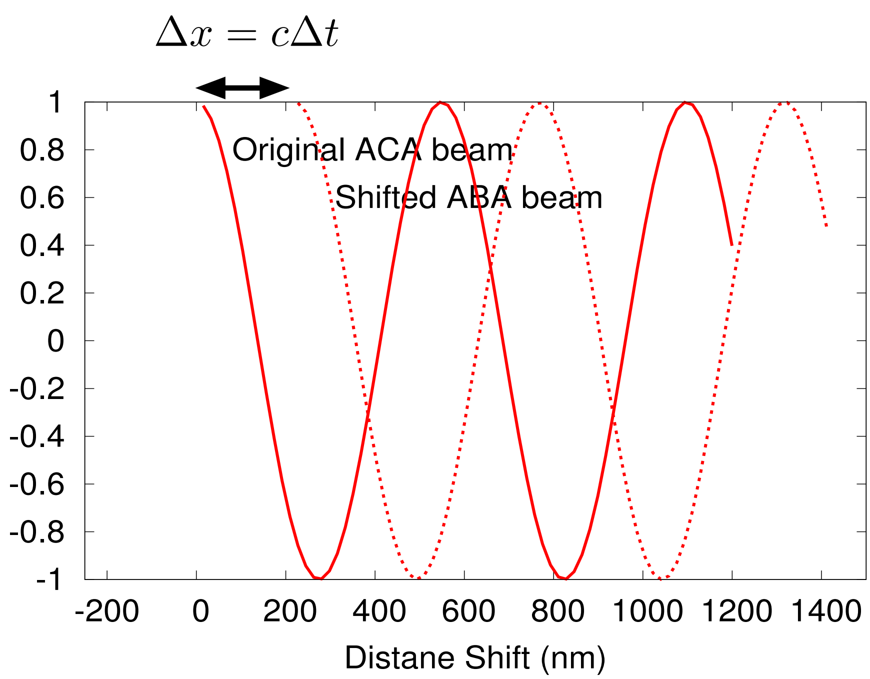


The classical (wrong) explanation is that the light moves with speed $c - v$ with it swims against the river (from A to B) and with a speed $c + v$ when it moves with the river (from B to C). This can be compared to the speed from A to C which is unaffected by aether. The time difference $\Delta t_{ABA} - \Delta t_{ACA}$ is measured by measuring the interference of light.

For easy of numbers we take $v_{\text{aether}} = v_{\text{earth}} = 3 \times 10^4\text{m/s}$ while $c = 3 \times 10^8 m/s$, so $(v/c)^2 \approx 10^{-8}$. And take $L = 10\,\text{m}$. ($L = 10\,m$ was achieved by bouncing the light back and forth several times.

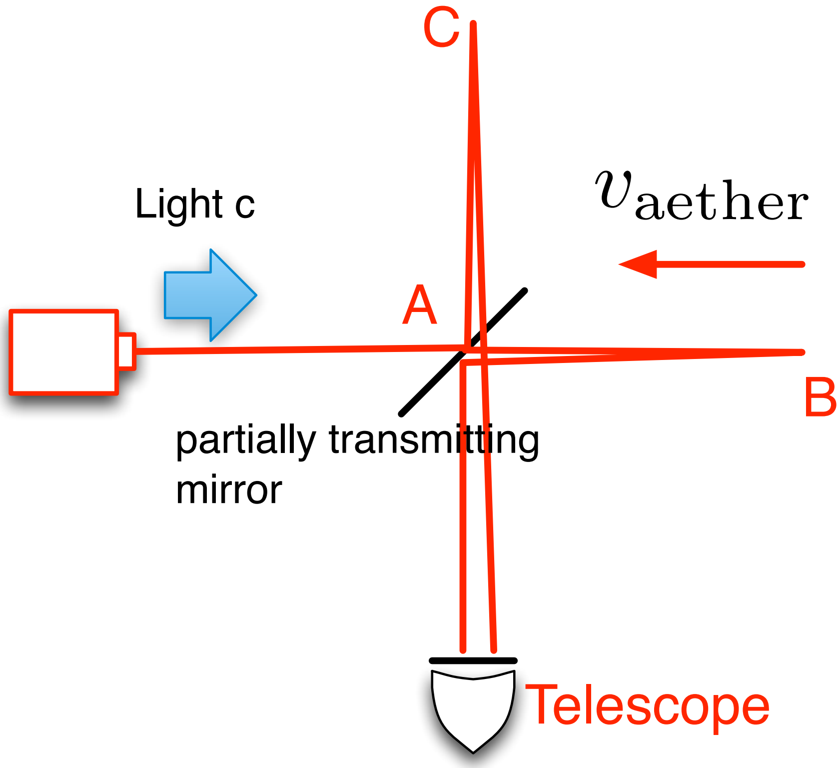
- After doing the homework, you realize that it takes longer to go from A to B and back, then it does to go from A to C and back.

- Therefore two light beams which the beams were started at A, and then went along the paths ACA and ABA, will be shifted in space relative to each other. When ACA returns to A $x = 0$, ABA light beam will be still need to travel a certain $\Delta x = c\Delta t$ before the head of the light reaches zero.



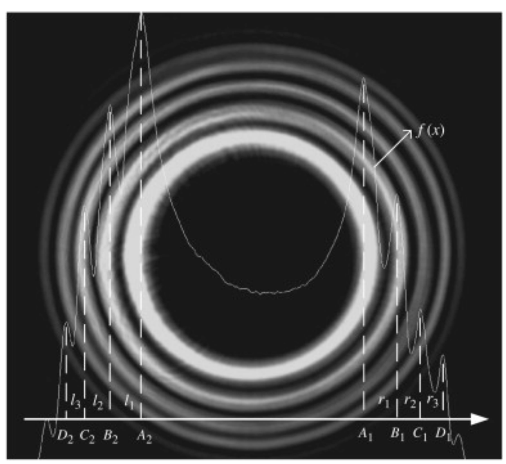
- When these two light beams are added together they interfere, and the interference depends on the time difference between the two beams. By measuring the pattern of intereference you can determine the time difference

- The experiment looks like this

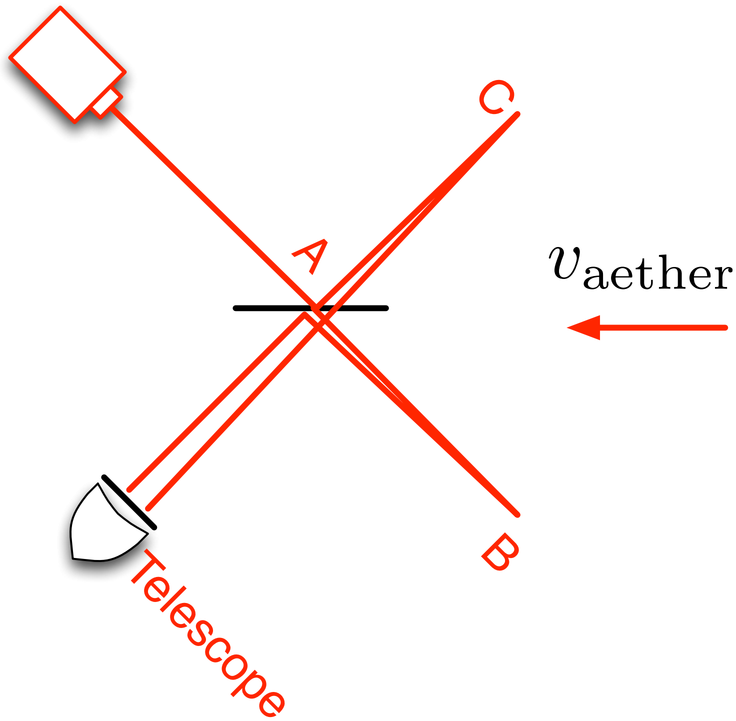


The two beams pass through the (two-way) mirror travel ABA and ACA and recombine in the telescope making an interference pattern

- In the telescope, you see an interference between the two beams which looks something like this



- In general you will always see such an interefrence pattern even without the aether, because the lengths ABA and ACA are not exactly equal. So, how do you tell the effect due to the "aether"? The way you do this is by rotating the apparatus by 45^o – In this case the "aether" effect goes away and the interference pattern changes. The change in the intereference pattern would then be unambiguously due to the light's longer time to traverse ABA.



- **Why its hard.** Well the time difference is rather small. Using the estimates we had we expect the correction to the length Δx to go like v^2/c^2

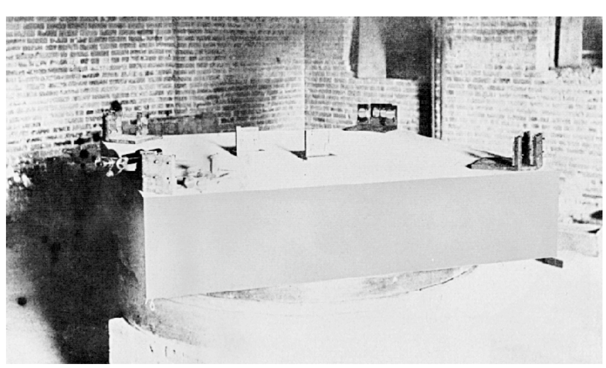
$$\frac{\Delta x}{L} \sim \frac{v^2}{c^2} \tag{4.1}$$

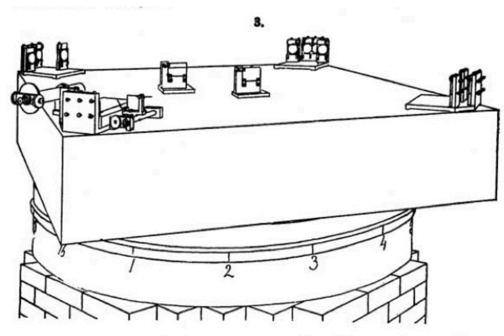
Or

$$\Delta x \sim L\frac{v^2}{c^2} \sim 10\,\mathrm{m} \times 10^{-8} \sim 100\,\mathrm{nm} \tag{4.2}$$

So we need have no vibrations at the level of 100 nm . Further, we need to be able to rotate the apparatus without changing any of the lengths at the level of 100 nm

- So the whole apparatus was rather involved and is shown below:





- A Lead slab was floated in a trough of mercury. When you pushed on the table (kind of like a water bed) it damps out the vibrations. Then you could rotate the lead slab by providing a slow push to set it in a slow steady rotation

- **Michelson saw nothing!** As he rotated the table the interference pattern did not change. It looked exactly the same. It seems that the earth is not moving with respect to the aether. The experiemnt was repeated at three month intervals and no changes of the fringes were ever seen.

V. EINSTEIN AND RELATIVITY

I The speed of light is constant for all observers.

II The laws of physics are the same for all observers and are the familiar Newton's Laws. But, each observer measures with his own coordinates

- Example: two observers measure coordinates Δt and Δx, and $\Delta t'$ and $\Delta x'$, respectively. Both observers measure speeds to be

$$v = \frac{\Delta x}{\Delta t} \qquad \text{and} \qquad v' = \frac{\Delta x'}{\Delta t'} \tag{5.1}$$

respectively. But, the relation between Δt, Δx, and v and $\Delta t'$,$\Delta x'$ and v' is not simple (but can be determined using (I))

A. Time Dilation & Length Contraction

To see how the times and distances are related we have some separate notes. See notes