Electrostatics in Media:

$$\nabla \cdot E = \rho$$

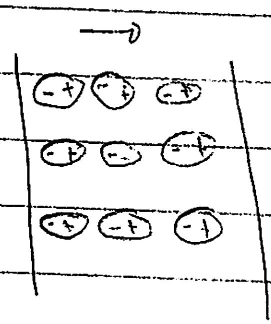
$$\nabla \times B = \frac{j}{c} + \frac{1}{c}\partial_t E \qquad \partial_t \rho + \nabla \cdot \vec{j} = 0$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{1}{c}\partial_t \vec{B}$$

Need to specify the currents in the medium in order to solve. Symmetry is key

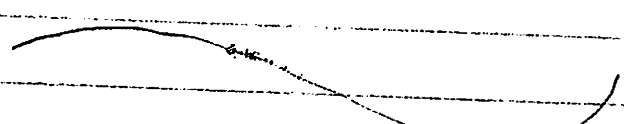
Basic Picture of Insulating Material



Electric field polarizes the insulating material

Key points:

① The wavelength of the external field is vastly longer than the microscopic scales



$\lambda_{light} \sim 500\ nm$

Where as size of atom $a_0 \sim \frac{1}{2} \mathring{A}$

$$\frac{\lambda_{light}}{a_0} \sim 10^4$$

(2) Fields are often weak compared to micro-fields

$$E_{ext} \sim 1\frac{kV}{cm} \sim 10^5 \frac{V}{m} \quad \text{(Largest man-made field)}$$

$$E_{atom} \sim \frac{13.6\ \cancel{e}V}{\cancel{e}\ a_0} \sim 13 \times 10^{10} \frac{V}{m}$$

Now we will need these points and Symmetry

Rotations:

rotated coords　　　rotation matrix

$$x \longrightarrow \underset{\sim}{x}^i = R^i{}_j \, x^j$$

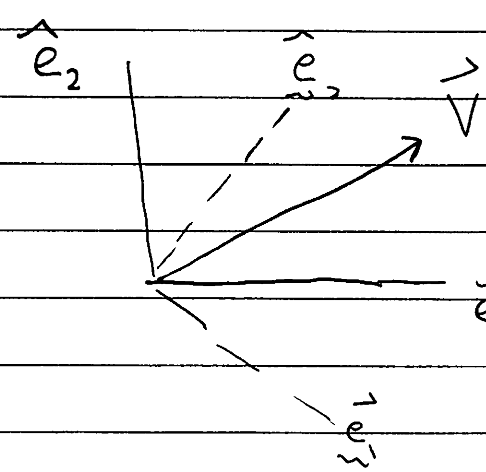
$$\begin{pmatrix} \underset{\sim}{x}^1 \\ \underset{\sim}{x}^2 \\ \underset{\sim}{x}^3 \end{pmatrix} = \begin{pmatrix} & & \\ & R & \\ & & \end{pmatrix} \begin{pmatrix} x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

- Repeated indices sum $\quad a_i b^i = a_1 b^1 + a_2 b^2 + a_3 b^3$
- Think of upper indices (covariant indices) as row indices, lower indices (contravariant indices) as column indices. (But x^i is a column vector, x_i is a row vector)

Scalars $\quad S \longrightarrow \underset{\sim}{S} = S \quad$ (invariant under rotations

$$Q \longrightarrow \underset{\sim}{Q} = Q \qquad \text{charge}$$
$$m \longrightarrow \underset{\sim}{m} = m \qquad \text{mass}$$

Vectors



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Take passive point of view

$$\vec{V} \longrightarrow \underset{\sim}{\vec{V}} = \vec{V}$$

Vectors are physical objects unchanged by coordinate rotations

$$\vec{V} = V^i \vec{e}_i = \underset{\sim}{V}^i \underset{\sim}{\vec{e}}_i$$

But the components and the basis vectors are changed by rotations (see figure on prev page)

$$\underset{\sim}{V}^i = R^i{}_j V^j$$

$$\underset{\sim}{\hat{e}}_j = \hat{e}_i (R^{-1})^i{}_j$$

i.e

$$(\hat{e}_{\sim 1}\ \hat{e}_{\sim 2}\ \hat{e}_{\sim 3}) = (\hat{e}_1\ \hat{e}_2\ \hat{e}_3)\begin{pmatrix} R^{-1} \end{pmatrix}$$

Now

$$\vec{V}_{\sim} = (\hat{e}_{\sim 1}\ \hat{e}_{\sim 2}\ \hat{e}_{\sim 3})\begin{pmatrix} v_{\sim 1} \\ v_{\sim 2} \\ v_{\sim 3} \end{pmatrix} = (e_1\ e_2\ e_3)\overbrace{\begin{pmatrix} R^{-1} \end{pmatrix}\begin{pmatrix} R \end{pmatrix}}^{\mathbb{1}}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$= (e_1\ e_2\ e_3)\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \vec{V}$$

For Rotation group $R^{-1} = R^T$ so:

$$(e_{\sim 1}\ e_{\sim 2}\ e_{\sim 3}) = (e_1\ e_2\ e_3)\begin{pmatrix} R^T \end{pmatrix}$$

Take Transpose

$$\underbrace{\begin{pmatrix} e_{\sim 1} \\ e_{\sim 2} \\ e_{\sim 3} \end{pmatrix} = \begin{pmatrix} R \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}}$$

This is the same transformation rule as for upstairs indices (covariant components). Thus for rotations there is no difference between up and down indices (covariant and contravariant indices.

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Scalar Product:

$$S = \vec{a} \cdot \vec{b} = a_i b^i$$ Takes two vectors and gives a scalar.

i.e.

$$S \longrightarrow \underset{\sim}{S} = \vec{\underset{\sim}{a}} \cdot \vec{\underset{\sim}{b}} = \vec{a} \cdot \vec{b} = S$$

Ex:

$$\nabla \cdot E = \frac{\partial}{\partial x^i} E^i \qquad \vec{\nabla} = \hat{e}^i \frac{\partial}{\partial x^i}$$

$$= \partial_i E^i \qquad \left(\partial_i \equiv \frac{\partial}{\partial x^i} \right)$$

Vector Product:

Takes two vectors and gives another vector

$$(\vec{a} \times \vec{b})^i = \varepsilon^{ijk} a_j b_k$$

Ex $$(\nabla \times \underset{\sim}{E})^i = \varepsilon^{ijk} \partial_j E_k$$

where $\frac{\partial}{\partial x^j} = \partial_j$

Tensors

$$\overleftrightarrow{T} = T^{ij}\, \vec{e}_i\, \vec{e}_j$$

They are invariant under rotations but the components change as:

$$\underset{\sim}{T}^{ij} = R^{i}{}_{i'}\, R^{j}{}_{j'}\, T^{i'j'}$$

Parity

$$t \longrightarrow \underset{\sim}{t} = t \qquad \text{(P-even)}$$

$$x \longrightarrow \underset{\sim}{x} = -x \qquad \text{(P-odd)}$$

$$\vec{p} \longrightarrow \underset{\sim}{\vec{p}} = -\vec{p} \qquad \text{(P-odd)}$$

Prf:

$$\underset{\sim}{\vec{p}} = m\frac{d\underset{\sim}{x}}{d\underset{\sim}{t}} = -m\frac{dx}{dt} = -p$$

$$F \longrightarrow -\vec{F} \qquad \text{(P-odd)}$$

$$\vec{L} \longrightarrow \underset{\sim}{\vec{L}} = \vec{L} \qquad \text{(P-even} \equiv \text{psuedo vector)}$$

(Use $L = \vec{r} \times \vec{p}$)

$$j \longrightarrow \vec{j} = -j \qquad \text{(P-odd)}$$

$$E \longrightarrow -\vec{E} \qquad \text{(P-odd)}$$

$$B \longrightarrow B \qquad \text{(P-even} \equiv \text{psuedo vector)}$$

Use $\underbrace{F_E}_{\text{odd}} = \underbrace{q}_{\text{even}}\, \underbrace{E}_{\text{odd}}$ and $\underbrace{F_B}_{\text{odd}} = \underbrace{q}_{\text{even}}\, \underbrace{\vec{v}}_{\text{odd}} \times \underbrace{\vec{B}}_{\text{even}}$

Time Reversal

$$t \longrightarrow \underset{\sim}{t} = -t \qquad \text{(T-odd)}$$

$$x \longrightarrow \underset{\sim}{x} = x \qquad \text{(T-even)}$$

$$\vec{p} \longrightarrow \underset{\sim}{p} = -p \qquad \text{(T-odd)}$$

Prf:

$$\underset{\sim}{p} = m \frac{d\underset{\sim}{x}}{d\underset{\sim}{t}} = -m \frac{dx}{dt} = -p$$

$$F \longrightarrow \underset{\sim}{F} = F \qquad \text{(T-even)}$$

(Use $\vec{F} = dp/dt$)

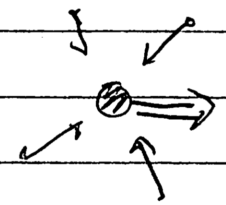
$$E \longrightarrow \underset{\sim}{E} = E \qquad \text{(T-even)}$$

(Use $F = qE$)

$$B \longrightarrow \underset{\sim}{B} = -B \qquad \text{(T-odd)}$$

(Use $\underbrace{F}_{\text{even}} = q \underbrace{\vec{v}}_{\text{odd}} \times \underbrace{\vec{B}}_{\text{odd}}$)

Dissipation



Put a heavy particle interacting with light particles in a computer simulation

$$m \frac{d^2x}{dt^2} = \sum_i F_i \qquad F_i = \frac{q\, q_i}{4\pi |\vec{r} - r_i|}$$

The heavy partice experiences an <u>effective</u> force

$$M \frac{d^2 x}{dt} = F_D$$

$$F_D = -\eta v$$

Now imagine running all particles in reverse $t \to \tilde{t} = -t$. The equations of motion are the same, and the heavy particle would still slow down. The effective force

$$\underset{\sim}{F_D} = -\underset{\sim}{\eta}\, \underset{\sim}{\vec{V}}$$

~~odd under t-reverse~~

even under time reversal

odd under t-reverse $\quad \underset{\sim}{\eta} = -\eta$

Summary

$$\eta \rightarrow \eta_{tr} = -\eta \quad \text{under time reversal}$$

Dissipative coëfficients are T-odd

Fourier Transforms:

$$\phi(t,\vec{r}) = \int \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i\omega t + i\vec{k}\cdot\vec{r}} \phi(\omega,k) \equiv (\phi(\omega,k))_{FT}$$

$$\phi(\omega,k) = \int dt\, d^3\vec{r}\; e^{i\omega t - i\vec{k}\cdot\vec{r}} \phi(t,r) \equiv (\phi(t,r))$$

Slow & Long wavelength means k, ω small

$$\phi(t,r) \longleftrightarrow \phi(\omega,k)$$

$$\partial_t \phi(t,r) \longleftrightarrow -i\omega\,\phi(\omega,k) = (\partial_t \phi)_{FT}$$

$$\cancel{\partial_i \phi(t,r) \longleftrightarrow i k_i\,\phi(\omega,k)}$$

Prf

$$\partial_t \phi(t,r) = \int \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} \partial_t (e^{-i\omega t + i\vec{k}\cdot\vec{r}})\, \phi(\omega,k)$$

$$= \int \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i\omega t + ik\cdot r} \, -i\omega\,\phi(\omega,k)$$

Thus!

$$\vec{\nabla}\cdot\vec{E} = \rho(\vec{x})$$

$$(\nabla\cdot E)_{FT} = (\rho(x))_{FT}$$

$$(\partial_i E^i)_{FT} = (\rho(x))_{FT}$$

$$i k_i E^i(\omega,k) = \rho(\omega,k)$$