Magnetic Matter

determined with electrostatics

$$\bullet \quad \nabla \times \vec{B} = \frac{\vec{j}}{c} + \frac{1}{c}\frac{\partial \vec{E}^{(0)}}{\partial t}$$

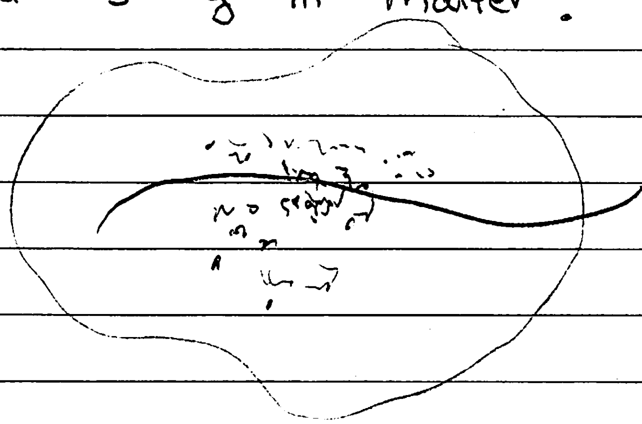
$$\nabla \cdot B = 0$$

For the moment set $E^{(0)} = 0$ (no net charge)

$$\nabla \times \vec{B} = \frac{j}{c} = \frac{j_{mat}}{c} + \frac{j_{free}}{c}$$

$$\nabla \cdot B = 0$$

What is $\vec{j}$ in matter?



long wavelength magnetic field

$$L \gg l_{micro}$$

Basic Principles

- Symmetry and long distance expansion
- very often weak

Consider a magnetic field slowly varying in time and space. Write $\vec{j}$ as some general fcn of $\vec{B}$ and its derivatives

$$\vec{j} = \sigma_B \vec{B} + \chi_1 \partial_t \vec{B} + \chi_m (\nabla \times \vec{B}) + \dots$$

+ higher derivatives
as before these are suppressed by powers of $\frac{l_{micro}}{L}$

Now recognize that symmetry forces many of these to be zero. Take the first term.

$$\underbrace{\vec{j}}_{\substack{\text{P-odd} \\ \text{T-odd}}} = \underbrace{\sigma_B}_{\substack{\text{P-odd} \\ \text{T-even}}} \underbrace{\vec{B}}_{\substack{\text{P-even} \\ \text{T-odd}}}$$

The coefficients σ_B reflect the microscopic interactions. If the microscopic forces are invariant under parity.

$$\vec{x} \longrightarrow -\vec{x}$$

Then σ_B will be zero $\sigma_B = 0$

(But in other cases such as high temperatue electro weak plasma, which violates parity, this coefficient will not be zero, It will not be dissipative)

Unless otherwise specified we will assume parity invariance of microscopic forces and set $\sigma_B = 0$

$$\vec{j} = \cancel{\sigma_B \vec{B}} + \underbrace{\cancel{x_1 \partial_t \vec{B}}}_{\text{P-odd}} + \chi_m^B c(\nabla \times \vec{B}) + \ldots$$

So $\boxed{\vec{j}_c = \chi_m^B \vec{\nabla} \times \vec{B}} \implies \boxed{j_c = \nabla \times \underbrace{(\chi_m^B \vec{B})}_{\equiv \vec{M} \,=\, \text{magnetization}}}$

Now our equations of magneto-statics becomes

$$\nabla \times B = j_{mat} + j^{free}_c \qquad \nabla \cdot B = 0$$

$$\nabla \times \vec{B} = \nabla \times \vec{m} + j^{free}_c$$

And

$$\nabla \times \underbrace{(\vec{B} - \vec{m})}_{\equiv \vec{H}} = j^{free}_c$$

Leading to our eqs - Magneto Statics in matter

$$\boxed{\begin{aligned} \nabla \times \vec{H} &= \frac{\vec{j}_{fr}}{c} \\ \nabla \cdot \vec{B} &= 0 \end{aligned}}$$

$$H[\vec{B}] = \vec{B} - \vec{M}(\vec{B})$$

$$\boxed{\vec{\nabla} \times \vec{m} = j_{mat}}$$

For a linear relation $\vec{M} = \chi^m_B \vec{B}$ and

$$\vec{H} = (1 - \chi^B_m)\vec{B}$$

$$\frac{1}{(1-\chi^B_m)} \vec{H} = \vec{B}$$

$$\boxed{\mu \vec{H} = \vec{B}}$$

$$\boxed{\mu = \frac{1}{1-\chi^B_m} = \text{permeability}}$$

and we get a set of equations; with $\vec{B} = \nabla \times \vec{A}$, for constant μ

$$\nabla \times \left(\frac{\nabla \times A}{\mu}\right) = j_{free}/c$$

$$\boxed{-\nabla^2 A = \mu \, j_{free}/c}$$

This assumes a linear relation

Non-linear Media - Ferromagnets and such

Non-linearly we still only keep one spatial derivative:

terms like this are even under parity

$$\frac{\vec{j}}{c} = \chi\, \partial_t \vec{B} (B^2)^n$$

terms like this can't satisfy $\nabla \cdot \vec{j} = 0$

$$+ \vec{\nabla} (B^2)^n + \underbrace{\nabla \times \vec{B}\, (B^2)^n}$$

P-odd & T-odd

terms like this are ok

So the general form is:

$$\frac{\vec{j}}{c} = \nabla \times \vec{M}[\vec{B}]$$

Often people will trade $\vec{B}$ for $\vec{H} = \vec{B} - \vec{m}$ and will write $\vec{M}[\vec{H}]$. This is more or less conventional. Usually the non-linear constituent relation is written

$$\vec{B} = \vec{F}(\vec{H})$$

Some non-linear function

or

$$\vec{B} = \mu(H)\, \vec{H}$$

assumes that $\vec{B}$ points in same direction as $\vec{H}$

• Then the dimensions

$$[B] \sim \frac{q}{m^3}$$

$$[\vec{j}] \sim \frac{q}{m^2 s}$$

So

$$[c\chi^B_m] = m/s \quad \text{so expect} \quad c\chi^B_m \sim \underset{\text{naive dimension}}{V_{micro}}$$

In fact we can anticipate that since the micro forces are $F \sim q\left(\frac{\vec{v}}{c}\right) \times \vec{B}$, the current which is generated is smaller than naive dimensions by a factor of V_{micro}/c.

small

i.e.

$$c\chi_m \sim \underset{\text{dimensions}}{V_{micro}} \left(\frac{V_{micro}}{c}\right)$$

knowledge that magnetic forces are of order (V/c) relative to electric ones.

So $$\boxed{\chi_m \sim \left(\frac{V_{micro}}{c}\right)^2}$$

Campe to the electric Polarizability

$$\chi_e \sim 1$$ (see previous lecture)

Indeed this is what one normally finds

$$\chi_e \sim 1$$

$$\boxed{\chi_m \sim \left(\frac{v}{c}\right)^2 \sim \left(\frac{1}{137}\right)^2 \sim 10^{-5}}$$

For Bohr model:

$\beta = \alpha$ ← fine structure constant

$$\frac{v}{c} = \frac{e^2}{4\pi\hbar c} = \frac{1}{137}$$

So: $\chi_m \sim 10^{-5}$.

This is usually what is found. For Ferromagnetic materials the magnetization can be much larger

$$\vec{B} = \mu(H)\,\vec{H}$$

← can be like 10^3.

Why? Because Ferromagnetic Substances involve <u>all</u> the atoms working collectively.