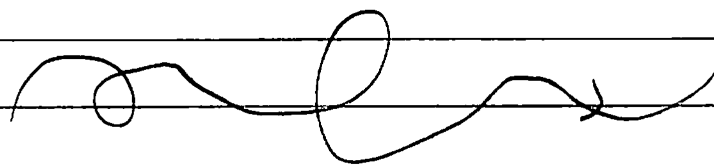
Last Time

Started From wave equation

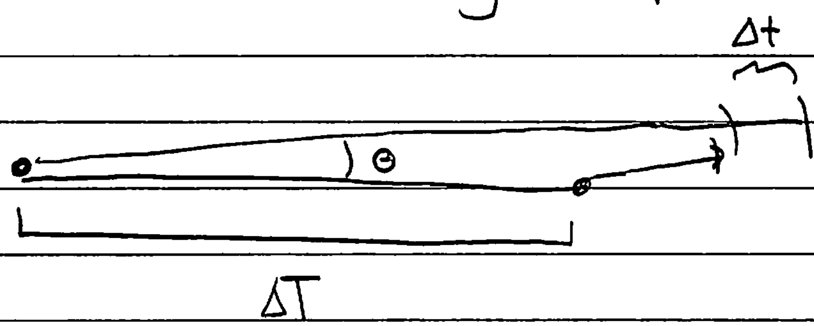


Derived the radiation for arbitrary relativistic motion

$$\vec{A}_{rad} = \frac{1}{4\pi r} \frac{q\,\vec{V}(T)/c}{(1 - n\cdot\beta)}$$

$$A_{rad} = \frac{q}{4\pi}\, \vec{V}(T)/c\, \frac{dT}{dt} \qquad T = t - \frac{r}{c} + \frac{\vec{n}\cdot r_0(T)}{c}$$

Again dT/dt is very important



if a waveform at infinity is observed to have a characteristic time scale Δt then it was formed over a time-scale $\Delta T = \frac{\Delta t}{(1 - n\cdot\beta)}$

Last Time Continued

Once we know A_{rad} we can find the electric field

$$\vec{E}_{rad} = \frac{e}{4\pi r} \frac{n \times (n - \vec{\beta}) \times \vec{a}(T)}{(1 - n \cdot \beta)^3} = n \times n \times \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

And we can find the radiated power

$$\frac{dW}{dt\,d\Omega} = \frac{dP(t)}{d\Omega} = c\,|E(t)|^2$$

Often care about the energy lost during a period of acceleration

$$\frac{dW}{dT\,d\Omega} = \frac{dP(T)}{d\Omega}$$

$$= c\,|E|^2 \frac{dt}{dT}$$

$$= c\,|E|^2 \,(1 - n \cdot \beta(T))$$

Problem - SHO

(1) A relativistic harmonic oscillator with charge q, moves on the z-axis

$$z = H \cos\omega_0 t$$

Show that the power emitted is

$$\frac{dW}{dT\,d\Omega} = \frac{dP(T)}{d\Omega} = \frac{e^2}{16\pi^2}\,\frac{c\beta^4}{H^2}\,\frac{\cos^2\omega_0 t\,\sin^2\theta}{(1+\beta\cos\theta\sin\omega_0 t)^5}$$

where $\beta \equiv \dfrac{\omega_0 H}{c}$, $\gamma \equiv \dfrac{1}{\sqrt{1-\beta^2}}$

(2) Most of the energy in the $\gamma \to \infty$ limit is radiated at particular points during its motion. Determine these points. The radiation is pulsed. Explain why.

(3) Show that for $\gamma \to \infty$ for $T = \dfrac{3\pi}{2\omega_0} + \Delta T$, small ΔT, have: is

$$\frac{dP(T)}{d\Omega} = \frac{2e^2}{\pi^2}\,\frac{c\beta^4\gamma^6}{H^4}\left[\frac{(\gamma\theta)^2(\gamma\omega_0\Delta T)^2}{(1+(\gamma\theta)^2+(\gamma\omega_0\Delta T)^2)^5}\right]$$

(4) Use the integral

$$\int_{-\infty}^{\infty}\frac{x^2\,dx}{(1+y^2+x^2)^5} = \frac{5\pi}{128}\,\frac{1}{(1+y^2)^{7/2}}$$

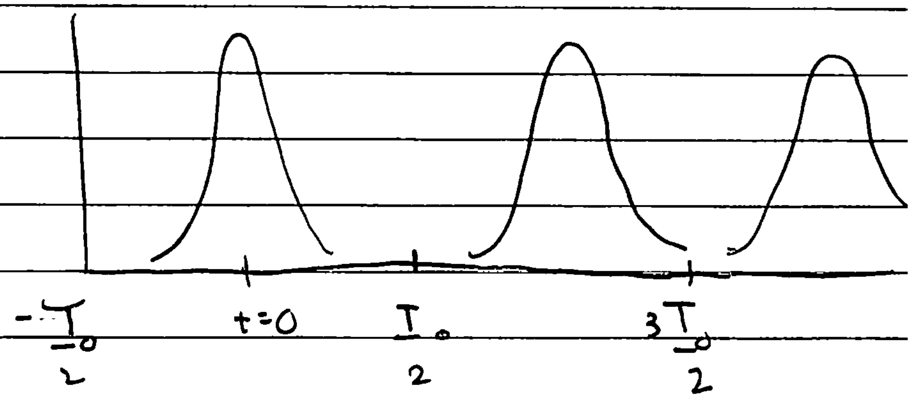
Problem - SHO continued

to show that the time averaged power is

$$\frac{d\overline{P}}{d\Omega} = \frac{e^2}{128\pi^2} \frac{c\beta^4}{H^2} \gamma^5 \left[\frac{5(\gamma\theta)^2}{(1+(\gamma\theta)^2)^{7/2}} \right]$$

A repeating sequence of pulses:

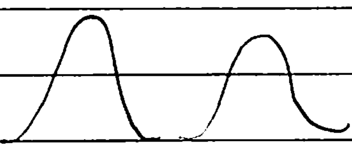
$E_1(t) \equiv$ The electric field in a single period, zero elsewhere.



$E_2(\omega)$ = Fourier transform $E_0(t)$

Problem 1

Suppose that the waveform repeats once. Show that the fourier transform is



$$E_2(\omega) \equiv E_1(\omega)\,(1 + e^{i\omega T_0})$$

And the power spectrum is

$$|E_2(\omega)|^2 = |E_1(\omega)|^2\,(2 + 2\cos(\omega T_0))$$

Problem 2

Show that the Electric field after a sum of n-pulses is A repeating Sequence of pulses pg. 2

$$E_N(\omega) = E_1(\omega)\, e^{-i\omega(n-1)T_0/2}\ \frac{\sin(\omega n T_0/2)}{\sin \omega T_0/2}$$

So that the power is

$$|E_N(\omega)|^2 = |E_1(\omega)|^2 \left(\frac{\sin(\omega n T_0/2)}{\sin \omega T_0/2}\right)^2$$

Problem 3

Show that the electric field appraches

$$E(\omega) \xrightarrow[N\to\infty]{} \sum_m E_1(\omega_m)\ \frac{2\pi}{T_0}\ \delta(\omega-\omega_m)$$

and

$$\frac{|E(\omega)|^2}{nT_0} \xrightarrow[N\to\infty]{} \sum_m |E_1(\omega_n)|^2\ \frac{2\pi}{T_0^2}\ \delta(\omega-\omega_m)$$

Use

$$\int_{-\infty}^{\infty} \frac{\sin(x/2)}{x/2}\, dx = 2\pi$$

and

$$\int_{-\infty}^{\infty} \left(\frac{\sin(x/2)}{x/2}\right)^2 dx = 2\pi$$

Problem 4

Use the results of this problem to claim

$\Delta_T(t) \equiv \sum_n \delta(t - nT_0)$; the ints fourier transform is

$$\Delta_T(\omega) = \sum_n e^{i\omega n T_0} = \frac{2\pi}{T_0} \sum_m \delta(\omega - \omega_m) \qquad \omega_m = \frac{2\pi}{T_0}$$

And

$$\frac{dP_m}{d\Omega} = \frac{1}{(\text{Time})} \int_{\omega_m - \text{bit}}^{\omega_m + \text{bit}} 2\pi \frac{dW}{d\omega\, d\Omega} \frac{d\omega}{2\pi}$$

$$\frac{dP_m}{d\Omega} = \frac{c}{T_0^2} |E_1(\omega_m)|^2$$